MTH240-02 DIFFERENTIAL EQUATIONS & COMPUTER EXPERIMENT1(English)

Department: ID Number: Name:

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1. Find the solution of the given initial value problem

$$y' + 3y = te^{-3t}, \qquad y(1) = 0.$$

2. The equation that has been used to model population growth is the Gompertz equation

$$\frac{dy}{dt} = ry \ln\left(\frac{K}{y}\right),$$

where r and K are positive constants.
(a) Sketch the graph of dy/dt versus y, find the critical points, and determine whether each is asymptotically stable or unstable.
(b) For $0 \leq y \leq K$, determine where the graph of y versus t is concave up and where is concave down.

3. Determine whether the equation is exact. If it is exact, find the solution.

$$(ye^{xy}\cos 2x - 2e^{xy}\sin 2x + 2x) + (xe^{xy}\cos 2x - 3)y' = 0$$

4. Find an integrating factor and solve the given equation

$$1 + \left(\frac{x}{y} - \cos y\right) y' = 0.$$

5. Solve the initial value problem

$$4y'' - y = 0, \qquad y(0) = 2,\ y'(0) = \beta.$$

Then find β so that the solution approaches zero as $t \to \infty$.

6. Find the solution of the given initial value problem.

$$y'' + y = 0, \qquad y(\pi/3) = 2,\ y'(\pi/3) = -2.$$