Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
    1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
    2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Conservative force

정의1: 임의의 닫힌 폐곡선을 따라 입자가 움직일 때 한 일이 0이면 이 힘을 conservative force라고 한다.

$$W = -\Delta U(x)$$

$$\Downarrow$$

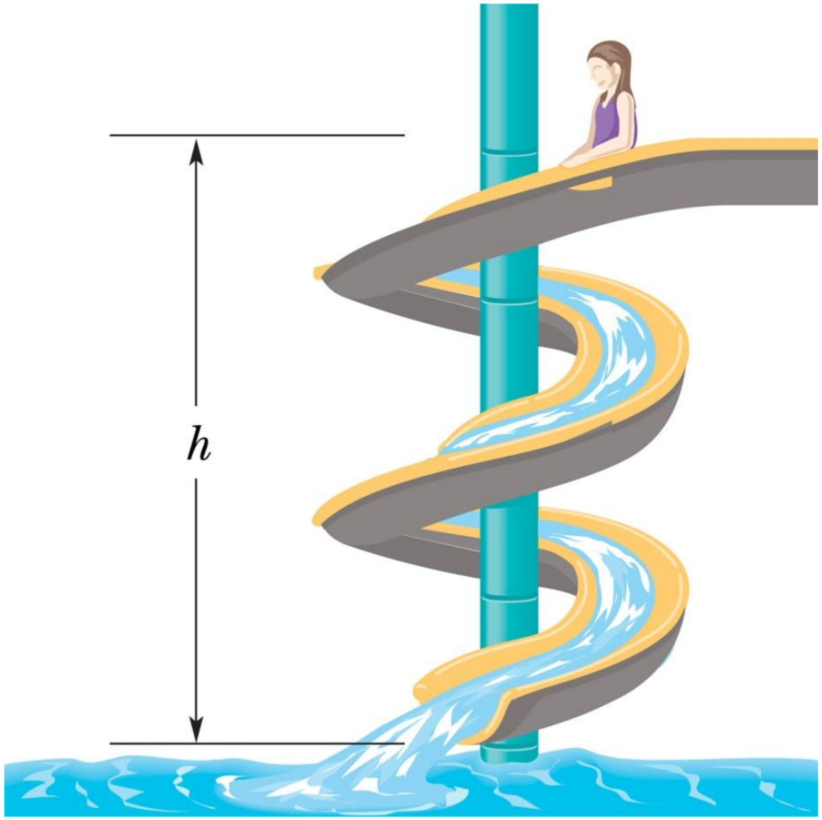
$$F(x)\Delta x$$

$$\therefore F(x) = -\frac{dU(x)}{dx}$$

역학에너지가 보존되는 형태로 potential energy 함수를 정의할 수 있을 때의 힘을 conservative force라고 부른다.

이때 운동에너지와 potential energy 사이의 전환이 양방향 모두 가능하므로 보존력이 한 일은 항상 가역적이다.

Sample problem



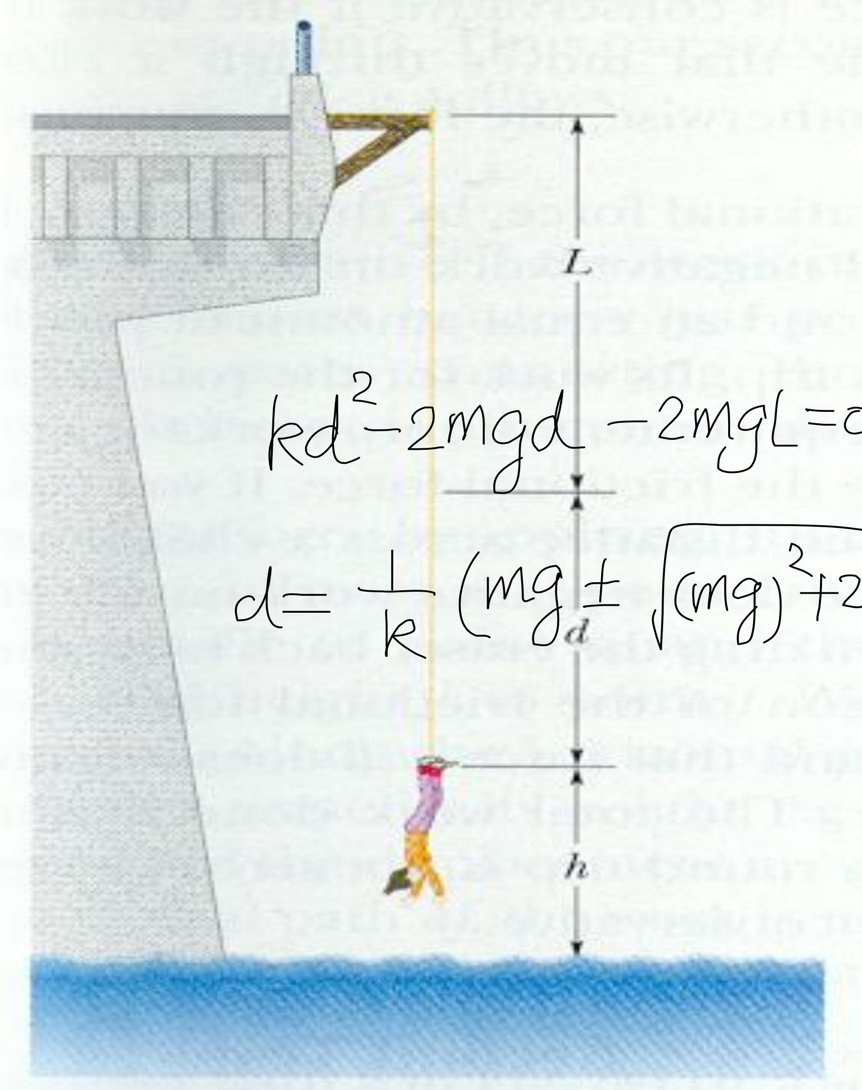
$$K_t + U_t = K_b + U_b$$

$$\tfrac{1}{2} m v_t^2 + mgy_t = \tfrac{1}{2} m v_b^2 + mgy_b$$

$$v_b^2 = v_t^2 + 2g(y_t - y_b)$$
$$= 0^2 + 2gh$$

$$v_b = \sqrt{2gh} = 13\text{m/s}$$

$m,\ h = 8.5\text{m}$



$$kd^2-2mgd-2mgL=0$$

$$d=\frac{1}{k}\left(mg\pm\sqrt{(mg)^2+2mgkL}\right)$$

$$m=61\text{kg}$$
$$L=25\text{m}$$
$$k=160\text{N/m}$$

sample

(1)

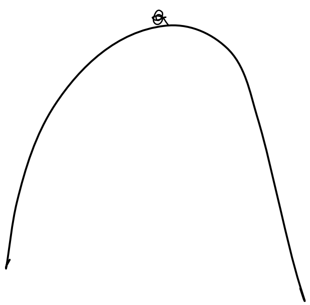
$$\Delta U_e=\tfrac{1}{2}kd^2$$
$$\Delta U_g=mg\Delta y=-mg(L+d)$$
$$\Delta U_e+\Delta U_g+\Delta K=0$$
$$\tfrac{1}{2}kd^2-mg(L+d)+0=0$$

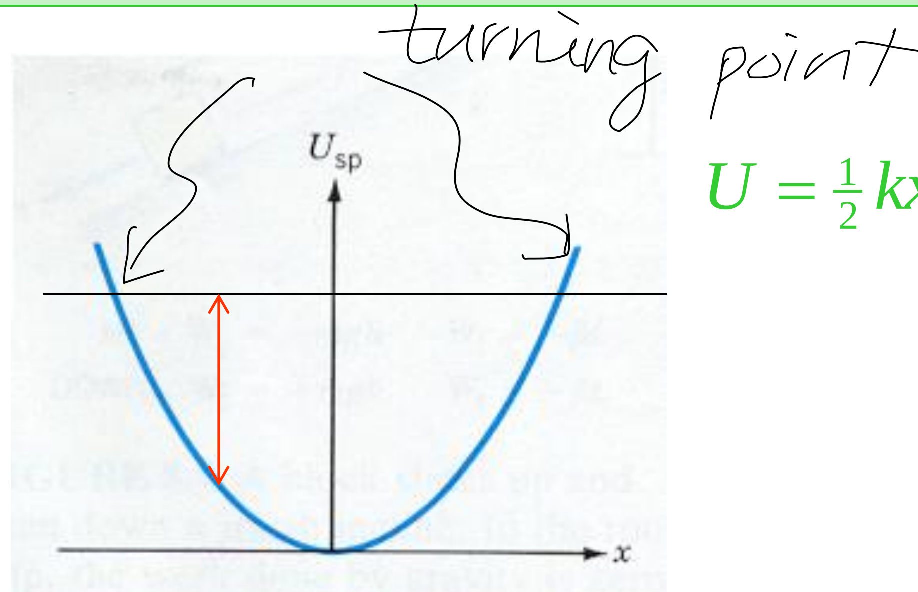
$$\therefore\ d=17.9\text{m}\rightarrow h=45-(L+d)=2.1\text{m}$$

(2)

$$mg=-597.8\text{N}$$
$$F=-kx=-k(-d)=2864\text{N}$$
$$\therefore\ F_t=kd-mg\approx 2270\text{N}$$

Potential energy curve





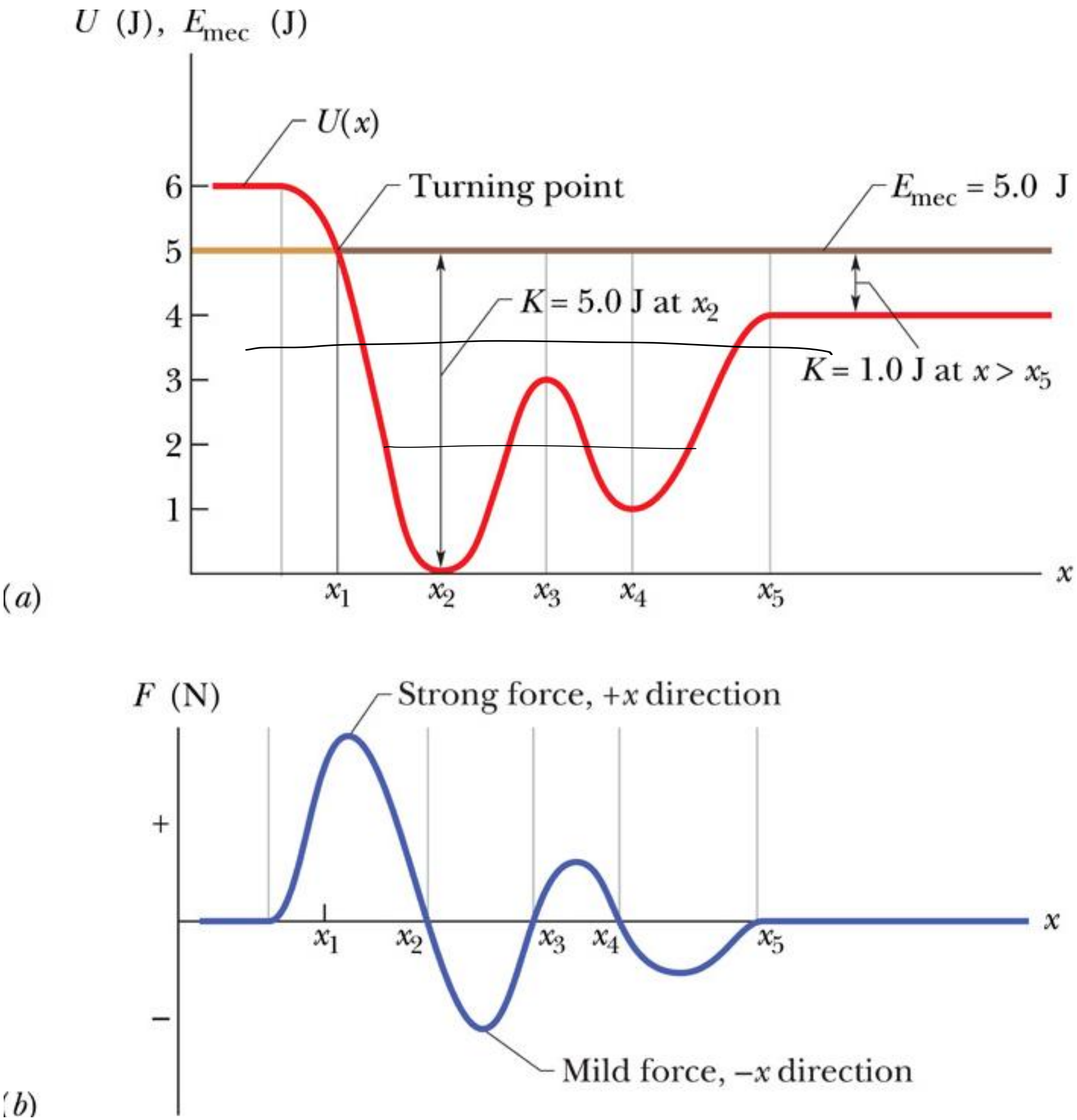
E

K

$U = \frac{1}{2} k x^2$

$$F(x) = -\frac{dU(x)}{dx} \Rightarrow \frac{dU(x)}{dx} = 0 \Rightarrow$$

Equilibrium point



turning point

Equilibrium point

$$F(x) = -\frac{dU}{dx} = 0$$

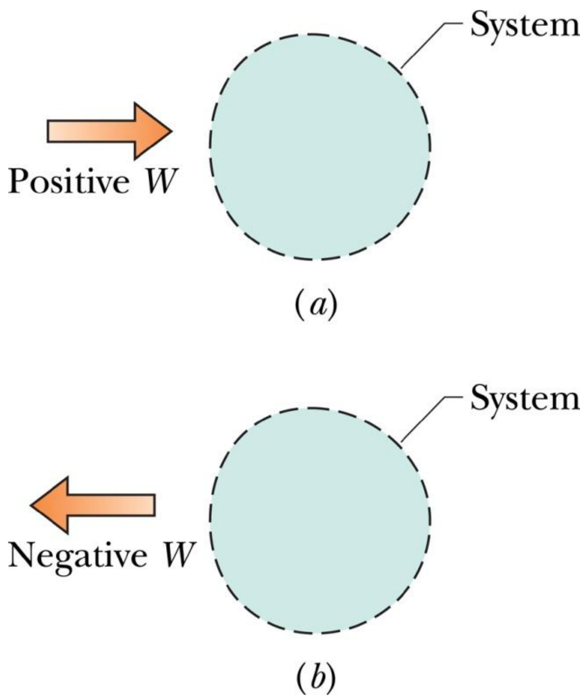
- stable equilibrium point

$$\frac{d^2U}{dx^2} > 0$$

- unstable equilibrium point

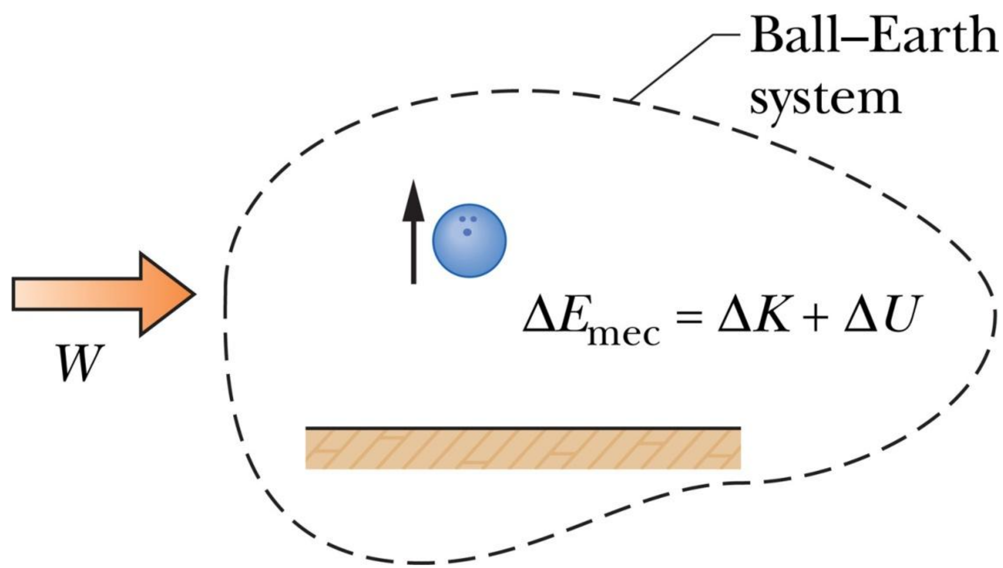
$$\frac{d^2U}{dx^2} < 0$$

외부 힘이 한 일



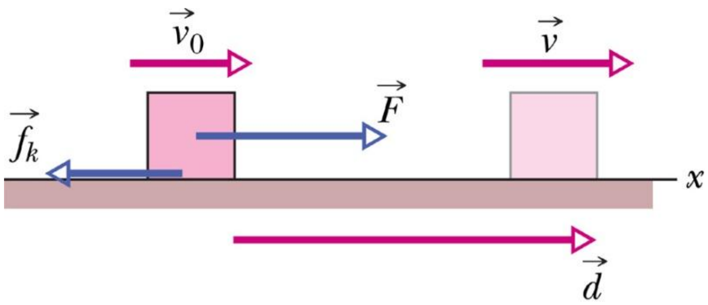
마찰력이 없는 경우

$$W = \Delta K + \Delta U = \Delta E_{\text{mech}}$$

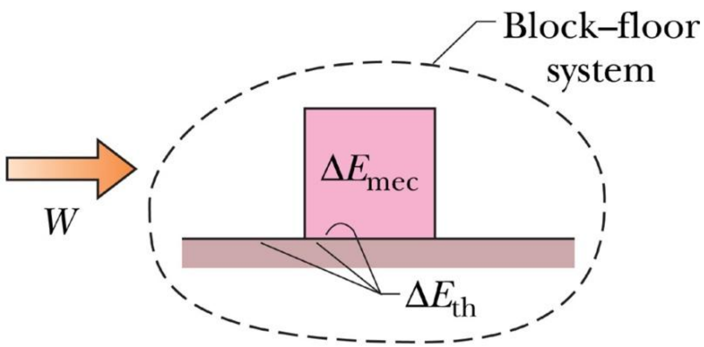


외부힘이 한 일 (마찰력이 있는 경우)

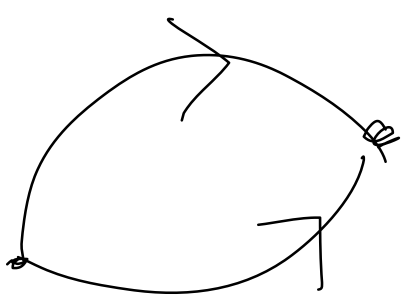
$Fd = mad + f_k d$



(a)



(b)



$$F - f_k = ma$$

$$v^2 - v_0^2 = 2ad$$

$$Fd = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 + f_k d$$

$$Fd = \Delta K + f_k d \rightarrow \Delta E_{\text{mech}} + f_k d$$

열에너지를 $\Delta E_{\text{th}} = f_k d$ 라고 정의하면

$$Fd = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$$

$$W = \Delta E_{\text{mech}} + \Delta E_{\text{th}}$$

토막

$$W_f = \Delta K + \Delta U + \Delta E_{in} = -f_k d + 0 + \Delta E_{in}$$

$$\therefore\ W_f \neq -f_k d$$

마찰력이 한 일중 일부는

토막의 내부에너지로 변환된다.

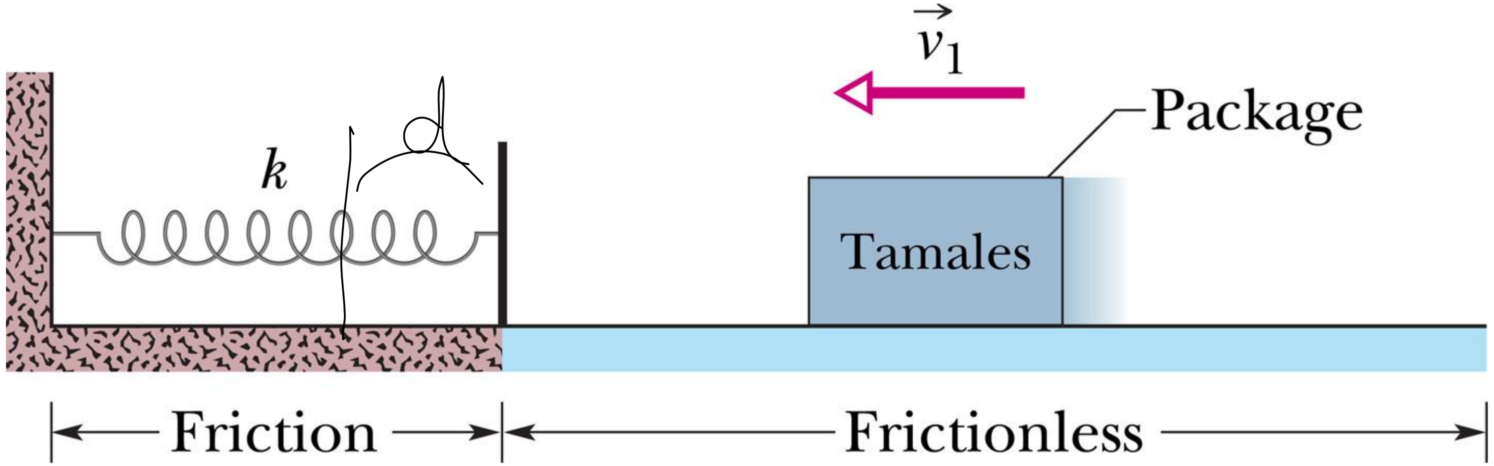
에너지 보존법칙

고립계에서 에너지는 한 형태에서 다른 형태로 변환될 수 있지만, 고립계의 총 에너지는 항상 보존된다.

$$\Delta E_t = \Delta K + \Delta U + \Delta E_{in} = 0$$

$$W = \Delta E = \Delta E_{\mathrm{mech}} + \Delta E_{\mathrm{th}} + \Delta E_{\mathrm{int}}$$

Sample problem



$m = 2.0\text{kg},\ v_1 = 4.0\text{m/s},\ F_f = 15\text{N},\ k = 10{,}000\text{N/m}$

$$E_1 = K_1 + U_1 = \tfrac{1}{2}mv_1^2 + 0$$

$$E_2 = K_2 + U_2 = 0 + \tfrac{1}{2}kd^2$$

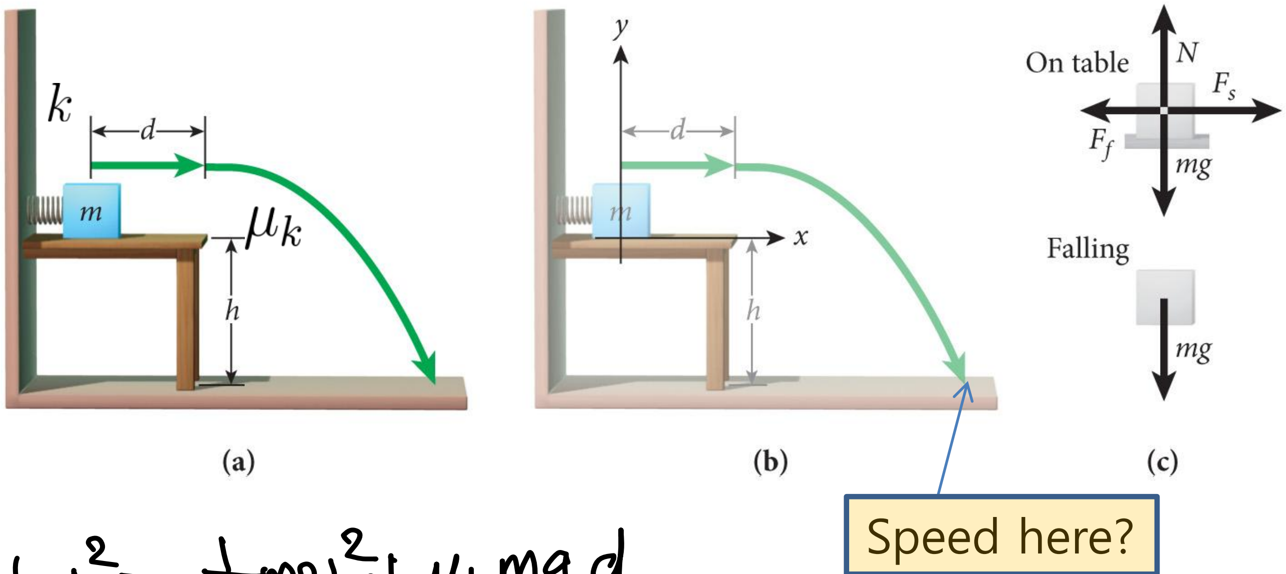
$$\Delta E_{th} = F_f d$$

$$E_2 = E_1 - \Delta E_{th}$$

$$5000d^2 + 15d - 16 = 0$$

$$\therefore d = 5.5\text{cm}$$

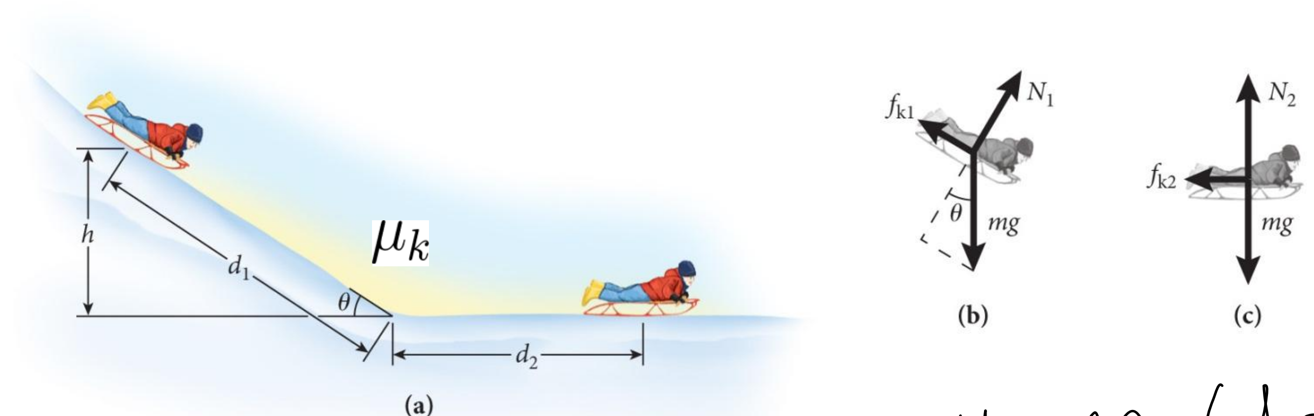
SP 6.3: block pushed off a table



$$\frac{1}{2}kd^2 = \frac{1}{2}mv^2 + \mu_k mgd$$

$$\frac{1}{2}mv^2 + mgh = \frac{1}{2}mv'^2$$

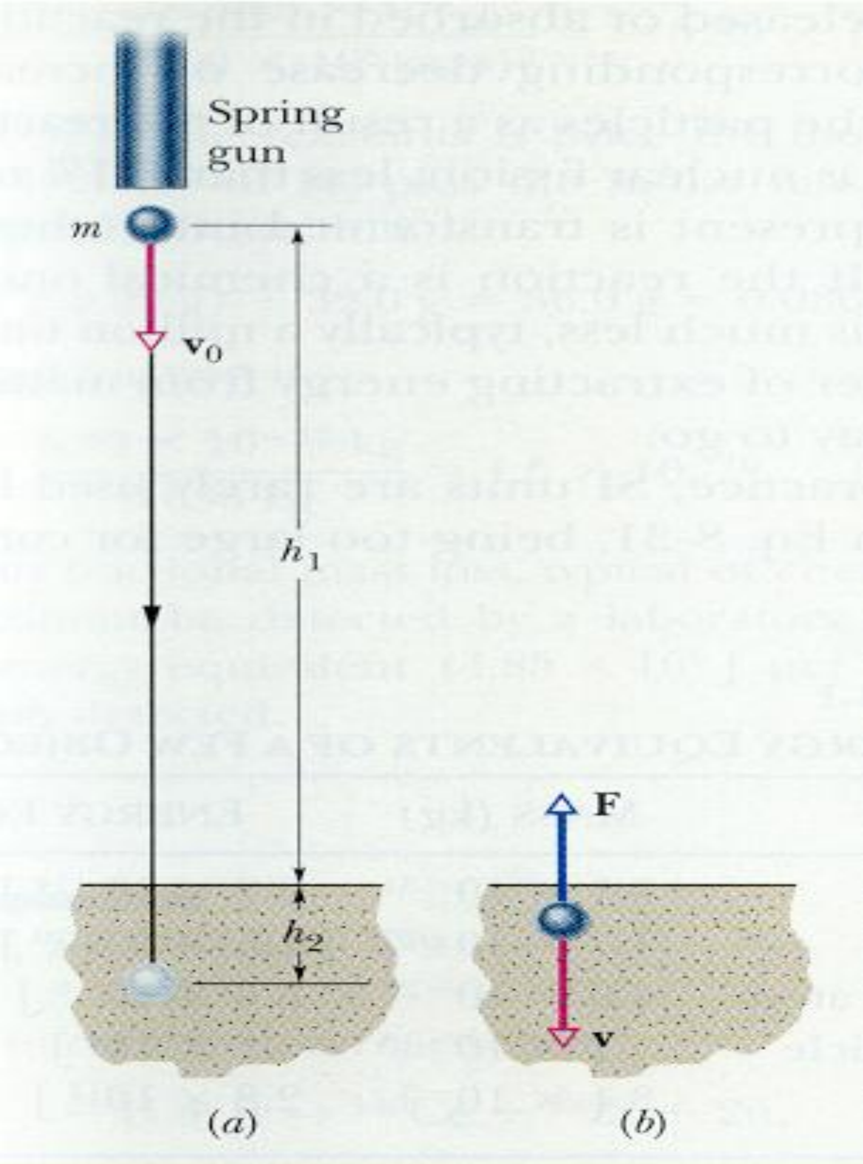
SP 6.5: Sledding on Mickey Mouse Hill



$$mgh = \frac{1}{2}mv^2 + \mu_k mg \cos\theta \, d_1 = \mu_k mg \, (d_2 + d_1 \cos\theta)$$

$$\frac{1}{2}mv^2 = \mu_k mg \, d_2 \qquad h = \mu_k (d_2 + d_1 \cos\theta)$$

$$d_2 = \frac{h}{\mu_k} - d_1 \cos\theta$$



$$m = 5.2\text{kg},\ \ v_0 = 14\text{m/s}$$
$$h_1 = 18\text{m},\ \ \ h_2 = 0.21\text{m}$$

Sample problem

(1) 총알의 mechanical E.

$$\Delta E = \Delta K + \Delta U$$
$$= (0 - \tfrac{1}{2} m v_0^2) - mg(h_1 + h_2)$$
$$= -1.44\text{J}$$

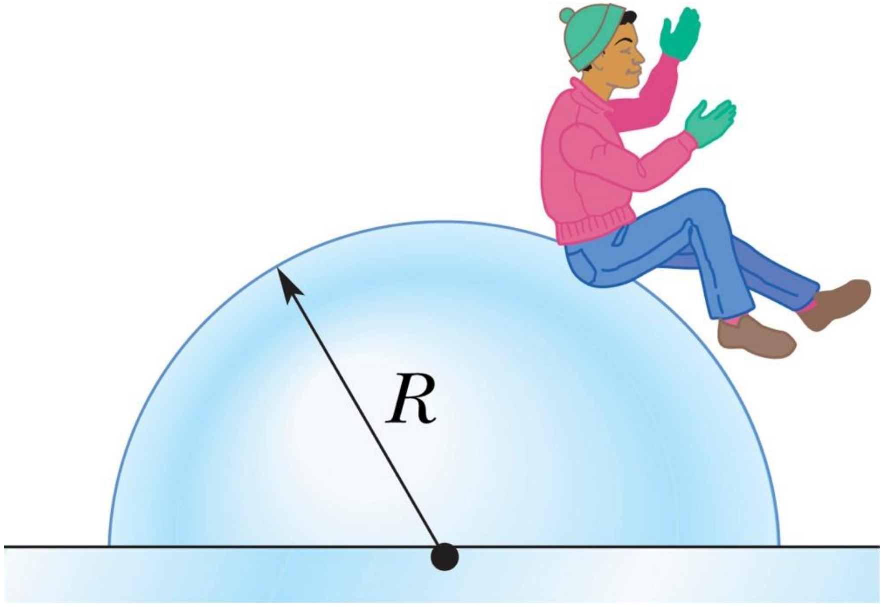
(2) 총알+지구+모래=고립계

$$\Delta E_t = \Delta E + \Delta E_{in} = 0$$
$$\therefore \Delta E_{in} = \Delta E = 1.44\text{J}$$

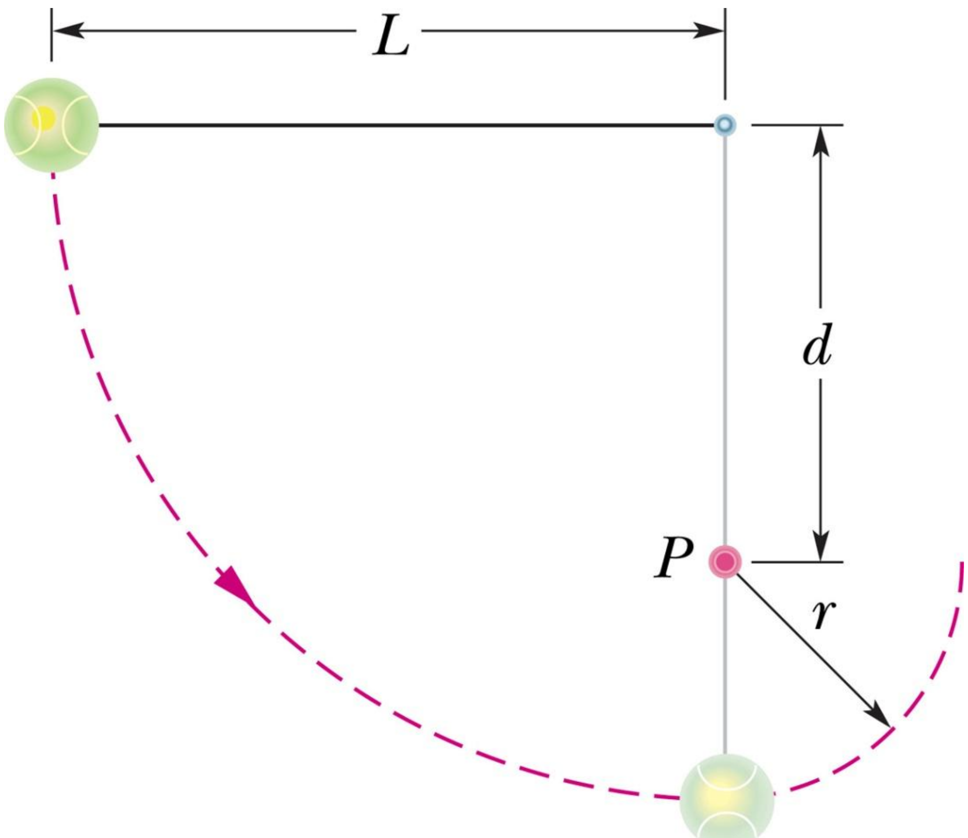
(3) 모래가 총알에 가한 힘

$$\Delta E = -fd$$
$$\therefore f = \frac{\Delta E}{-h_2} = 6.84\text{N}$$

얼음공에서 미끄러지기



최저점에서, 최고점에서의 속도



Ch. 7 Momentum and collisions

linear momentum

선운동량의 정의:

$$\mathbf{p} \equiv m\mathbf{v}$$

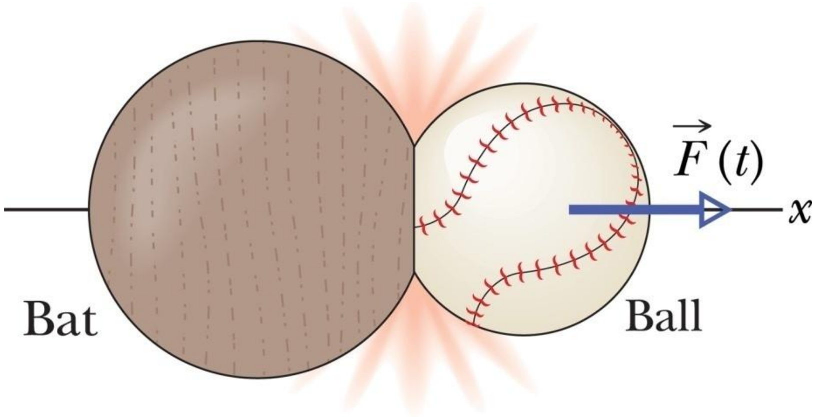
따라서 뉴턴의 방정식은

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt}$$

가 된다.

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

Collision and impulse



$$d\mathbf{p} = \mathbf{F}(t)dt$$

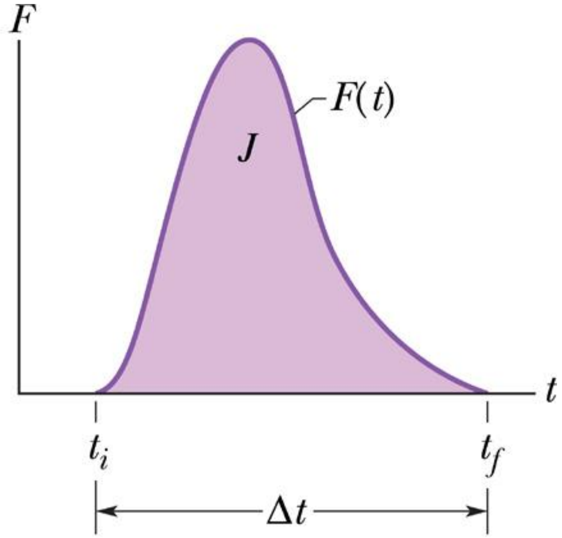
$$\Delta\mathbf{p} = \int_{t_i}^{t_f} d\mathbf{p} = \int_{t_i}^{t_f} \mathbf{F}(t)dt$$

impulse

$$\mathbf{J} = \int_{t_i}^{t_f} \mathbf{F}(t)dt$$

Dimension:

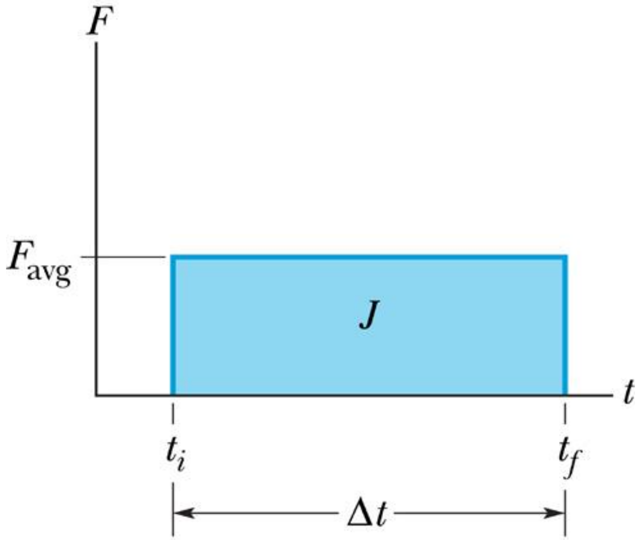
$$[J] = [F \cdot t] = MLT^{-2}T = MLT^{-1}$$



(a)

$$\Delta \mathbf{p} = \mathbf{p}_f - \mathbf{p}_i = \mathbf{J}$$

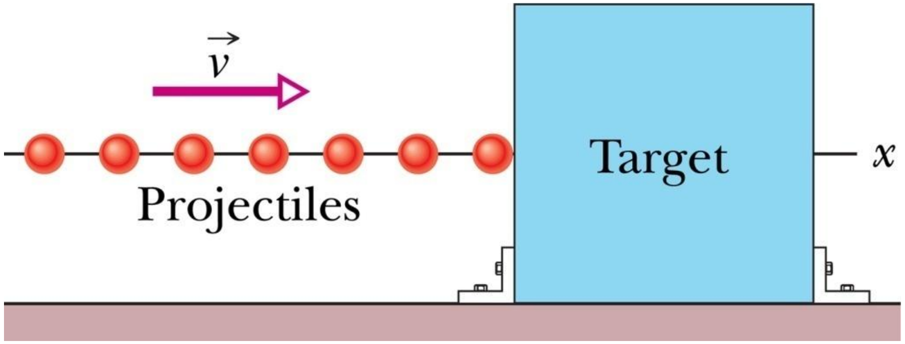
$$p_{fx} - p_{ix} = \int_{t_i}^{t_f} F_x dt$$



(b)

$$J = F_{\text{avg}} \Delta t$$

연속충돌



$$J = -n\Delta p$$

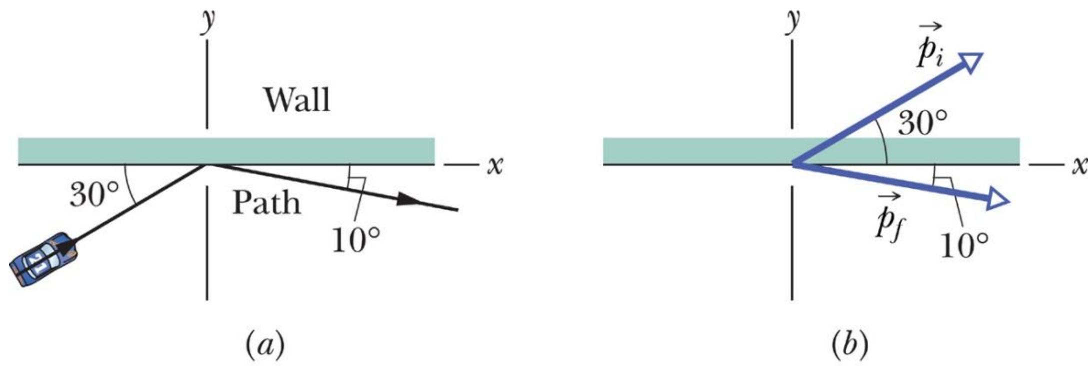
$$F_{\text{avg}} = \frac{J}{\Delta t} = -\frac{n}{\Delta t}\Delta p = -\frac{n}{\Delta t} m \Delta v$$

$$\Delta v = v_f - v_i = \begin{cases} 0 - v = -v, & \text{발사체가 붙을 경우} \\ -v - v = -2v, & \text{발사체가 튕겨 나올 경우} \end{cases}$$

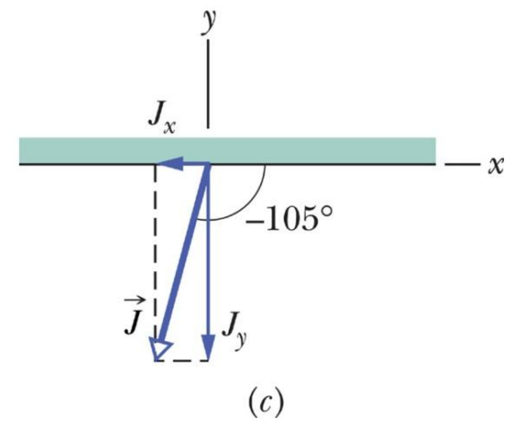
$$\Delta m = nm$$

$$F_{\text{avg}} = -\frac{\Delta m}{\Delta t}\Delta v$$

Sample prob.



$$v_i = 70 \text{ m/s},\ v_f = 50 \text{ m/s},\ m = 80 \text{ kg}$$



(a) 충격량 J

$$\mathbf{J} = \mathbf{p}_f - \mathbf{p}_i = m(\mathbf{v}_f - \mathbf{v}_i)$$

(b) 평균 힘 $\Delta t = 14 \text{ ms}$

$$F_{\text{avg}} = \frac{J}{\Delta t}$$

Conservation of linear momentum

외부의 알짜힘이 없으면 ($\mathbf{F}_{\text{net}} = 0$),

$$\frac{d\mathbf{P}}{dt} = 0$$

이다. 따라서 시간에 대해

$$\mathbf{P} = \text{일정}$$

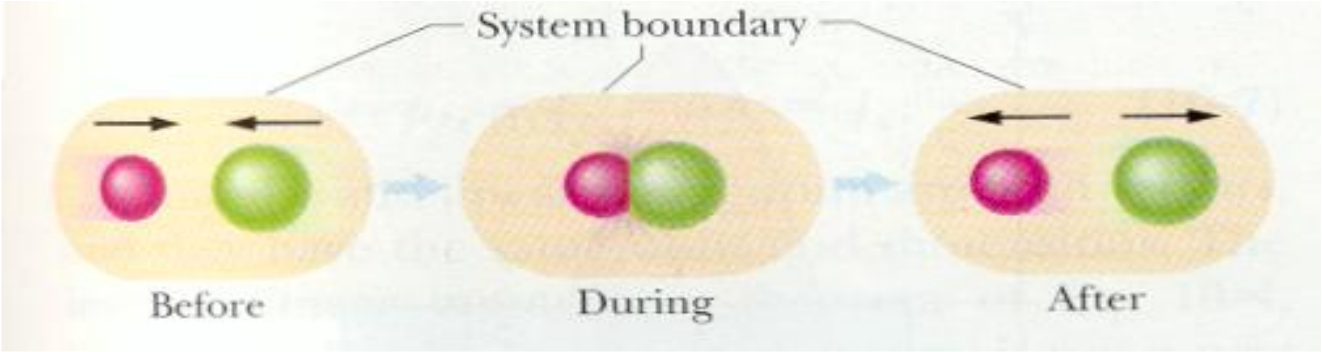
하고,

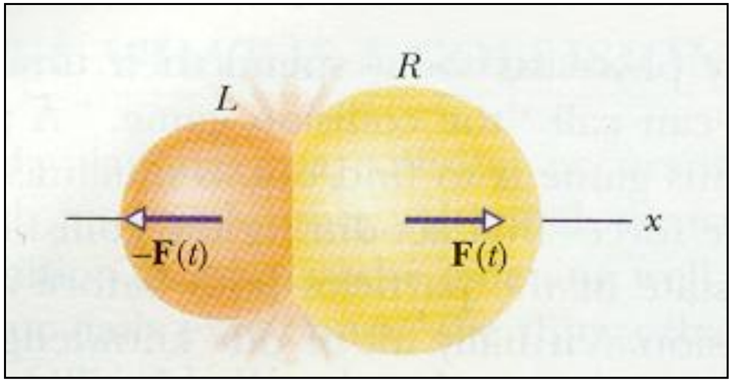
$$\mathbf{P}_i = \mathbf{P}_f$$

로 선운동량이 보존된다.

* 한 방향에 대해서만 외부의 힘이 없으면 그 방향의 운동량은 보존된다.

Collision





$$\Delta p_1 = \int_i^f \mathbf{F}_1(t)dt = -\int_i^f \mathbf{F}(t)dt$$

$$\Delta p_2 = \int_i^f \mathbf{F}_2(t)dt = \int_i^f \mathbf{F}(t)dt$$

$$\therefore\ \Delta p_1 + \Delta p_2 = \Delta P = 0$$

collision의 종류

(1) Elastic collision: 충돌 과정에서 운동에너지가 보존
(2) Inelastic collsion: 충돌과정에서 운동에너지가 보존되지 않음.

*완전 비탄성충돌: 두 물체가 붙어버리는 경우

충돌의 종류에 따라 운동에너지는 보존되거나 보존되지 않지만, 운동량은 항상 보존된다.