

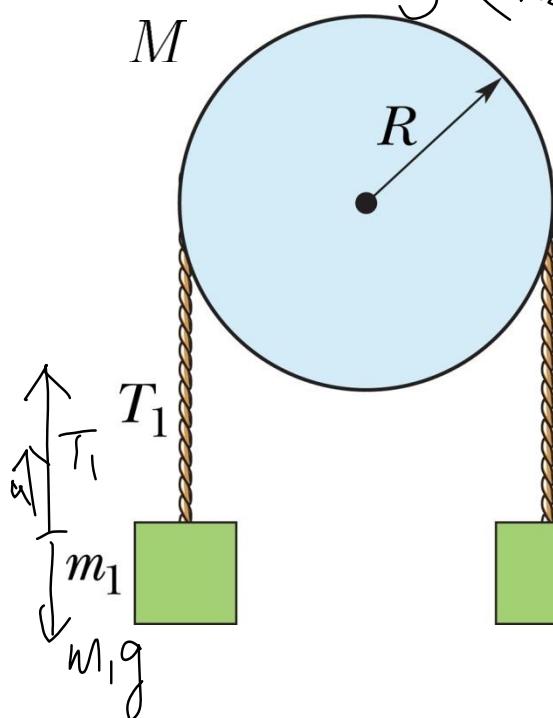
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- The images and the pictures in this lecture are provided by the CDs accompanied by the books
 1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
 2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

$$T_1 = m_1 g \left(1 + \frac{a}{g}\right) = m_1 g \left(1 + \frac{\frac{M}{2} - m_1}{\frac{M}{2} + m_1 + m_2}\right) = m_1 g \cdot \frac{\frac{M}{2} + 2m_2}{\frac{M}{2} + m_1 + m_2}$$

Prob. 2

$$T_2 = m_2 g \left(1 - \frac{a}{g}\right) \leq m_2 g \cdot \frac{\frac{M}{2} + 2m_1}{\frac{M}{2} + m_1 + m_2}$$



$$\frac{M}{2} + m_1 + m_2 \cdot m_2 g - T_2 = m_2 a$$

$$\cdot T_1 - m_1 g = m_1 a$$

$$R(T_2 - T_1) = \frac{1}{2} MR^2 \frac{a}{R} = \frac{MRa}{2}$$

$$\cdot T_2 - T_1 = \frac{Ma}{2}$$

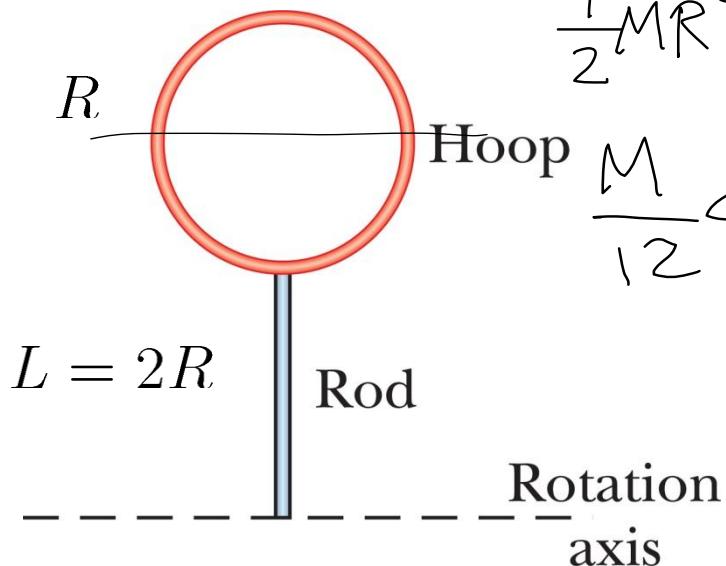
$$T_1 - T_2 + (m_2 - m_1)g = (m_1 + m_2)a$$

$$\left(\frac{M}{2} + m_1 + m_2\right) a = (m_2 - m_1)g$$

$$m_2 - m_1$$

$$a = \frac{\frac{M}{2} + m_1 + m_2}{m_2 - m_1} g$$

Prob. 3

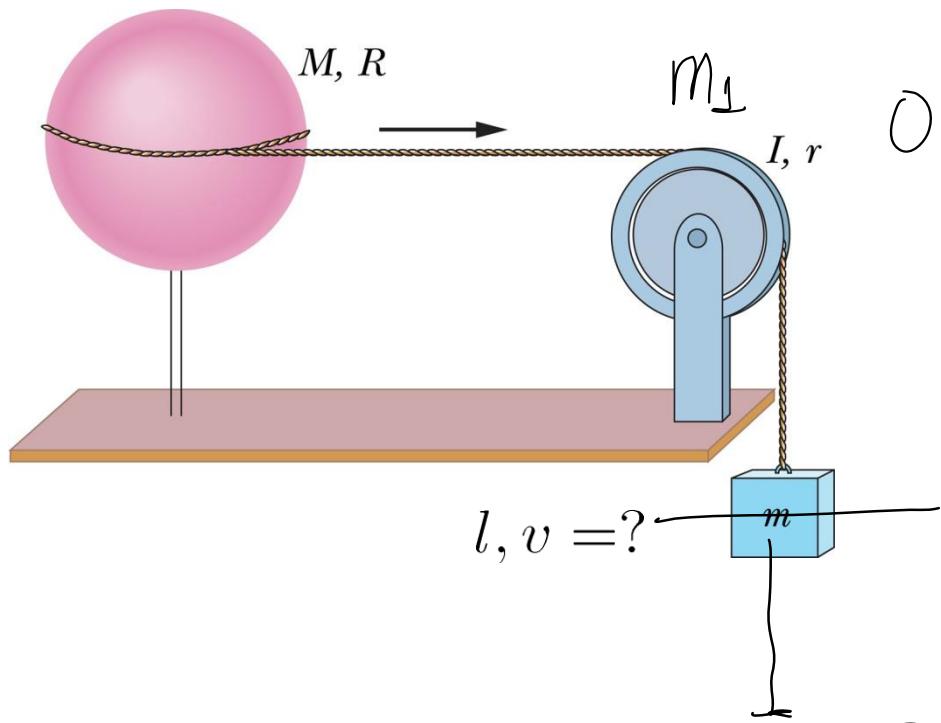


$$\frac{1}{2}MR^2 + M9R^2 = \frac{19}{2}MR^2$$

$$\frac{M}{12}4R^2 + MR^2 = \frac{4}{3}MR^2$$

$$I = MR^2 \frac{57+8}{6} = \frac{65}{6}MR^2$$

Prob. 4

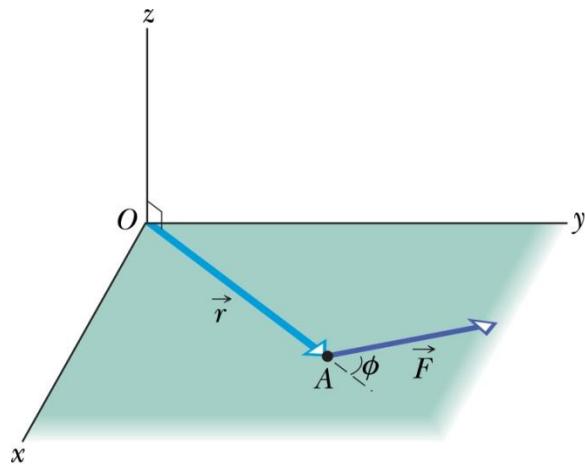


$$0 = -mgl + \frac{1}{2}mv^2 + \frac{1}{2}m_1(R^2\omega_1^2)v^2 + \frac{2}{5}M(R^2\omega_2^2)v^2$$

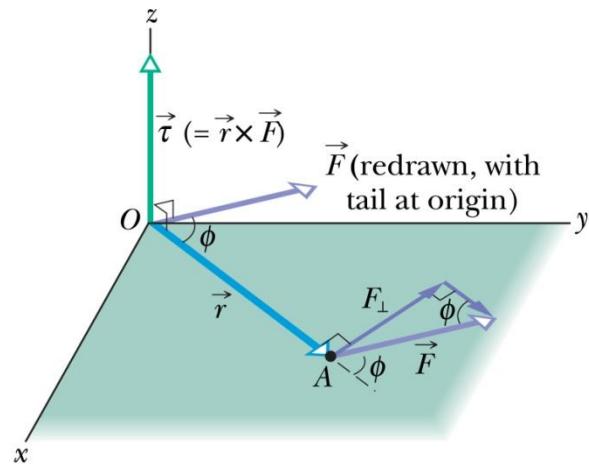
$$mgl = \left(\frac{1}{2}m_1 + \frac{1}{2}m + \frac{2}{5}M \right) v^2$$

$$v = \sqrt{\frac{mgl}{\frac{1}{2}m + \frac{1}{2}m_1 + \frac{2}{5}M}}$$

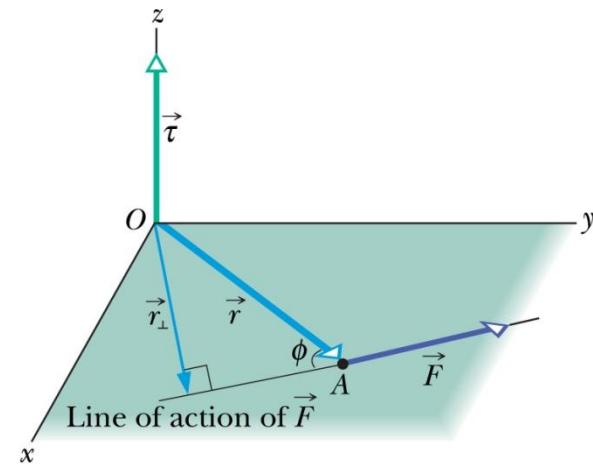
Generalized torque



(a)



(b)



(c)

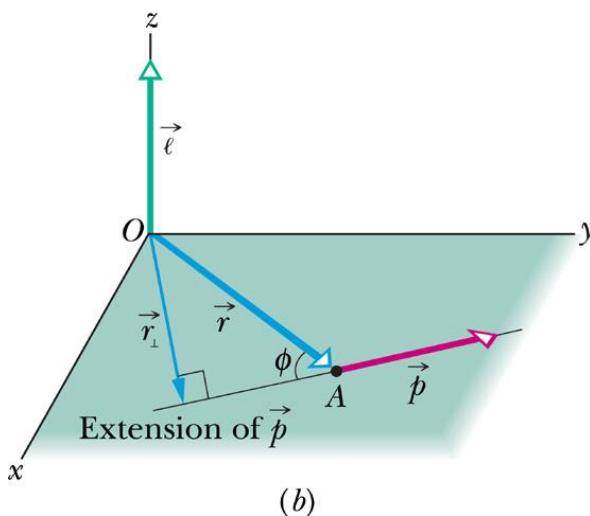
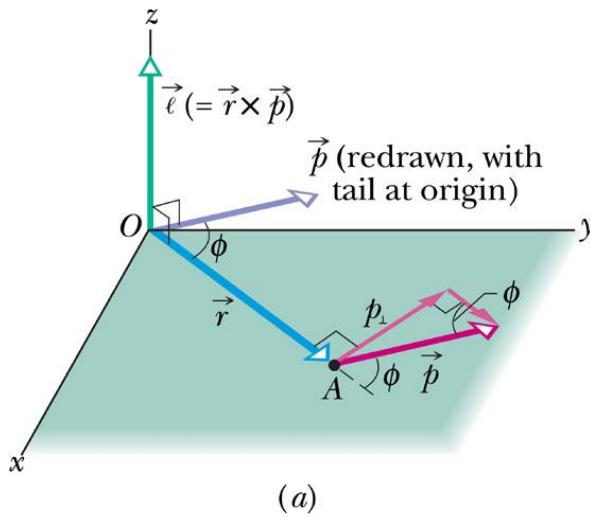
$$\vec{\tau} = \vec{r} \times \vec{F}$$

$$\tau = r F \sin \phi$$

$$= r F_{\perp}$$

$$= r_{\perp} F$$

angular momentum



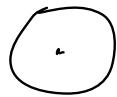
Definition of angular momentum

$$\vec{l} = \vec{r} \times \vec{p} = m(\vec{r} \times \vec{v})$$

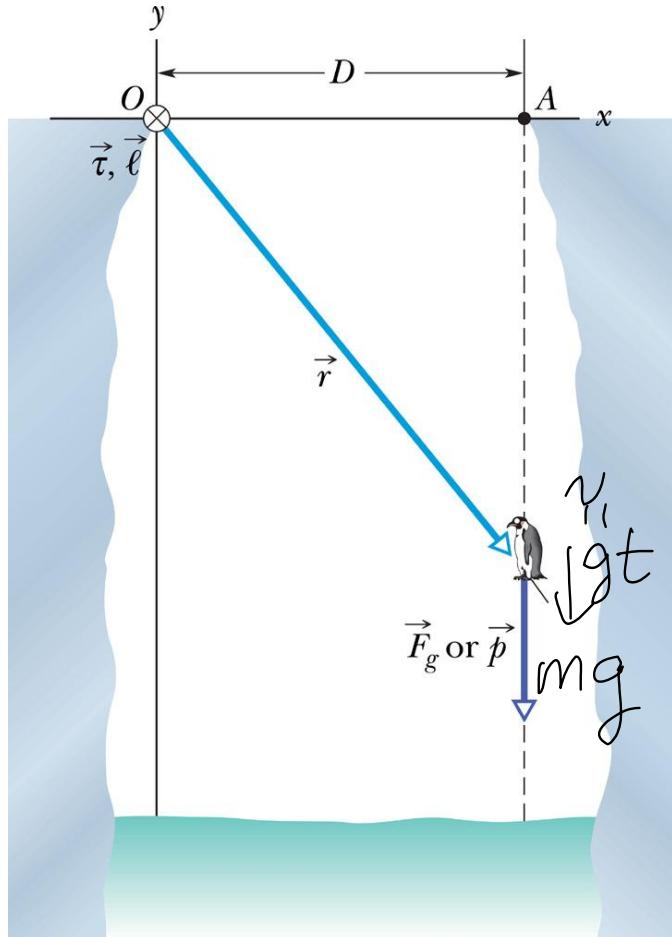
$$\begin{aligned} l &= rmv \sin \phi \\ &= rp_{\perp} = rmv_{\perp} \\ &= r_{\perp}p = r_{\perp}mv \end{aligned}$$

$$\begin{aligned} \frac{d\vec{l}}{dt} &= \frac{d}{dt}(\vec{r} \times m\vec{v}) \\ &= m\left(\frac{d\vec{r}}{dt} \times \vec{v} + \vec{r} \times \frac{d\vec{v}}{dt}\right) \\ &= \vec{r} \times m\frac{d\vec{v}}{dt} = \vec{r} \times \vec{F} \end{aligned}$$

$$\sum \vec{\tau} = \frac{d\vec{l}}{dt}$$



Sample prob.



(a) O에 대한 펭귄의 angular momentum

$$v = v_0 - gt = -gt$$

$$l = D m g t \quad \text{X}$$

(b) O에 대한 중력에 의한 torque

⊗ $\tau = D m g$

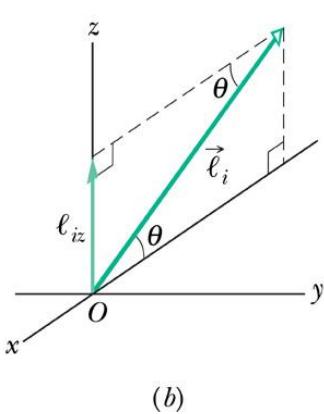
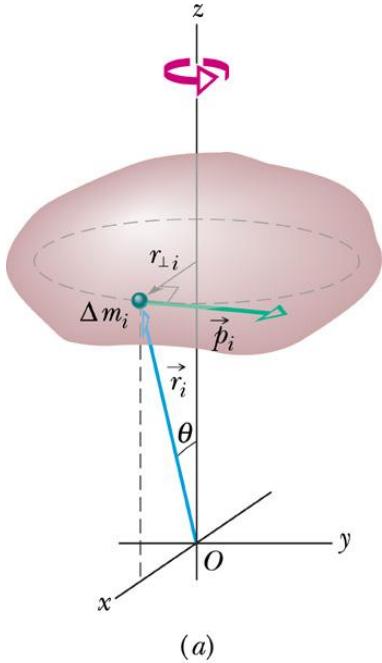
$$\frac{dl}{dt} = \tau$$

Angular momentum of particles

$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_n = \sum_{i=1}^n \vec{l}_i$$

$$\frac{d\vec{L}}{dt} = \sum_{i=1}^n \frac{d\vec{l}_i}{dt} = \sum_{i=1}^n \vec{\tau}_{\text{net},i} = \vec{\tau}_{\text{net}}$$

Angular momentum of a rigid body



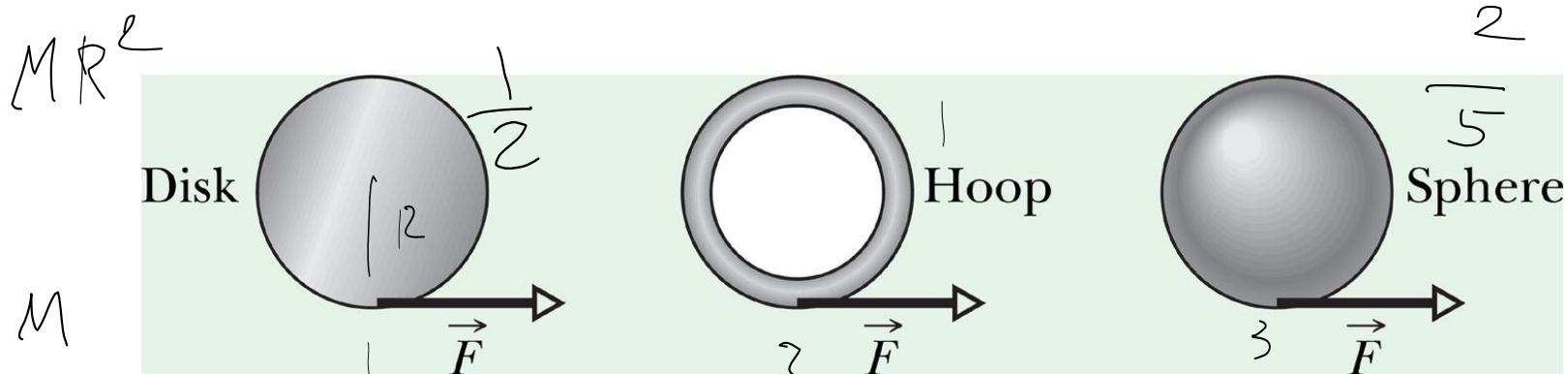
$$l_i = r_i p_i \sin 90^\circ = r_i \Delta m_i v_i$$

$$l_{iz} = l_i \sin \theta = (r_i \sin \theta)(\Delta m_i v_i) = r_{\perp i} \Delta m_i v_i$$

$$\begin{aligned} L_z &= \sum_i l_{iz} = \sum \Delta m_i v_i r_{\perp i} \\ &= \sum_i \Delta m_i (\omega r_{\perp i}) r_{\perp i} \\ &= \omega \left(\sum_i \Delta m_i r_{\perp i}^2 \right) \end{aligned}$$

$$L = I\omega$$

판, 고리, 공에 일정한 힘 F 를 가할 때



$$\tau = RF = \frac{dL}{dt} = \frac{d}{dt} I\omega = I \left(\frac{d\omega}{dt} \right)$$

$$\alpha_1 : \alpha_2 : \alpha_3 = \frac{1}{I_1} : \frac{1}{I_2} : \frac{1}{I_3}$$

$$= 2 : 1 : \frac{5}{2} = 4 : 2 : 5$$

More Corresponding Variables and Relations for Translational and Rotational Motion^a

Translational	Rotational
Force	\vec{F}
Linear momentum	\vec{p}
Linear momentum ^b	$\vec{P} (= \sum \vec{p}_i)$
Linear momentum ^b	$\vec{P} = M\vec{v}_{\text{com}}$
Newton's second law ^b	$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt}$
Conservation law ^d	$\vec{P} = \text{a constant}$
Torque	$\vec{\tau} (= \vec{r} \times \vec{F})$
Angular momentum	$\vec{\ell} (= \vec{r} \times \vec{p})$
Angular momentum ^b	$\vec{L} (= \sum \vec{\ell}_i)$
Angular momentum ^c	$L = I\omega$
Newton's second law ^b	$\vec{\tau}_{\text{net}} = \frac{d\vec{L}}{dt}$
Conservation law ^d	$\vec{L} = \text{a constant}$

^aSee also Table 10-3.

^bFor systems of particles, including rigid bodies.

^cFor a rigid body about a fixed axis, with L being the component along that axis.

^dFor a closed, isolated system.

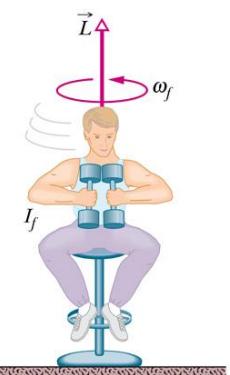
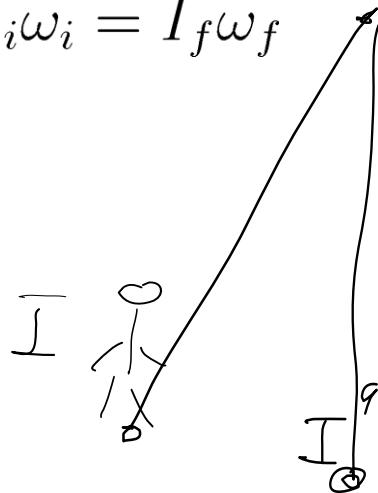
Conservation of angular momentum

Net torque가 없으면 $\vec{\tau}_{\text{net}} = 0$ $\vec{L} = \text{constant}$



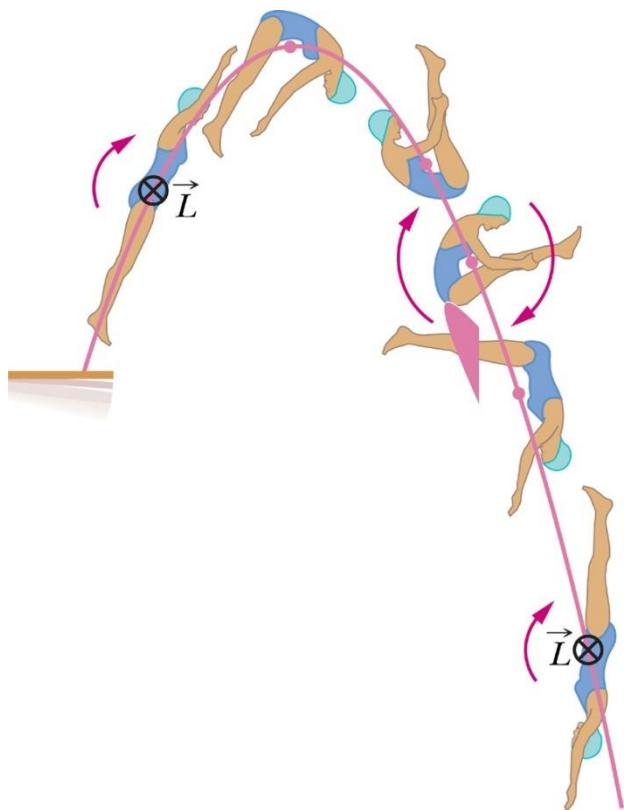
$$L_i = L_f$$

$$I_i \omega_i = I_f \omega_f$$

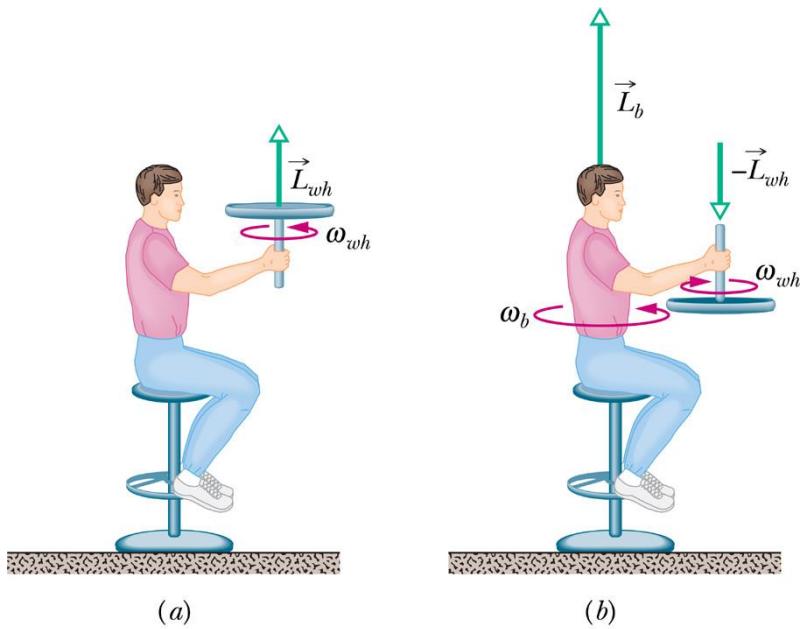


* 그네

(b)



Sample prob.



$$\begin{array}{ccc} \uparrow \vec{L}_{wh} & = & \uparrow \vec{L}_b + \downarrow -\vec{L}_{wh} \\ \text{Initial} & & \text{Final} \end{array}$$

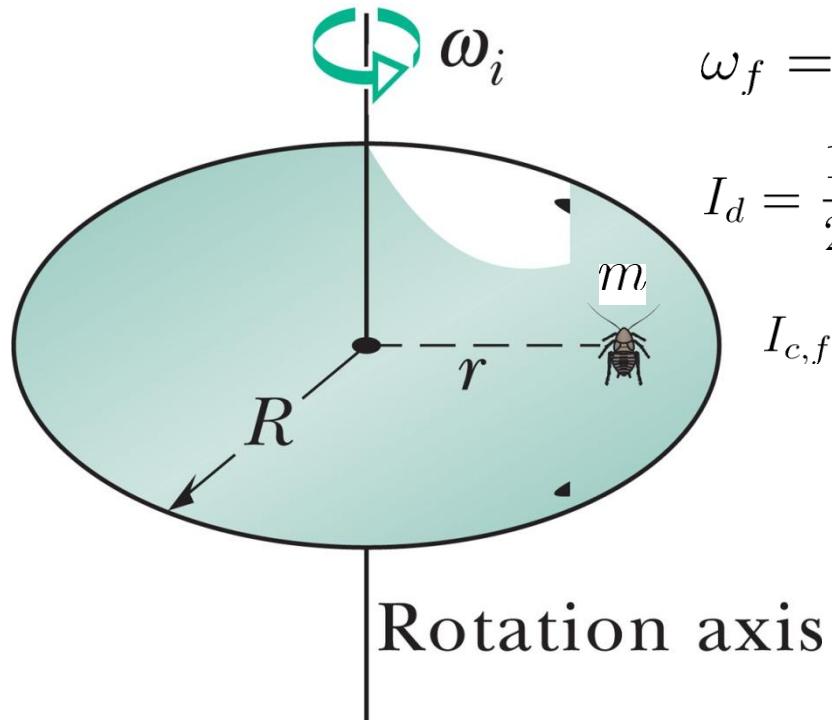
(c)

$$L_{b,f} + L_{wh,f} = L_{b,i} + L_{wh,i}$$

$$L_{b,f} = 2L_{wh,i}$$

$$\omega_b = \frac{2I_{wh}}{I_b}\omega_{wh}$$

Sample prob.



$$\omega_f = ?$$

$$I_d = \frac{1}{2}MR^2 = 3.00mR^2, \quad I_{c,i} = mr^2 = 0.64mR^2$$

$$I_{c,f} = mR^2$$

$$I_i = I_d + I_{c,i} = 3.64mR^2$$

$$I_f = I_d + I_{c,f} = 4.00mR^2$$

$$I_f \omega_f = I_i \omega_i$$

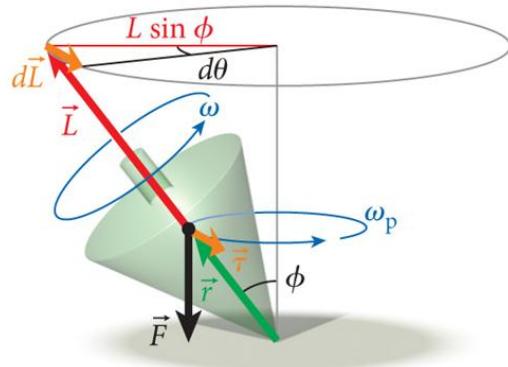
$$R, \quad M = 6.00m, \quad r = 0.800R$$

$$4.00mR^2 \omega_f = 3.64mR^2 \omega_i$$

$$\omega_f = 0.91\omega_i$$

Gyroscope

$$\vec{\tau} = \frac{d\vec{L}}{dt} \quad \vec{L} \parallel \vec{r} \rightarrow \vec{\tau} \perp \vec{L}$$



$$d\vec{L} = \vec{\tau} dt \quad (\text{torque는 각운동량의 방향만 바꾼다})$$

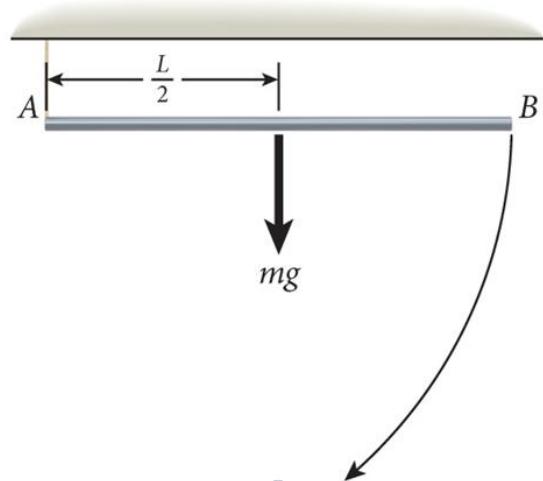
$$dL = L \sin \phi d\theta \rightarrow \frac{dL}{dt} = L \sin \phi \frac{d\theta}{dt}$$

$$rmg \sin \phi = \tau = \frac{dL}{dt} = L \sin \phi \omega_p$$

$$\omega_p = \frac{rmg}{I\omega} \quad (\text{precession 운동의 각속도})$$

Angular velocity of precession

SP 10.4 Falling horizontal rod



a?

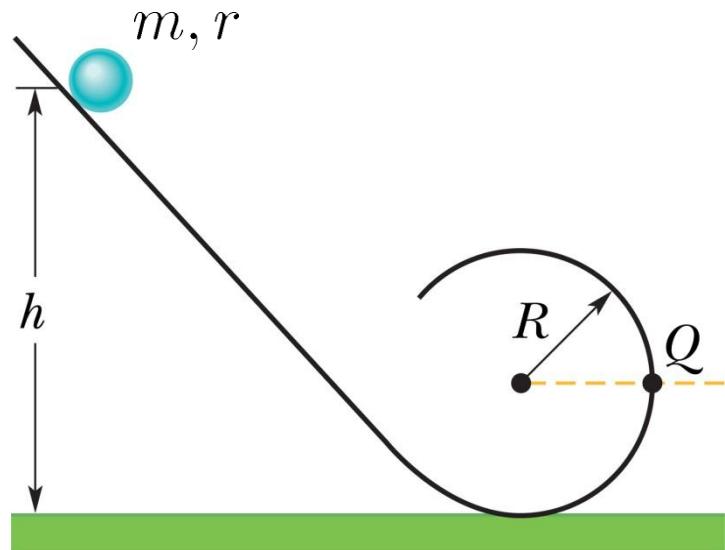
$$\tau = \frac{d\ell}{dt} = I\alpha$$

$$\frac{L}{2} \cancel{mg} = \frac{\cancel{I} L^2}{3} \alpha$$

$$\alpha = \frac{3g}{2L}$$

$$a = L\alpha = \frac{3g}{2}$$

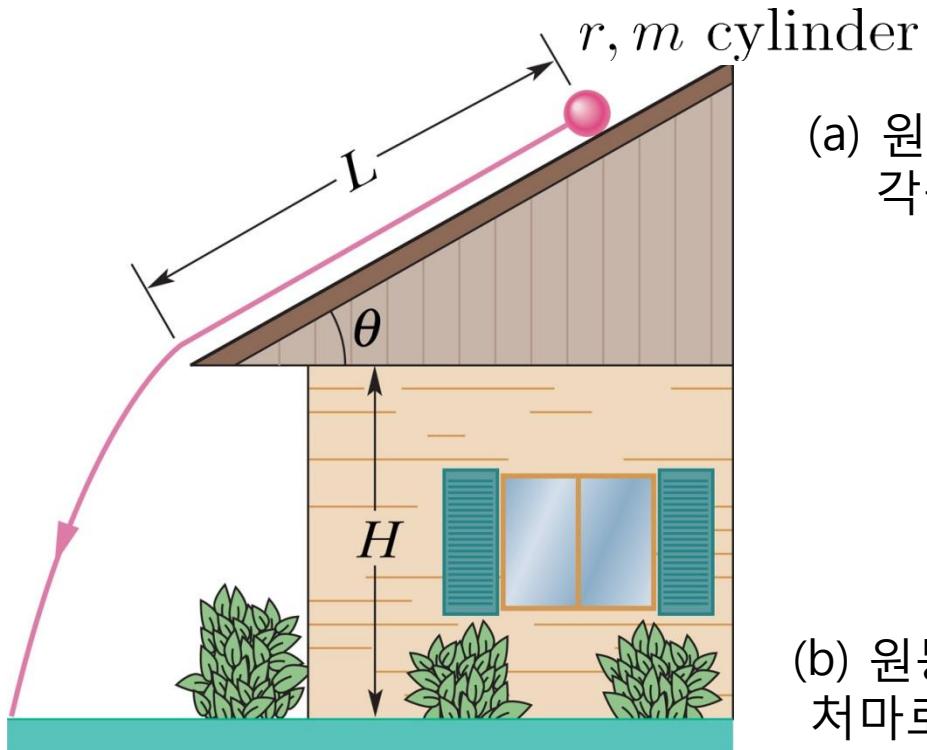
Prob. 5



(a) 원의 꼭대기에서 궤도를 이탈하지 않으려면
 h 는?

(b) $h=6R$ 일 때 점 Q 에서 수평으로 작용하는 힘은?

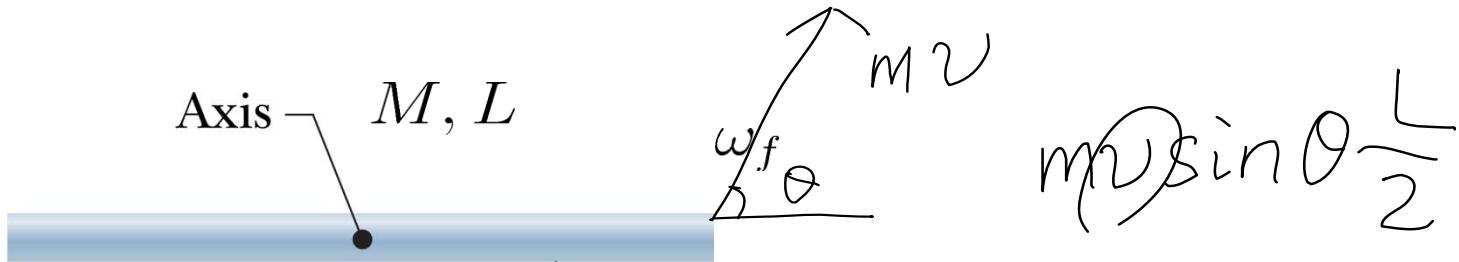
Prob. 6



(a) 원통이 지붕을 떠날 때 원통 중심에 대한
각속도는?

(b) 원통이 지면에 떨어졌을 때
처마로부터 수평거리는?

Prob. 7

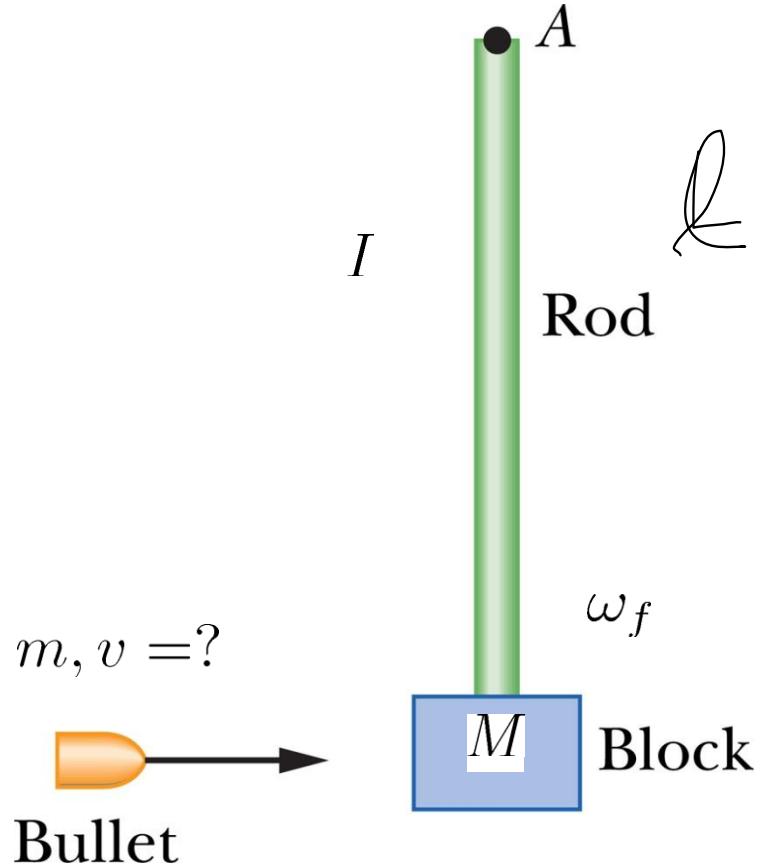


$$I = \frac{1}{12}ML^2 + m\left(\frac{L}{2}\right)^2 = I\omega_f = \left(\frac{1}{12}ML^2 + \frac{mL^2}{4}\right)\omega_f$$

$$\omega_f = \sqrt{\frac{ML^2}{12} + \frac{mL^2}{4}}$$

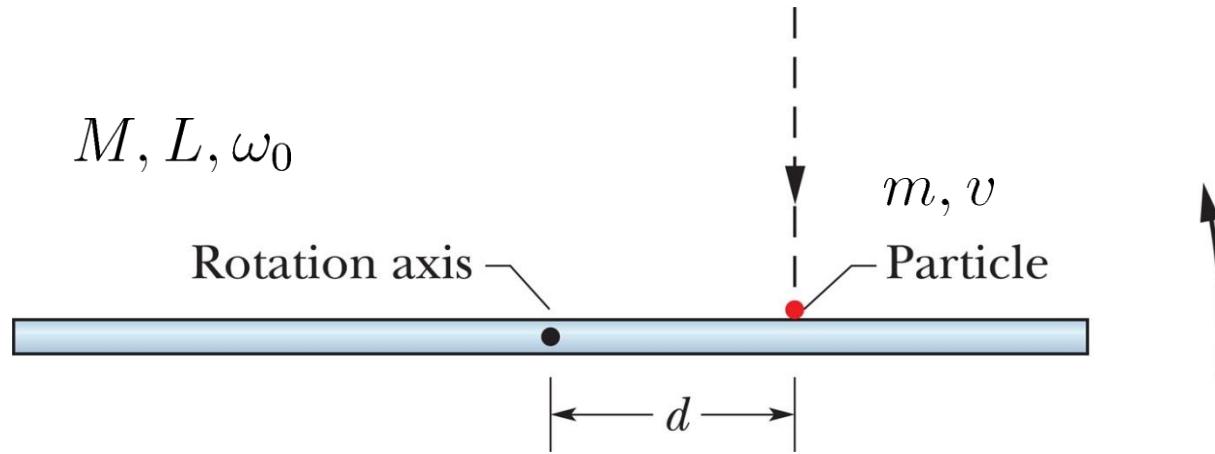
$$v = \frac{\left(\frac{M}{12} + \frac{m}{4}\right)L^2\omega_f}{m \sin \theta \frac{L}{2}} = \frac{\left(\frac{M}{6} + \frac{m}{2}\right)L\omega_f}{m \sin \theta}$$

Prob. 8

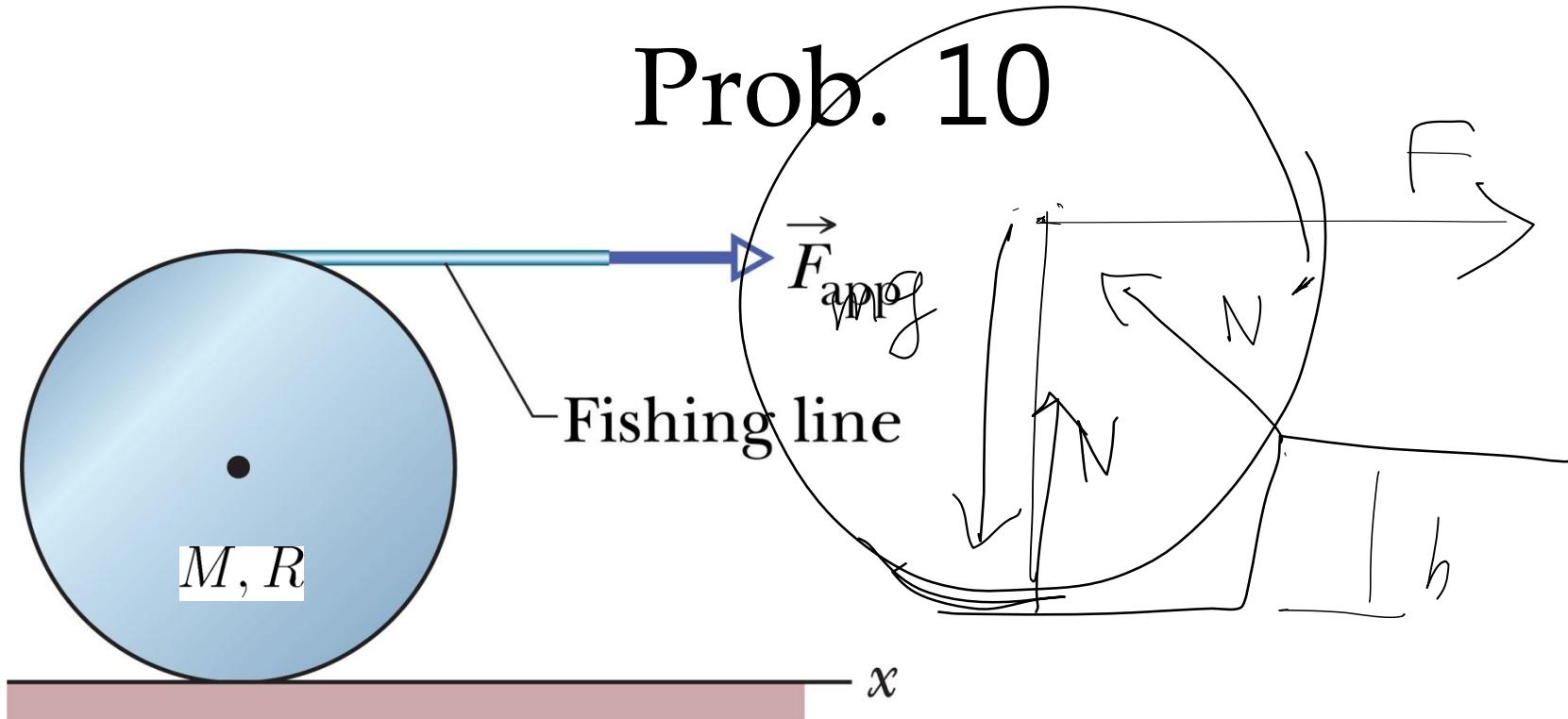


$$\cancel{Rmv} \\ = [I + (M+m)\ell^2] \omega_f \\ v = \frac{I + (M+m)\ell^2}{\cancel{\ell m}} \omega_f$$

Prob. 9



Prob. 10



J