System Models

KEEE343 Communication Theory

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Review

- Signal classification
- Basic continuous-time signals
  - Unit step function
  - Unit impulse function

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau$$

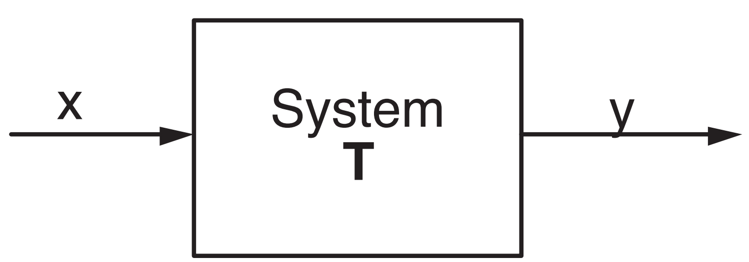
  - Complex exponential signals

Summary

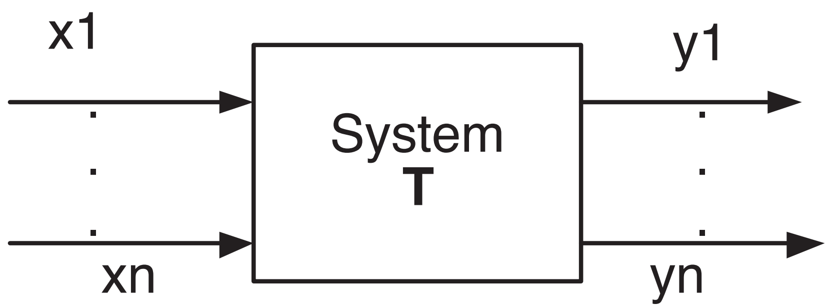
- System classification
  - System requirement
  - Continuous-time and Discrete-time systems
  - System and Classification of Systems
  - Memory and Memoryless systems
  - Causal and Noncausal systems
  - Linear and nonlinear systems
  - Time variant and Time varying systems
  - Stable systems
- Linear-Time Invariant (LTI) systems

System Requirement

- **System:**
  - *Mathematical* model of a physical process that relates the *input* (excitation) signal to the *output* (or response) signal.



[Single-Input Single-Output (SISO) System]



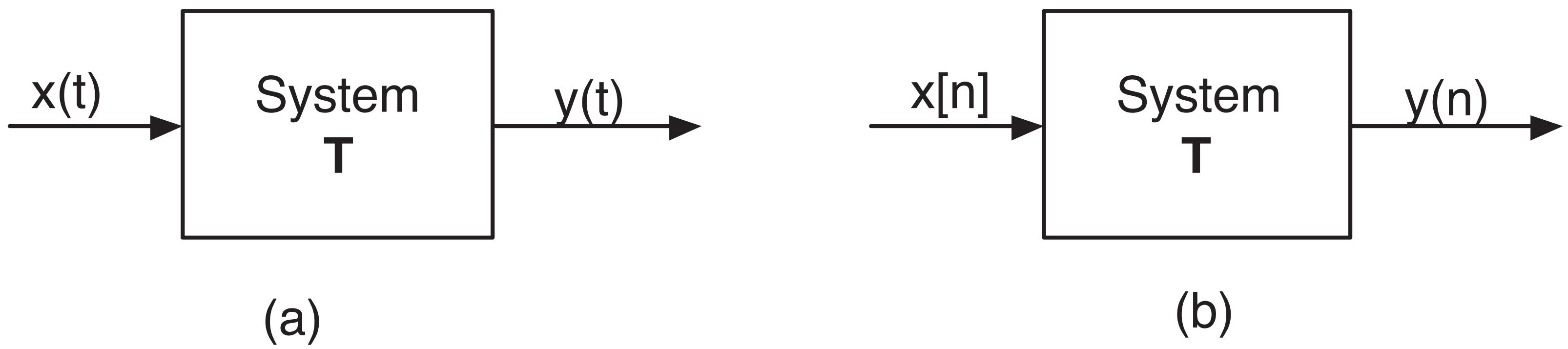
[Multiple-Input Multiple-Output (MIMO) System]

  - System: transformation of x into y

$$y = \mathbf{T}x$$

Continuous and Discrete Time Systems

- If input and output signals are continuous-time signals, the system is continuous time system.
- If input and output signals are discrete-time signals, the system is discrete time system.



Memory vs. Memoryless and Causal vs. Noncausal System

- **Memoryless and Memory systems**
  - Memoryless system: if the output signal at any time depends on only the input signal at the same time.
  - Memory system: if the output signal depends on the past values of the input signal.
- **Causal and Noncausal systems**
  - Causal system: if its output signal at an arbitrary time $t = t_0$ depends on only the input signal for $t < t_0$, not on its future values.
  - Noncausal system: if its output signal depends on the future values of the input signal.
    - Example of Noncausal system: $y(t) = x(t+1)$
  - Note that all memoryless systems are causal, but not vice versa.

Linear and Nonlinear Systems

- Linear system if the system has "additivity" and "homogeneity (or scaling)"

1. Additivity: Given that $\mathbf{T}x_1 = y_1$ and $\mathbf{T}x_2 = y_2$, then

$$\mathbf{T}\{x_1 + x_2\} = y_1 + y_2$$

2. Homogeneity (or Scaling):

$$\mathbf{T}\{\alpha x\} = \alpha y$$

- Additivity and homogeneity condition to be linear system

$$\mathbf{T}\{\alpha_1 x_1 + \alpha_2 x_2\} = \alpha_1 y_1 + \alpha_2 y_2$$

- Example of Nonlinear system

$$y = x^2$$

$$y = \cos x$$

Time-Invariant, Time-Varying, Stable Systems

- Time-invariant system if

$$\mathbf{T}\{x(t-\tau)\} = y(t-\tau)$$

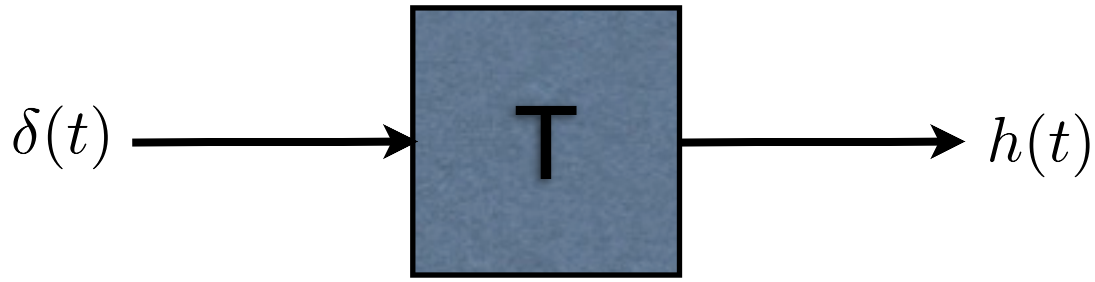
- Liner Time-Invariant system: if the system is linear and also time-invariant.
- Stable system: if the system is bounded-input bounded-output (BIBO).

$$|x| \leq k_1 \longrightarrow |y| \leq k_2$$

$k_1,\ k_2$ are finite real constants.

Response of Linear-Time Invariant Systems

- Impulse response of a continuous-time LTI system



$$h(t) = \mathbf{T}\delta(t)$$

- Response to an arbitrary input, $x(t)$

$$x(t) = \int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)\, d\tau$$

  - Since the system is linear

$$\begin{aligned} y(t) &= \mathbf{T}\{x(t)\} = \mathbf{T}\left\{\int_{-\infty}^{\infty} x(\tau)\delta(t-\tau)d\tau\right\} \\ &= \int_{-\infty}^{\infty} x(\tau)\mathbf{T}\{\delta(t-\tau)\}\, d\tau \\ &= \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau \end{aligned}$$

Convolution Integral

- **Convolution of two continuous-time signals** $x(t)$, and $h(t)$

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)\, d\tau$$