Angle Modulation

KEEE343 Communication Theory

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Summary

- Frequency Division Multiplexing (FDM)
- Angle Modulation

Frequency-Division Multiplexing

- To transmit a number of communication signals over the same channel, the signals must be kept apart so that they do not interfere with each other, and thus they can be separated at the receiving end.
- FDM (Frequency division multiplexing)
- TDM (Time division multiplexing)
- SDM (Space division multiplexing)
- CDM (Code division multiplexing)

Block Diagram of FDM

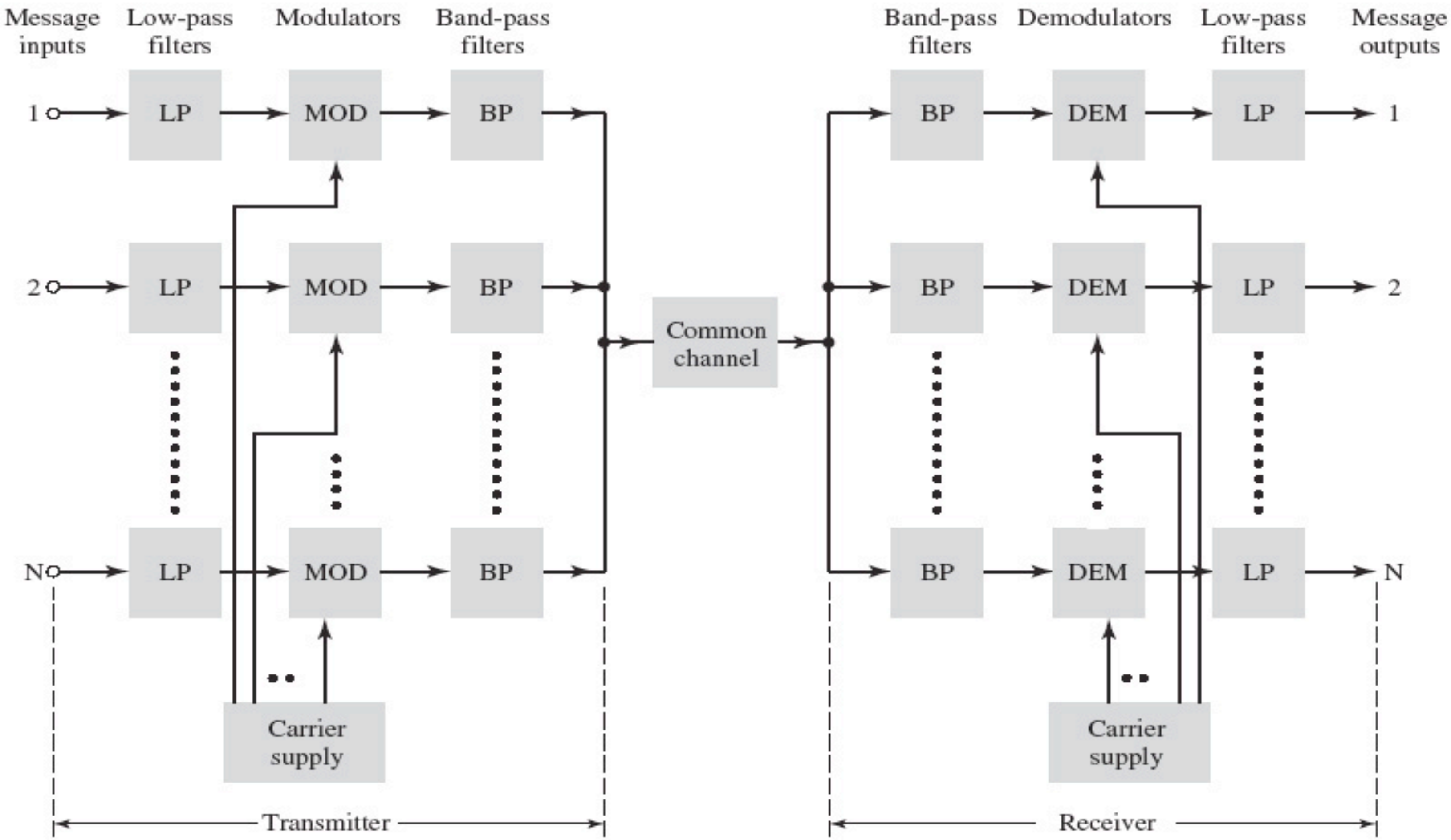

FIGURE 3.29 Block diagram of frequency-division multiplexing (FDM) system.

[Ref: Haykin & Moher, Textbook]

Angle Modulation

- Basic Definition of Angle Modulation

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + \phi_c]$$

  - Phase modulation (PM) if

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

  - Frequency modulation (FM) if

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

Angle Modulation

- The angle of the carrier wave is varied according to the information-bearing signal.
- Lesson 1: Angle modulation is a nonlinear process
  - In analytic terms, the spectral analysis of angle modulation is complicated
  - In practical terms, the implementation of angle modulation is demanding
- Lesson 2: Whereas the transmission bandwidth of an amplitude-modulated wave is of limited extent, the transmission bandwidth of an angle-modulated wave may an infinite extent, at least in theory.
- Lesson 3: Given that the amplitude of the carrier wave is maintained constant, we would intuitively expect that additive noise would affect the performance of angle modulation to a lesser extent than amplitude modulation.

Basic Definitions

- Angle-modulated wave

$$s(t) = A_c \cos[\theta_i(t))]$$

  - Average frequency in hertz

$$f_{\Delta t} = \frac{\theta(t + \Delta t) - \theta(t)}{2\pi \Delta t}$$

  - Instantaneous frequency of the angle-modulated signal

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

$$\theta_i(t) = 2\pi f_c t + \phi_c, \qquad \text{for } m(t) = 0$$

- Phase modulation (PM) is that form of angle modulation in which instantaneous angle is varied linearly with the message signal

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

- Frequency modulation (FM) is that form of angle modulation in which the instantaneous frequency is varied linearly with the message signal

$$f_i(t) = f_c + k_f m(t)$$

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

TABLE 4.1 ***Summary of Basic Definitions in Angle Modulation***

	Phase modulation	*Frequency modulation*	*Comments*
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$	A_c: carrier amplitude f_c: carrier frequency $m(t)$: message signal k_p: phase-sensitivity factor k_f: frequency-sensitivity factor
Instantaneous frequency $f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{d}{dt}m(t)$	$f_c + k_f m(t)$	
Modulated wave $s(t)$	$A_c \cos[2\pi f_c t + k_p m(t)]$	$A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$	

[Ref: Haykin & Moher, Textbook]

Properties of Angle-Modulated Wave

- Property 1: Constancy of transmitted wave
  - The amplitude of PM and FM waves is maintained at a constant value equal to the carrier amplitude for all time.
  - The average transmitted power of angle-modulated wave is a constant

$$P_{av} = \frac{1}{2}A_c^2$$

- Property 2: Nonlinearity of the modulated process

$$m(t) = m_1(t) + m_2(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p(m_1(t) + m_2(t))\right]$$

$$s_1(t) = A_c \cos(2\pi f_c t + k_p m_1(t)), \qquad s_2(t) = A_c \cos(2\pi f_c t + k_p m_2(t))$$

$$s(t) \neq s_1(t) + s_2(t)$$

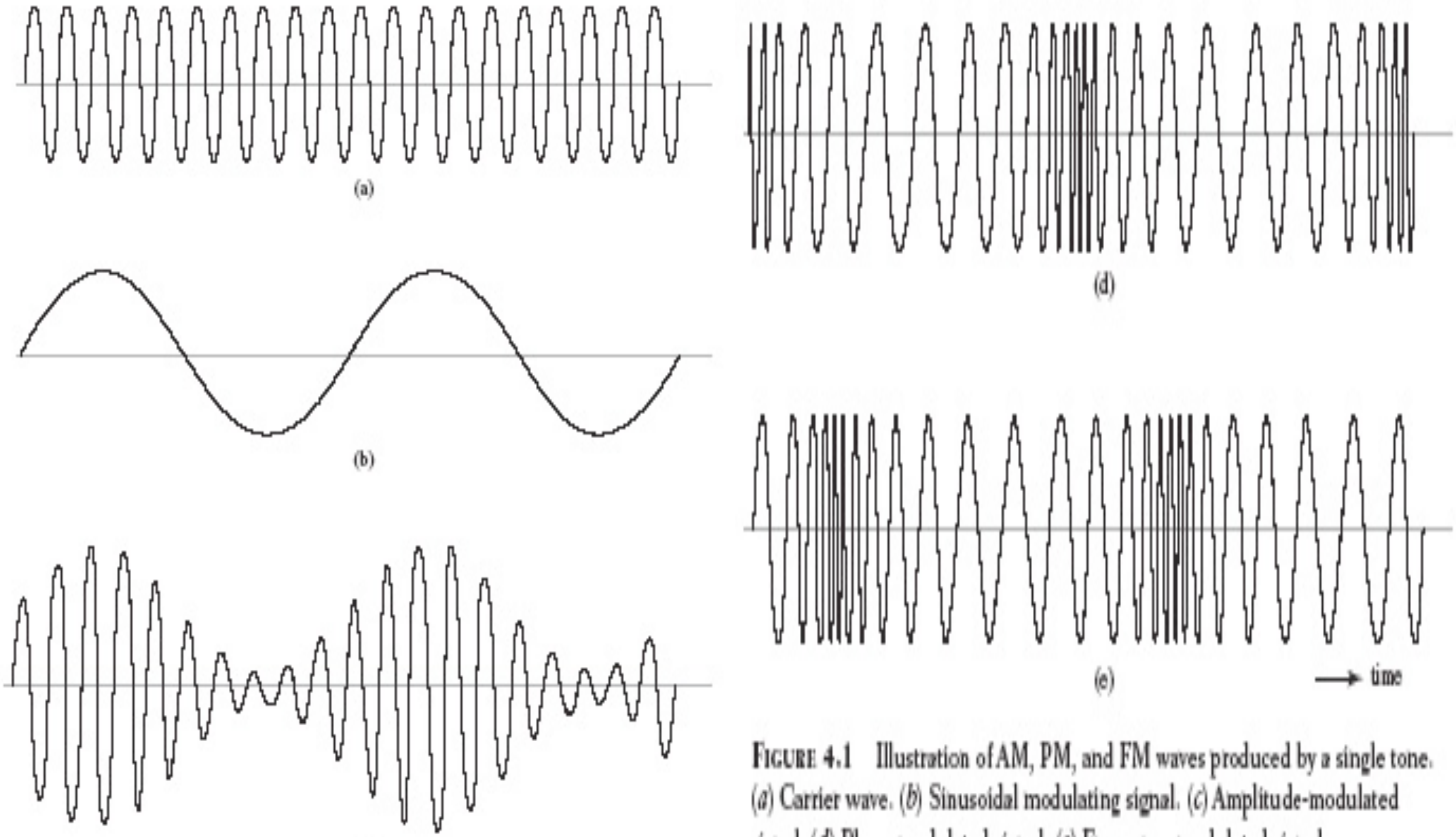

FIGURE 4.1 Illustration of AM, PM, and FM waves produced by a single tone. (*a*) Carrier wave. (*b*) Sinusoidal modulating signal. (*c*) Amplitude-modulated signal. (*d*) Phase-modulated signal. (*e*) Frequency modulated signal.

[Ref: Haykin & Moher, Textbook]

- Property 3: Irregularity of zero-crossings

  - Zero-crossings are defined as the instants of time at which a waveform changes its amplitude from a positive to negative value or the other way around

  - The irregularity of zero-crossings in angle-modulation wave is attributed to the nonlinear character of the modulation process.

    - The message signal m(t) increases or decreases linearly with time t, in which case the instantaneous frequency $f_i(t)$ of the PM wave changes form the unmodulated carrier frequency f_c to a new constant value dependent on the constant value of $m(t)$

- Property 4: Visualization difficulty of message waveform

    - The difficulty in visualizing the message waveform in angle-modulated waves is also attributed to the nonlinear character of angle-modulated waves.

- Property 5: Tradeoff of increased transmission bandwidth for improved noise performance

    - The transmission of a message signal by modulating the angle of a sinusoidal carrier wave is less sensitive to the presence of additive noise

Example of Zero-Crossing

- Consider a modulating wave $m(t)$ given as

$$m(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$$

where a is the slope parameter. In what follows we study the zero-crossing of PM and FM waves for the following set of parameters

$$f_c = \frac{1}{4}\ [\mathrm{Hz}]$$

$$a = 1\ \mathrm{volt/s}$$

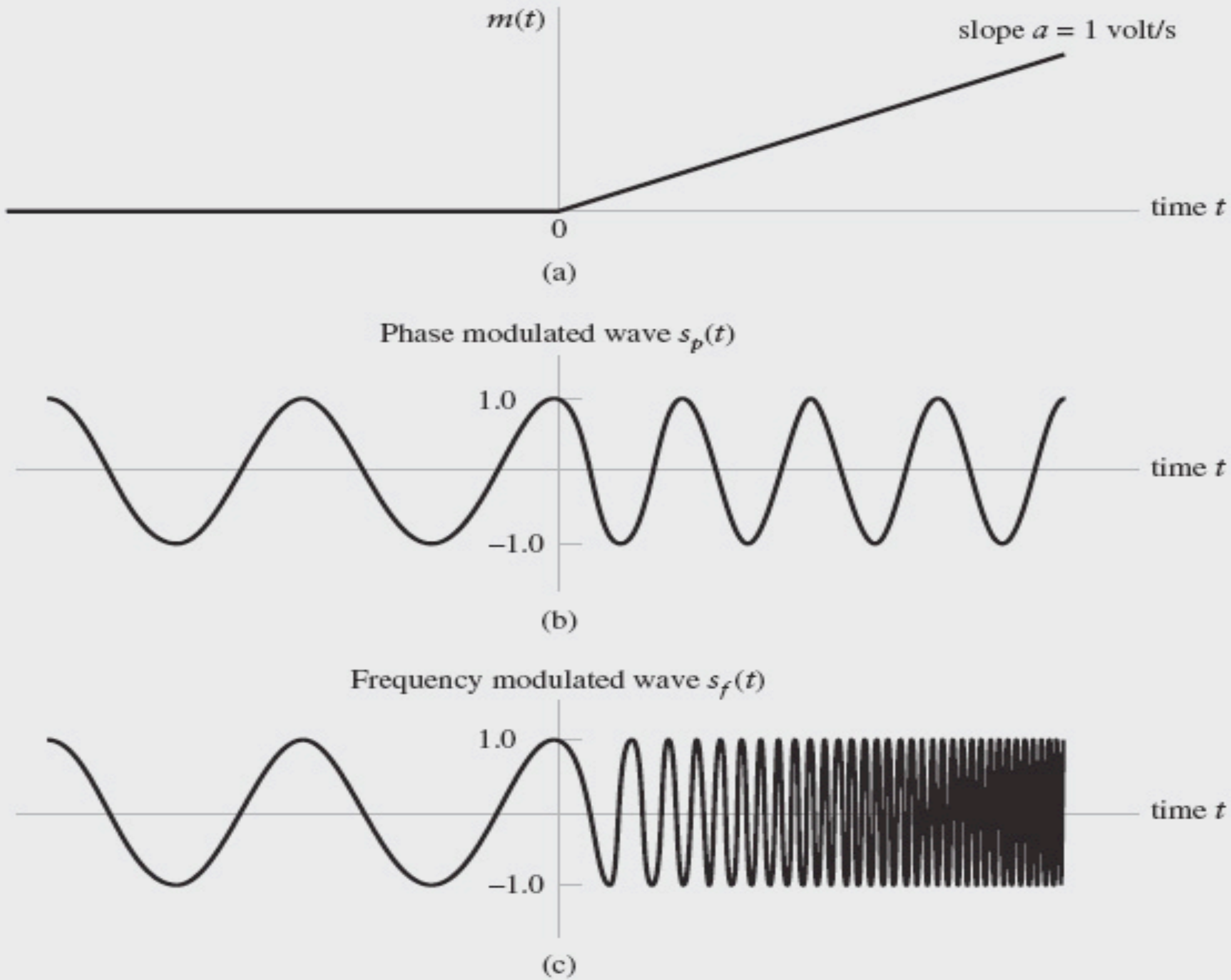

FIGURE 4.2 Starting at time $t = 0$, the figure displays (*a*) linearly increasing message signal $m(t)$, (*b*) phase-modulated wave, and (*c*) frequency-modulated wave.

[Ref: Haykin & Moher, Textbook]

- Phase modulation: phase-sensitivity factor $k_p = \frac{\pi}{2}$ radians/volt. Then, the PM wave is

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

Let t_n denote the instant of time at which the PM wave experiences a zero-crossing; this occurs whenever the angle of the PM wave is an odd multiple of $\pi/2$. Then we may set up

$$2\pi f_c t_n + k_p a t_n = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, ...$$

as the linear equation for t_n. Solving this equation for t_n, we get the linear formula

$$t_n = \frac{\frac{1}{2} + n}{2f_c + \frac{k_p}{\pi} a} = \frac{1}{2} + n, \qquad n = 0, 1, 2, ...$$

$$f_c = 1/4 \text{ [Hz] and } a = 1 \text{ volt/s}$$

- Frequency modulation

  - Let $k_f = 1$. Then the FM wave is

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

  - Invoking the definition of a zero-crossing, we may set up

$$2\pi f_c t_n + \pi k_f a t_n^2 = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, ...$$

$$t_n = \frac{1}{a k_f} \left(-f_c + \sqrt{f_c^2 + a k_f \left(\frac{1}{2} + n \right)} \right), \qquad n = 0, 1, 2, ...$$

$$t_n = \frac{1}{4} \left(-1 + \sqrt{9 + 16n} \right), \qquad n = 0, 1, 2, ...$$

$$f_c = 1/4 \text{ [Hz] and } a = 1 \text{ volt/s}$$

- Comparing the zero-crossing results derived for PM and FM waves, we may make the following observations once the linear modulating wave begins to act on the sinusoidal carrier wave:

  - For PM, regularity of the zero-crossing is maintained; the instantaneous frequency changes from the unmodulated value of $f_c = 1/4$ Hz to the new constant value of

$$f_c + k_p(a/2\pi) = \frac{1}{2}\ \mathrm{Hz}.$$

  - For FM, the zero-crossings assume an irregular form; as expected, the instantaneous frequency increases linearly with time t

Relationship between PM and FM

- An FM wave can be generated by first integrating the message signal m(t) with respect to time t and thus using the resulting signal as the input to a phase modulation.
- A PM wave can be generated by first differentiating m(t) with respect to time t and then using the resulting signal as the input to a frequency modulator.
    - We may deduce the properties of phase modulation from those frequency modulation and vice versa.

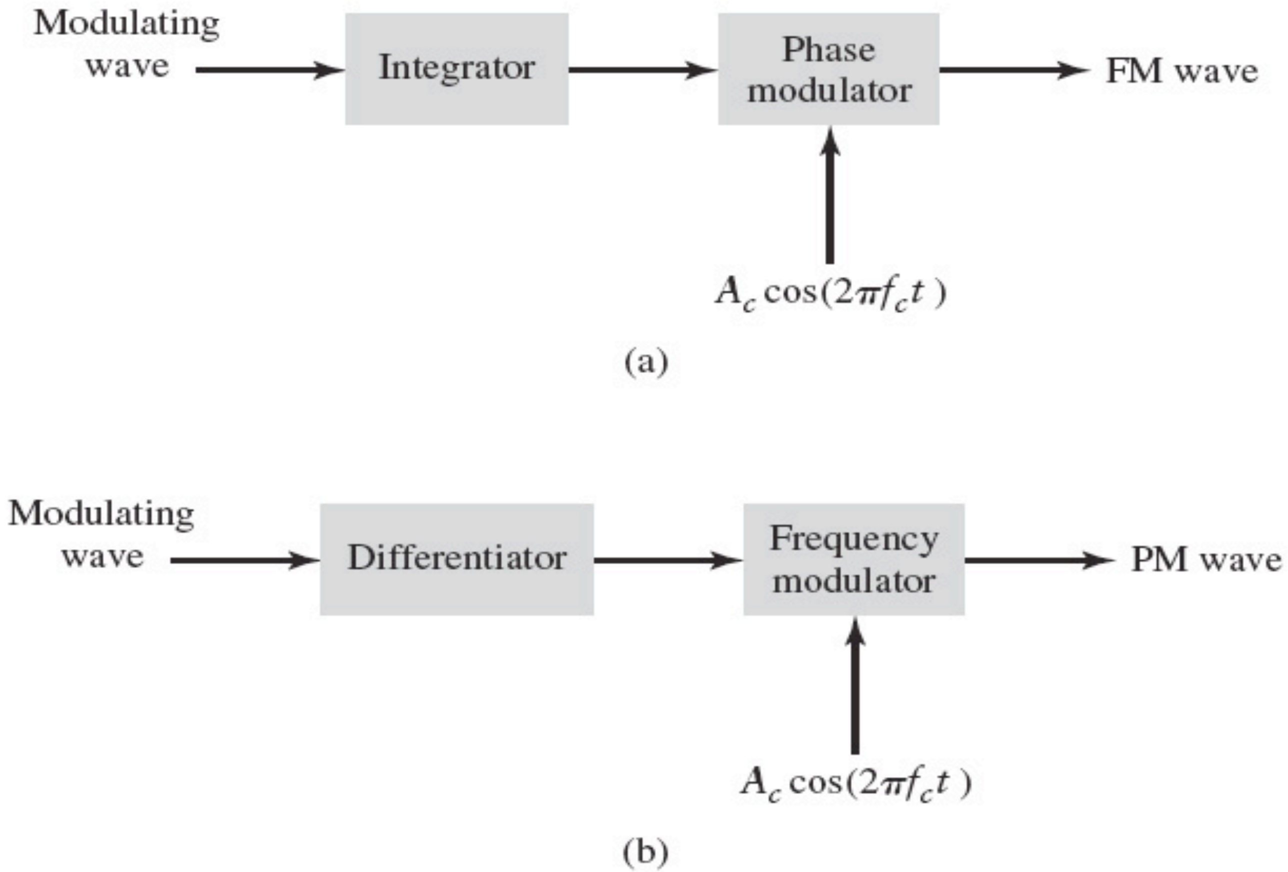

FIGURE 4.3 Illustration of the relationship between frequency modulation and phase modulation. (*a*) Scheme for generating an FM wave by using a phase modulator. (*b*) Scheme for generating a PM wave by using a frequency modulator.

[Ref: Haykin & Moher, Textbook]