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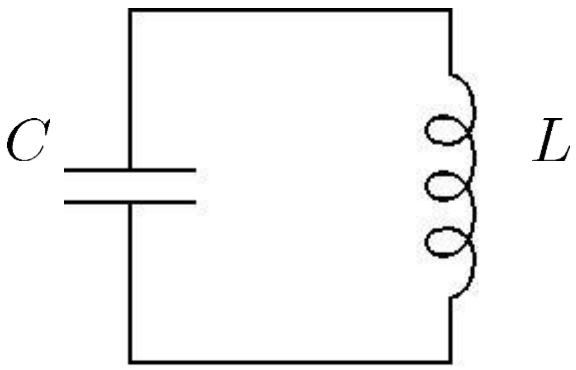
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8^{th} and 9^{th} Ed.
- The rest is made by me.

LC circuits



Energy stored in E and B fields

$$U_E = \frac{q^2}{2C}, \quad U_B = \frac{Li^2}{2}$$

Total energy is conserved.

electromagnetic oscillation

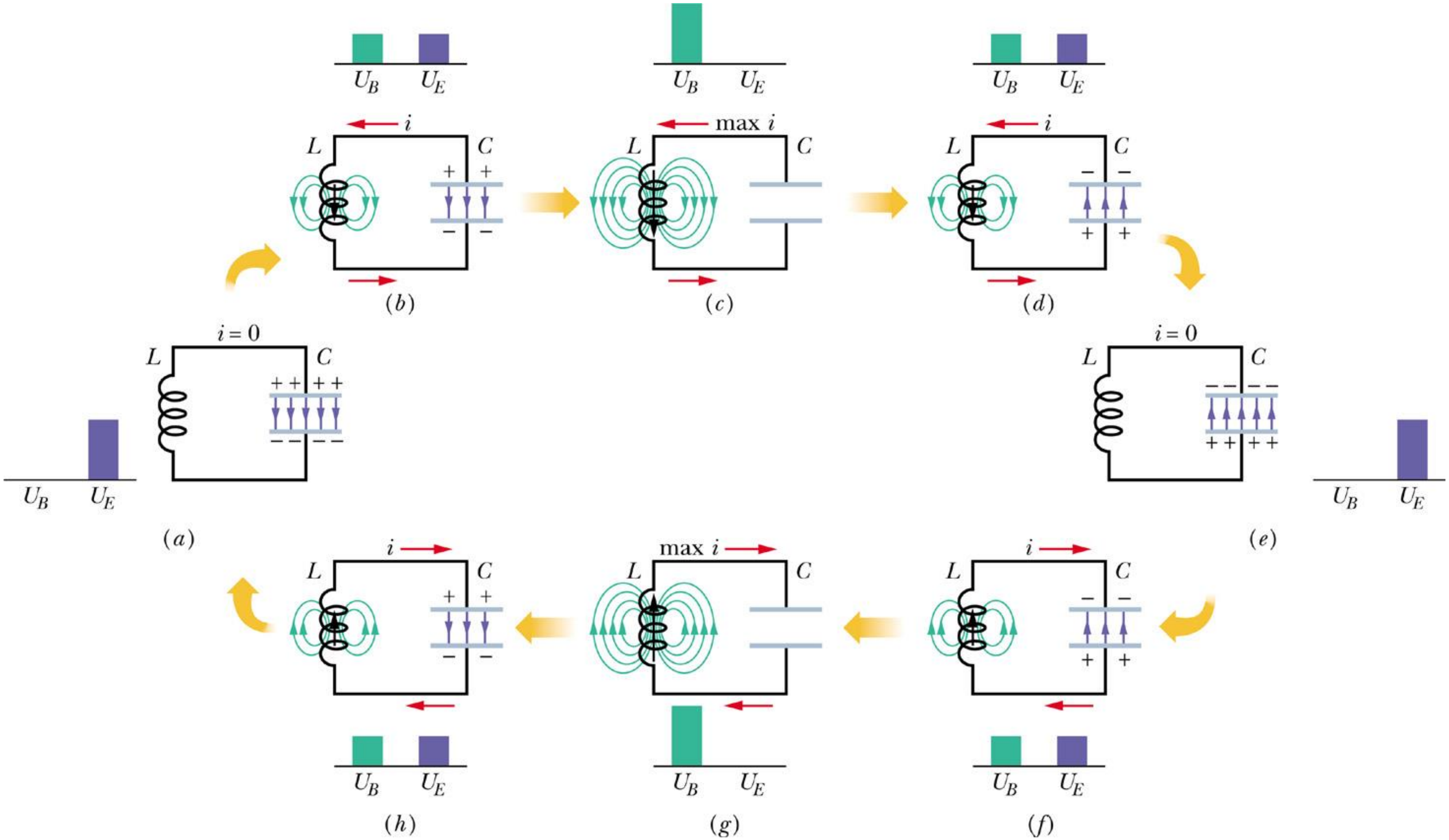
N. B. potential and kinetic energies of a spring

$$U = \frac{1}{2}kx^2, \quad T = \frac{1}{2}mv^2$$

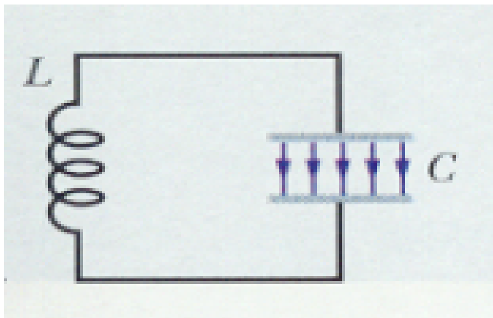
퍼텐셜에너지와 운동에너지 사이의 교환

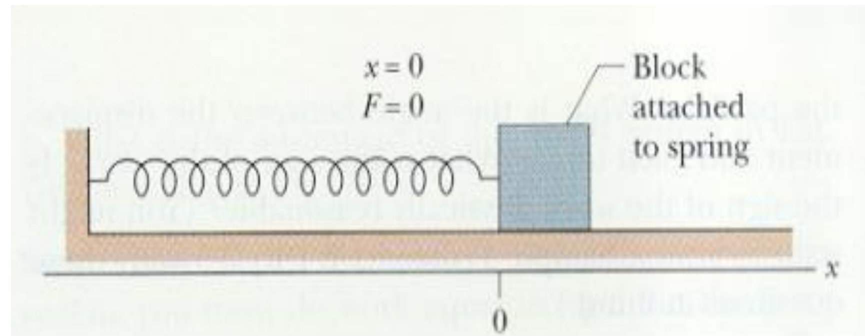
-> 용수철의 진동

LC oscillation



LC oscillator & simple harmonic oscillator





용수철 진동		LC 진동	
요소	에너지	요소	에너지
용수철	퍼텐셜 $kx^2/2$	축전기	전기 $q^2/(2C)$
질량	운동 $mv^2/2$	인덕터	자기 $Li^2/2$
$v = dx/dt$		$i = dq/dt$	
$\omega = \sqrt{k/m}$		$\omega = 1/\sqrt{LC}$	

Analysis of LC oscillations

1. 용수철 진동

$$\text{전체 에너지} \quad U = U_b + U_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

에너지가 보존되므로

$$\frac{dU}{dt} = 0 \longrightarrow \frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\frac{dv}{dt} + kx\frac{dx}{dt} = 0.$$

$$m\frac{d^2x}{dt^2} + kx = 0 \quad (\text{용수철의 진동 방정식})$$

$$x(t) = X\cos(\omega t + \phi)$$

2. LC 진동

에너지

전체 에너지는

$$U = U_B + U_E = \frac{Li^2}{2} + \frac{q^2}{2C}.$$

에너지는 보존되므로

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{Li^2}{2} + \frac{q^2}{2C}\right) = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = 0.$$

따라서 LC 진동에 대한 운동방정식은

$$L\frac{d^2q}{dt^2} + \frac{1}{C}q = 0$$

전하와 전류의 진동

$$\begin{aligned} q &= Q\cos(\omega t + \phi) \quad \text{전하} \\ i &= \frac{dq}{dt} = -\omega Q\sin(\omega t + \phi) = -I\sin(\omega t + \phi) \quad \text{전류.} \end{aligned}$$

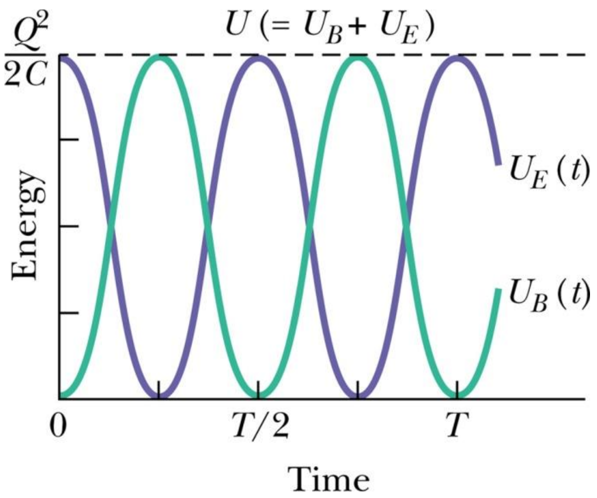
각진동수

$$\frac{d^2q}{dt^2} = -\omega^2 Q\cos(\omega t + \phi)$$

이므로,

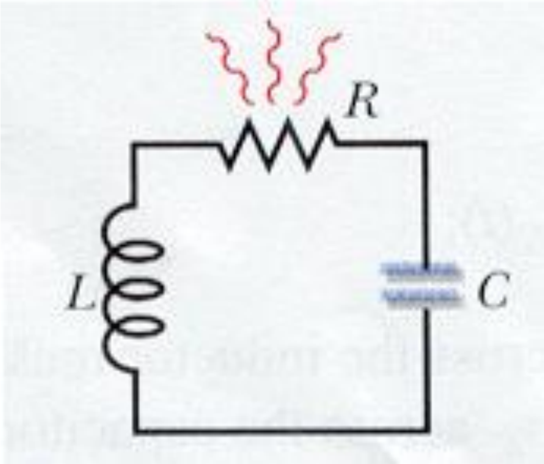
$$(-L\omega^2 + \frac{1}{C})Q\cos(\omega t + \phi) \longrightarrow \omega = \frac{1}{\sqrt{LC}}$$

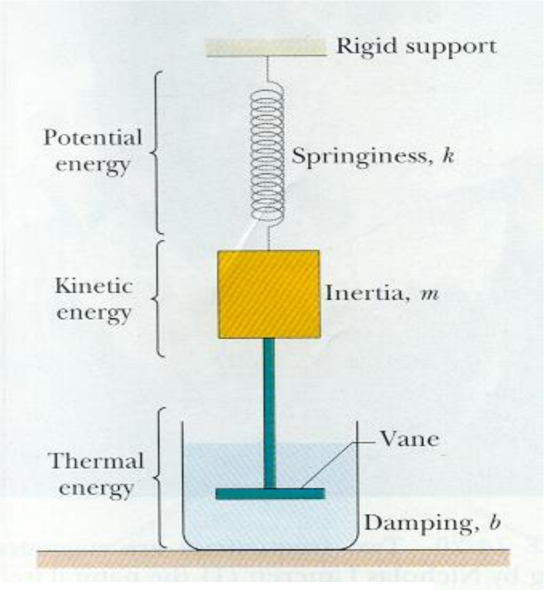
전기, 자기에너지의 진동



$$\begin{aligned} U_E &= \frac{q^2}{2C} = \frac{Q^2}{2C}\cos^2(\omega t + \phi), \\ U_B &= \frac{1}{2}Li^2 = \frac{1}{2}L\omega^2 Q^2\sin^2(\omega t + \phi) \\ &= \frac{Q^2}{2C}\sin^2(\omega t + \phi). \end{aligned}$$

RLC circuits & damped harmonic oscillator





$$U = U_B + U_E = \frac{1}{2}Li^2 + \frac{q^2}{2C}$$

한편 저항에서의 열에너지 손실은

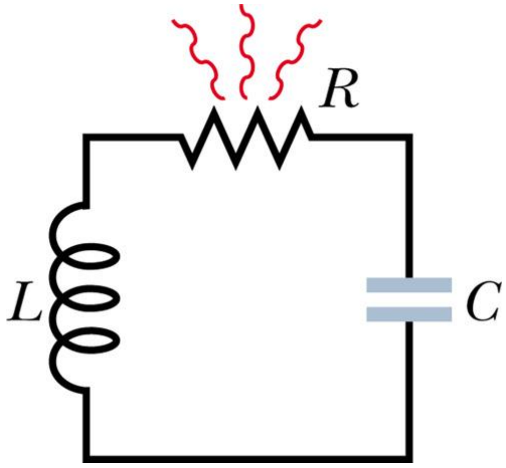
$$\frac{dU}{dt} = -i^2 R$$

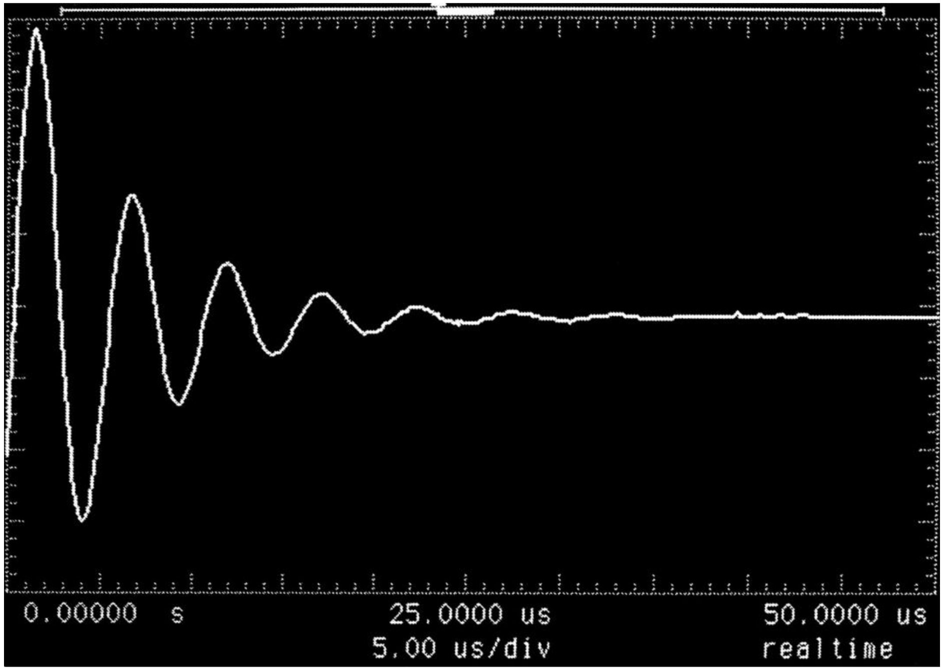
따라서

$$\frac{dU}{dt} = Li\frac{di}{dt} + \frac{q}{C}\frac{dq}{dt} = -i^2 R$$

$$L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = 0 \quad \text{RLC 회로}$$

RLC oscillation



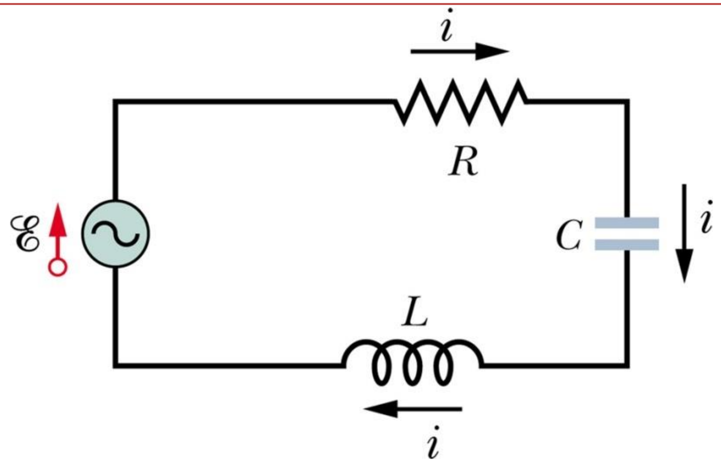


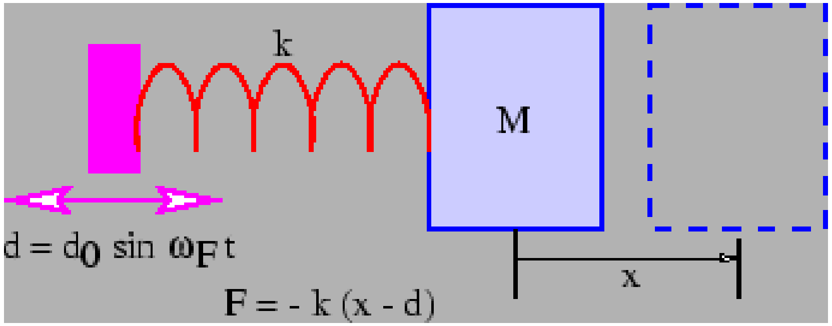
$$\frac{d^2q}{dt^2} + \frac{R}{L}\frac{dq}{dt} + \frac{1}{LC}q = 0$$

$$q(t) = Qe^{-\frac{R}{2L}t}\cos(\omega' t + \phi)$$

$$\omega' = \sqrt{\frac{1}{LC} - \left(\frac{R}{2L}\right)^2} < \omega = \frac{1}{\sqrt{LC}}$$

RLC forced oscillator and forced harmonic oscillator





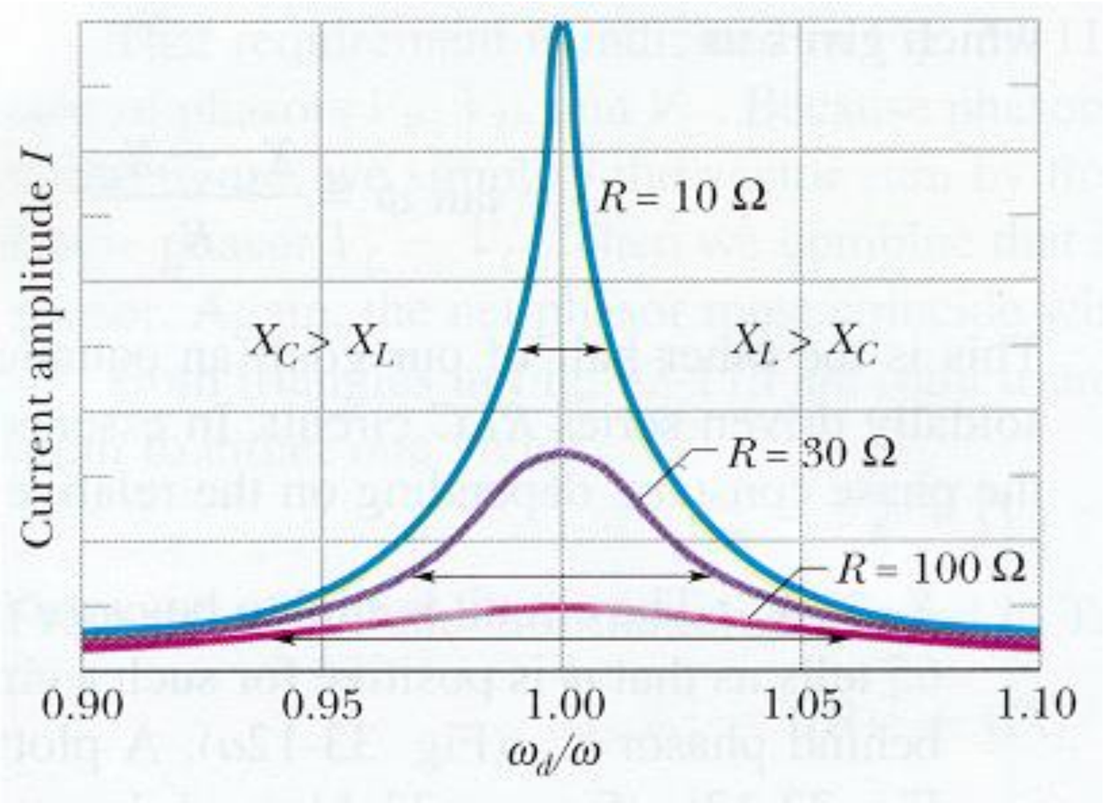
교류기전력

$$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$$

이 때 전류는

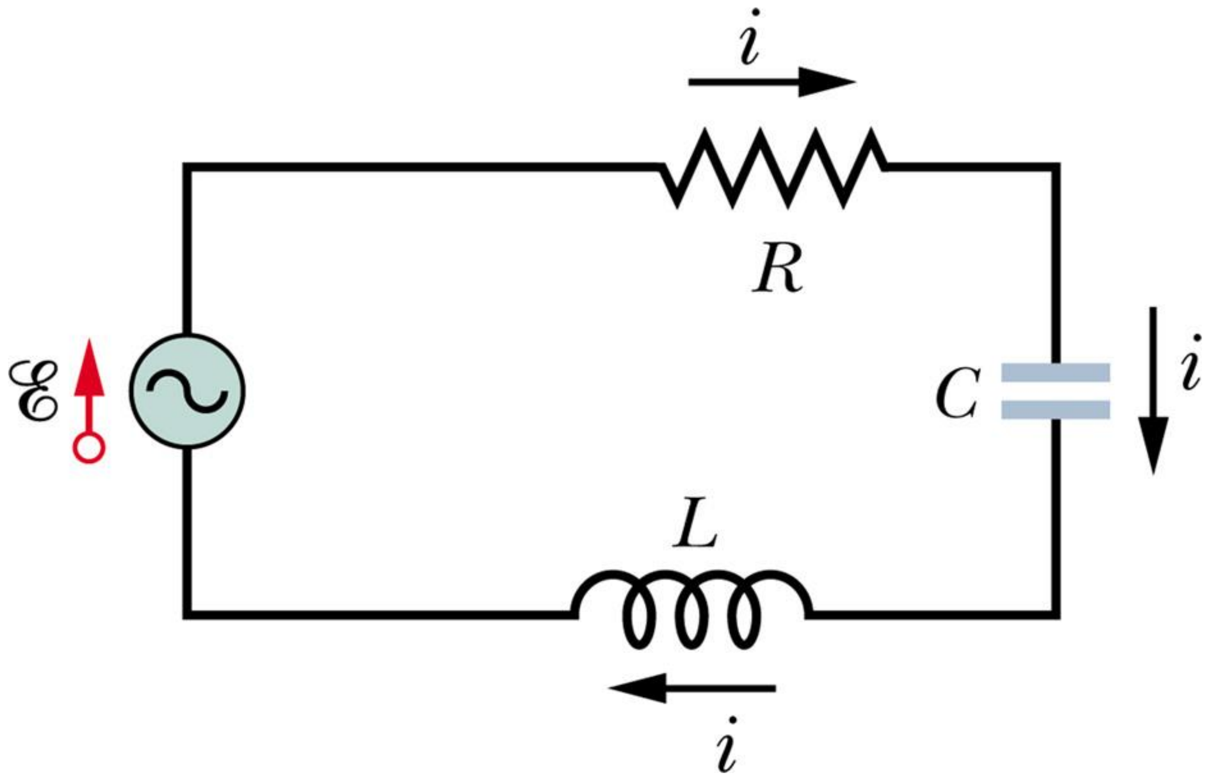
$$i = I \sin(\omega_d t - \phi)$$

로 쓸 수 있다.



$$\omega_d = \frac{1}{\sqrt{LC}} = \omega$$

RLC circuits

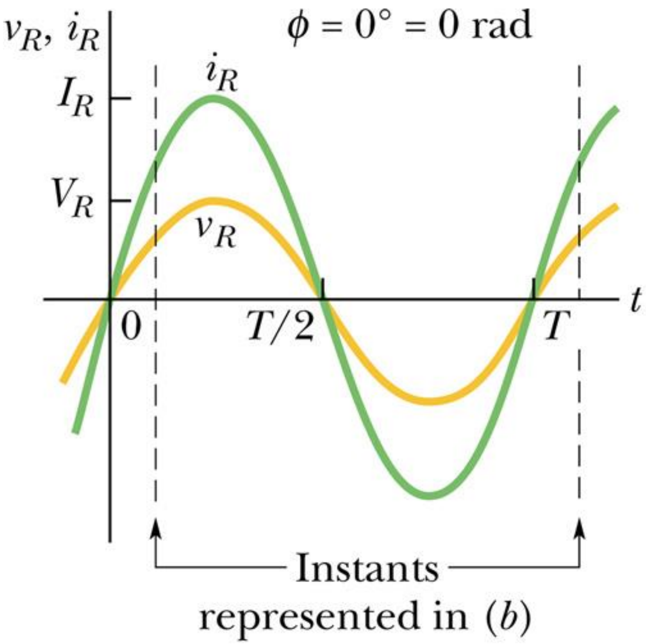


$\mathcal{E} = \mathcal{E}_m \sin \omega_d t$

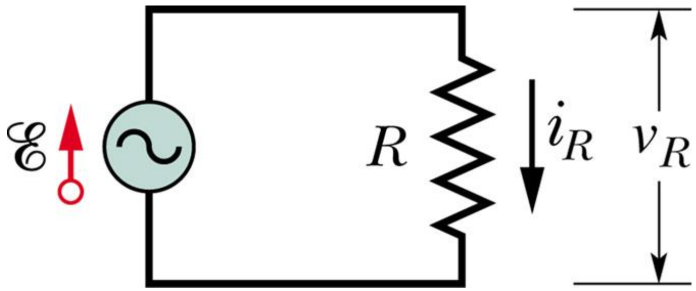
$i = I \sin(\omega_d t - \phi)$

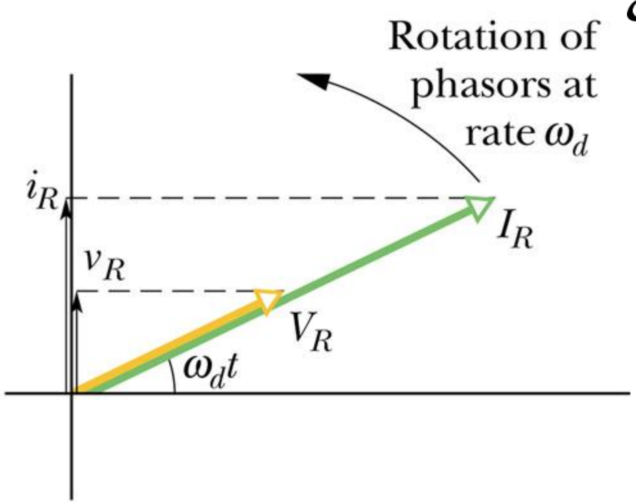
I와 ϕ구하기

Circuit with R



(a)





(b)

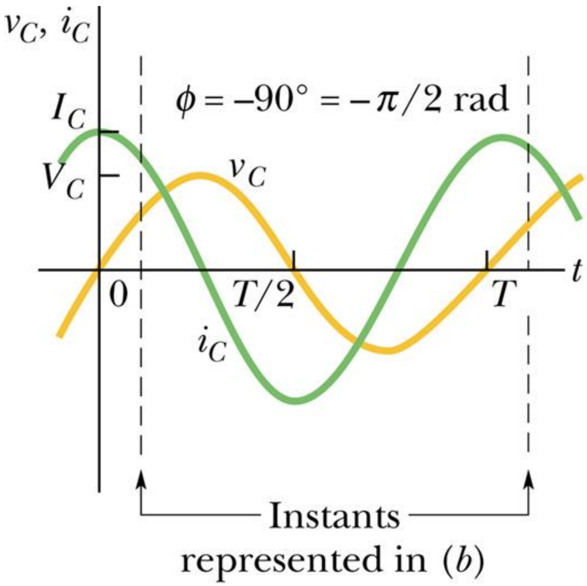
$$\mathcal{E} - v_R = 0 \longrightarrow v_R = \mathcal{E}_m \sin \omega_d t = V_R \sin \omega_d t$$

$$i_R = \frac{v_R}{R} = \frac{V_R}{R} \sin \omega_d t \rightarrow \phi = 0$$

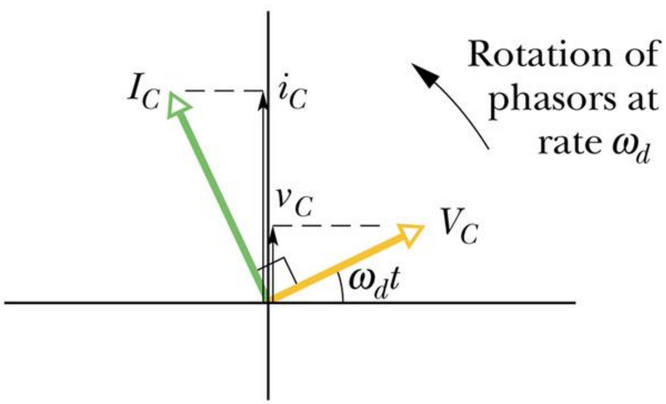
$$i_R = I_R \sin(\omega_d t - \phi)$$

$$V_R = I_R R$$

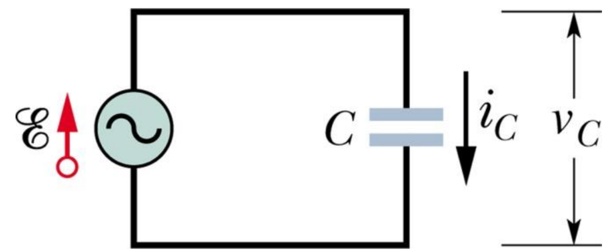
Circuit with C



(a)



(b)



$$v_C = V_C \sin \omega_d t$$

$$q_C = C v_C = C V_C \sin \omega_d t$$

$$i_C = \frac{dq_C}{dt} = \omega_d \cos \omega_d t$$

축전기형 저항 (capacitive resistance)

$$X_C = \frac{1}{\omega_d C}$$

$$\cos \omega_d t = \sin(\omega_d t + 90°)$$

$$i_C = \frac{V_C}{X_C} \sin(\omega_d t + 90°) = I_C \sin(\omega_d t - \phi) \longrightarrow \phi = -\frac{\pi}{2}$$

$$V_C = I_C X_C$$

Circuit with L

관계 ID가 rId3인 이미지 부분을 파일에서 찾을 수 없습니다.

관계 ID가 rId3인 이미지 부분을 파일에서 찾을 수 없습니다.

RLC 회로

관계 ID가 rId3의 이미지 부분을 파일에서 찾을 수 없습니다.

관계 ID가 rId3의 이미지 부분을 파일에서 찾을 수 없습니다.

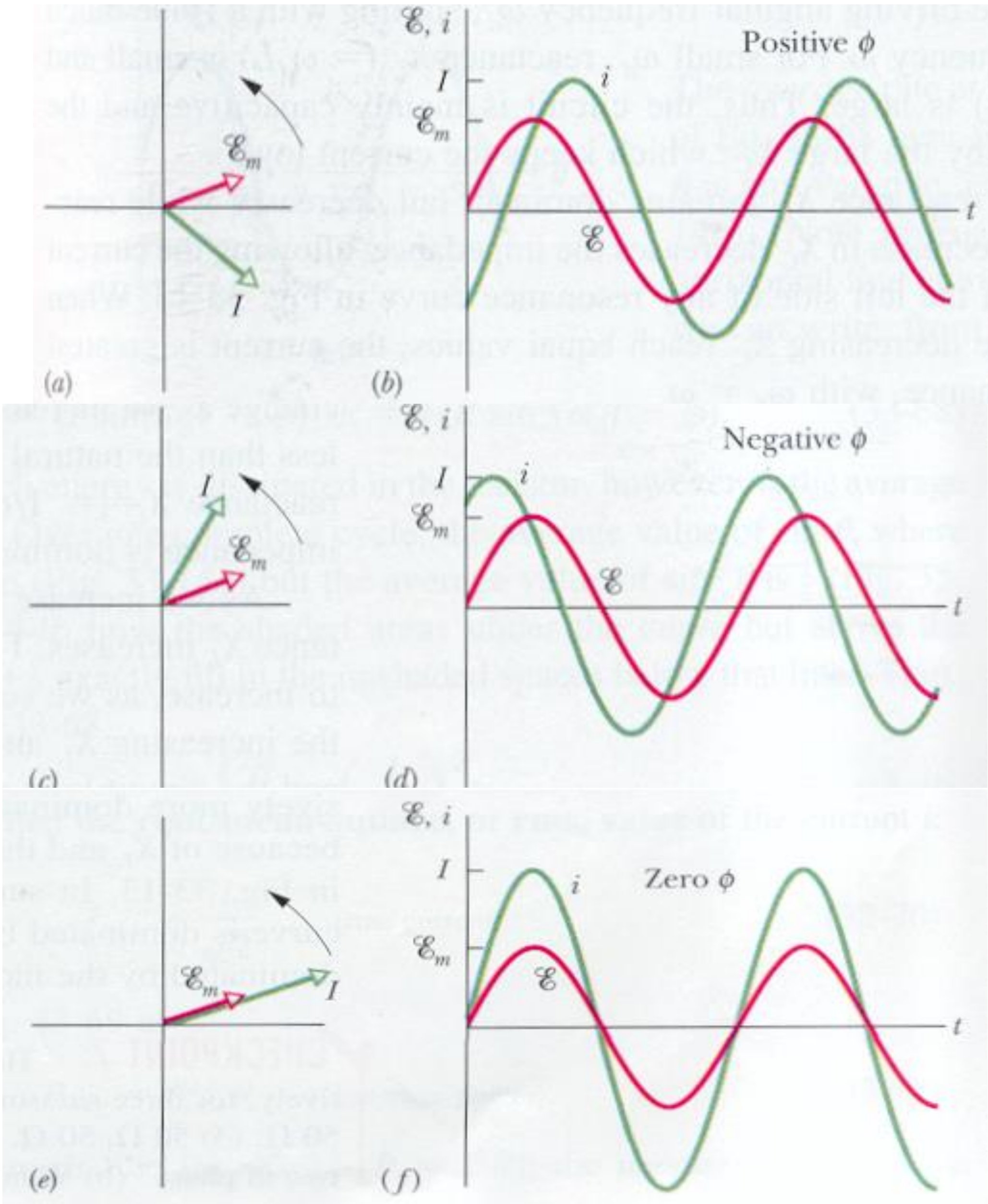
Series RLC circuit

impedance $$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{\mathcal{E}_m}{Z} = \frac{\mathcal{E}_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{X_L - X_C}{R}$$

Phase constants and resonance



$$X_L > X_C$$

$$X_L < X_C$$

$$X_L = X_C$$

phase

$$\tan\phi = \frac{V_L - V_C}{V_R} = \frac{IX_C - IX_L}{IR} = \frac{X_L - X_C}{R}.$$

공명현상 (resonance)
흐르는 전류가 최대일 조건

$$\omega_d = \omega = \frac{1}{\sqrt{LC}}$$