Fourier Series and Transform

KEEE343 Communication Theory

Lecture #9, March 31, 2011

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Summary

- Amplitude modulation
  - Introduction
  - Carrier wave
  - Modulation
  - Envelope detector

Introduction to Amplitude Modulation

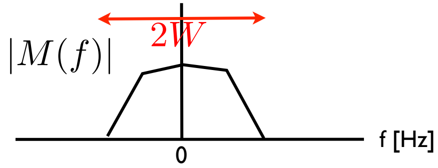
- Consideration of communication system design
  - Complexity
  - Two primary communication resources
    - Transmit power
    - Channel bandwidth

- Amplitude modulation family
  - Amplitude modulation
  - Double sideband-suppressed carrier (DSB-SC)
  - Single sideband (SSB)
  - Vestigial sideband (VSB)

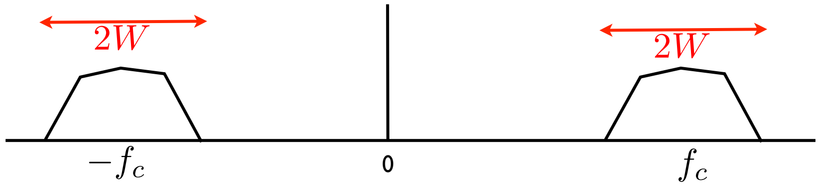
Carrier Wave

- Recall that the cosine function (or sine function) can shift the band by the frequency of the cosine function.

$$\mathcal{F}[m(t)] = M(f)$$



$$\mathcal{F}[m(t)\cos(2\pi f_c t)] = \frac{1}{2}\left[M(f-f_c) + M(f+f_c)\right]$$



- Carrier wave (or signal)
  - Carrier wave is a signal to move (translate) the baseband signal to the passband signal.
  - A commonly used carrier is a sinusoidal wave.

Amplitude Modulation

- Theory
  - Consider a sinusoidal carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$

  - Denote the message signal (information bearing signal) as $m(t)$

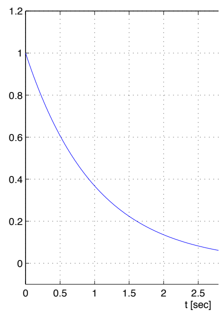
  - Transmission: Then an amplitude-modulated (AM) wave is

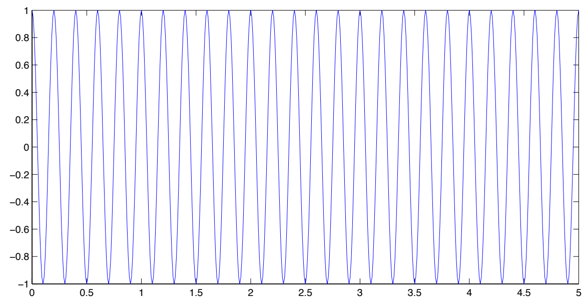
$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

  - Receiver: Envelope detector

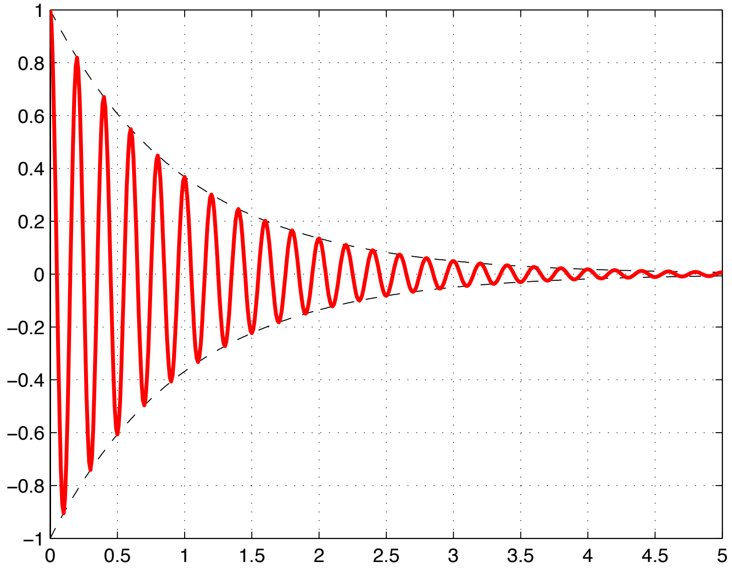
Waveform of the multiplication of the signal and the sinusoidal function

- Consider $m(t) = e^{-t}$ of which the waveform is illustrated as below:
- Now consider the carrier wave given as $c(t) = \cos(10\pi t)$

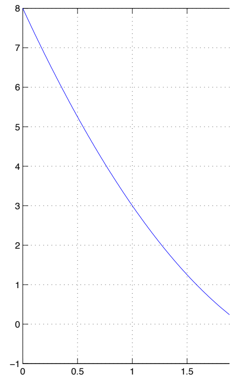




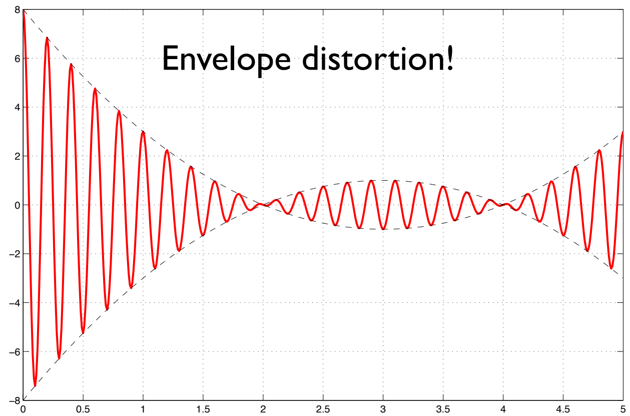
- Waveform of $m(t) \times c(t)$



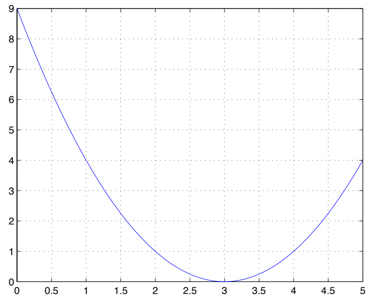
- Now consider $m(t) = t^2 - 6t + 8$



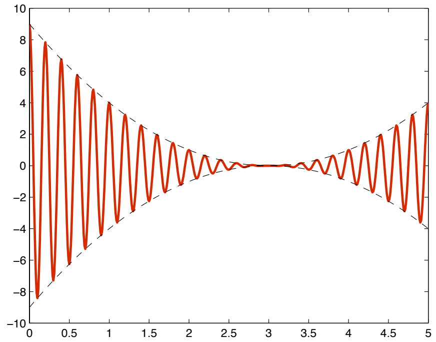
- Waveform of $m(t) \times c(t)$



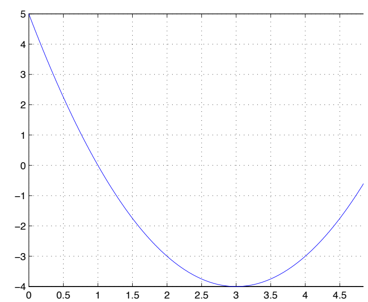
- Now consider $m(t) = t^2 - 6t + 9$



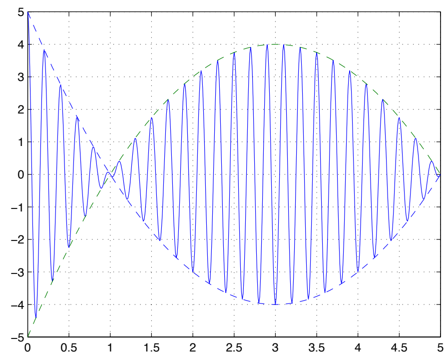
- Waveform of $m(t) \times c(t)$



- Now consider $m(t) = t^2 - 6t + 5$



- Waveform of $m(t) \times c(t)$

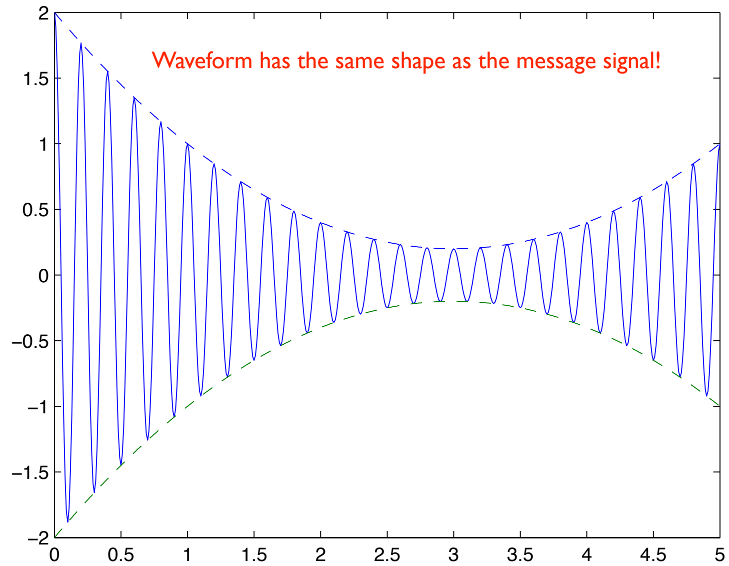


- Now let us do "amplitude modulation" such as

$$s(t) = [1 + k_a m(t)]c(t)$$

where we set k_a to be $|k_a m(t)| < 1$ such as

$$k_a = \frac{1}{\max|m(t)|}$$


Waveform has the same shape as the message signal!
2
1.5
1
0.5
0
−0.5
−1
−1.5
−2
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

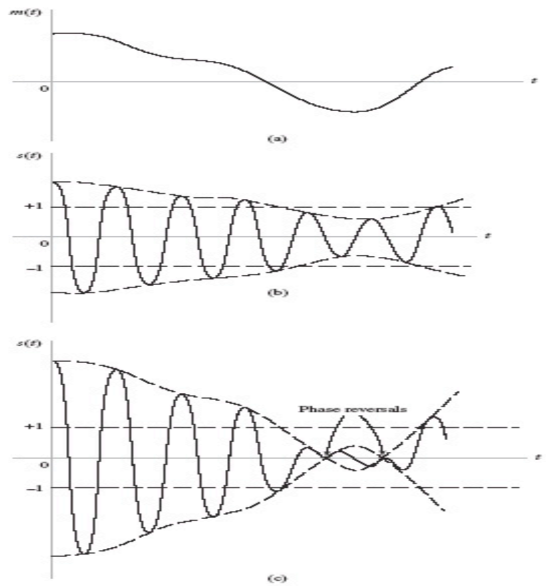
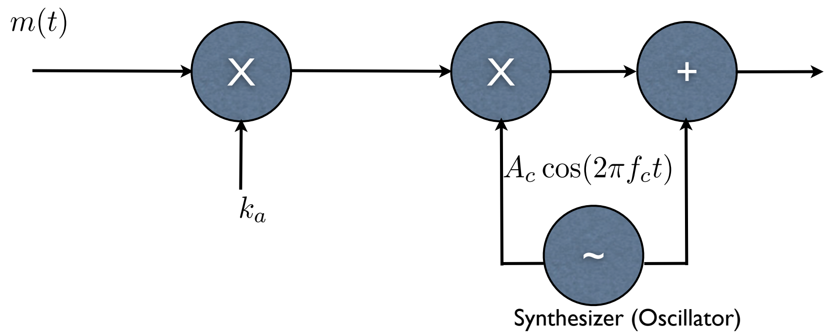


Figure 3.1 Illustration of the amplitude modulation process. (*a*) Message signal $m(t)$. (*b*) AM wave for $k_a m(t) < 1$ for all t. (*c*) AM wave for $|k_a m(t)| > 1$ for some t.

- If the amplification factor is considered, the amplitude modulated signal can be written as

$$s(t) = A_c[1 + k_a m(t)]c(t)$$

Note that A_c is just amplification factor.



- Envelop of $s(t)$ has essentially the same shape as the message signal $m(t)$ provided that two conditions are satisfied:

1. The amplitude of $k_a m(t)$ is always less than unity; that is

$$|k_a m(t)| < 1, \qquad \text{for all } t$$

   - In this case, the function $1 + k_a m(t)$ is always positive !

2. The carrier frequency f_c is much greater than the highest frequency component W of the message signal $m(t)$ - that is,

$$f_c >> W$$

We call W the message bandwidth.

Frequency-Domain Description of AM

- AM transmit signal

$$\begin{aligned} s(t) &= A_c[1 + k_a m(t)]\cos(2\pi f_c t) \\ &= A_c \cos(2\pi f_c t) + A_c k_a m(t)\cos(2\pi f_c t) \end{aligned}$$

- Fourier transform of AM transmitted signal

$$S(f) = \frac{A_c}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right] + \frac{k_a A_c}{2}\left[M(f - f_c) + M(f + f_c)\right]$$

where we make use of

$$\mathcal{F}[\cos(2\pi f_c t)] = \frac{1}{2}\left[\delta(f - f_c) + \delta(f + f_c)\right]$$

Message bandwidth = W

Transmission bandwidth: $B_T = 2W$

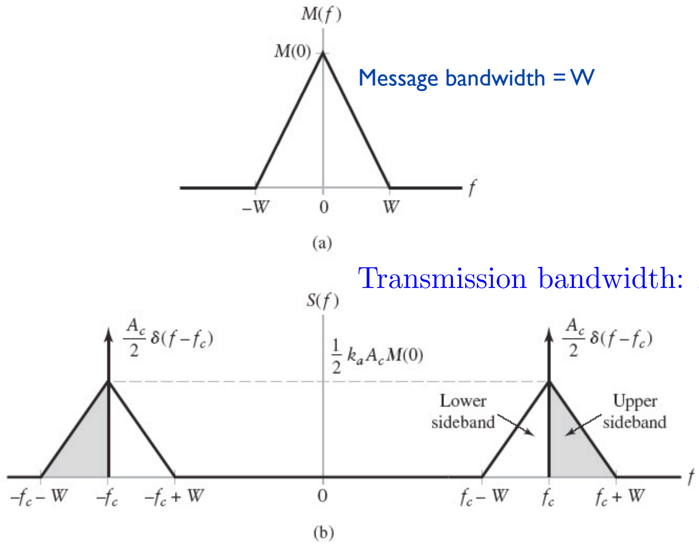
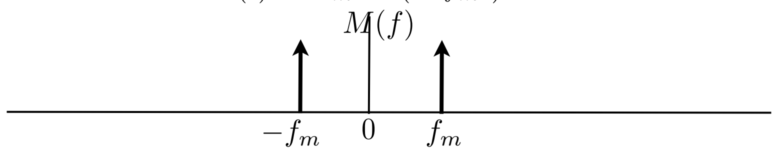


FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

Single -Tone Modulation

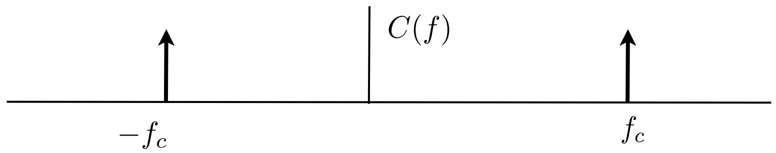
- Consider single-tone modulating wave (message signal)

$$m(t) = A_m \cos(2\pi f_m t)$$



- carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$



- AM wave

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

where $\mu = k_a A_m$

- Maximum envelope value and Minimum envelope value

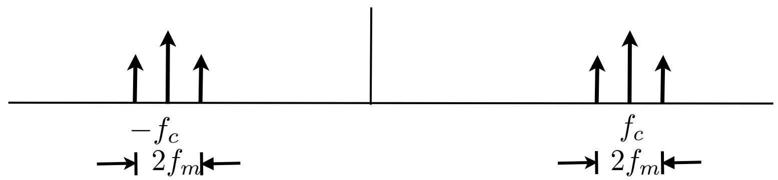
$$A_{\max} = A_c(1 + \mu), \quad A_{\min} = A_c(1 - \mu)$$

- Ratio between the max and min values

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1 + \mu)}{A_c(1 - \mu)} \qquad \Longrightarrow \mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

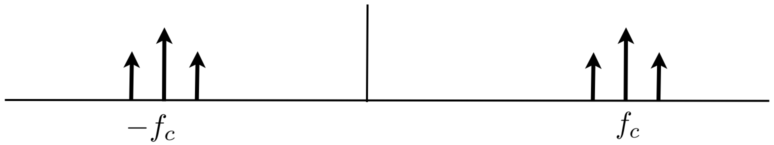
- Fourier transform

$$
\begin{aligned}
s(t) &= A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t] \\
S(f) &= \frac{1}{2}A_c \left[\delta(f - f_c) + \delta(f + f_c)\right] \\
&\quad + \frac{1}{4}\mu A_c \left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right] \\
&\quad + \frac{1}{4}\mu A_c \left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]
\end{aligned}
$$



- Recall the single-tone modulated signal

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2} A_c \mu \cos[2\pi(f_c + f_m)t)] + \frac{1}{2} A_c \mu \cos[2\pi(f_c - f_m)t]$$



- Power calculation
  - Carrier power = $\frac{1}{2} A_c^2$
  - Upper-side-frequency power = $\frac{1}{8} \mu^2 A_c^2$
  - Lower-side-frequency power = $\frac{1}{8} \mu^2 A_c^2$

- Power ratio
  - Total power = $\frac{1}{2}A_c^2 + \frac{1}{8}\mu^2 A_c^2 + \frac{1}{8}\mu^2 A_c^2 = 0.25\left(2+\mu^2\right)A_c^2$
  - Power portion of carrier signal

$$\frac{\text{Carrier power}}{\text{Total power}} = \frac{2}{2+\mu^2}$$

  - Power portion of message signal

$$\frac{\text{Uppersideband+Lowersideband}}{\text{Total power}} = \frac{\mu^2}{2+\mu^2}$$

  - If $\mu = 1$, only 1/3 out of total power is allocated to the message signal.

Envelope Detector

- The narrowband message signal modulated by AM can be recovered at the receiver by a simple envelope detector circuit

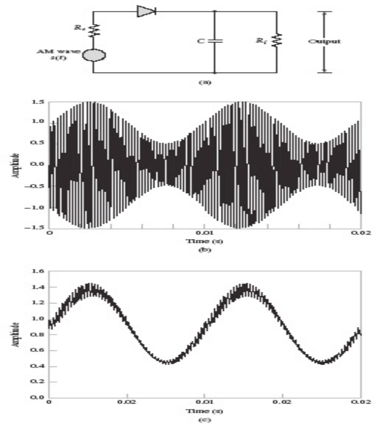


FIGURE 3.9 Envelope detector. (a) Circuit diagram. (b) AM wave input. (c) Envelope detector output

$$\frac{1}{f_c} << R_l C << \frac{1}{W}$$

[Ref: Haykin & Moher, Textbook]