Rubrics

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What is a **problem**?

- "A mathematical problem, to be effective, should confront the student with an authentic but manageable difficulty – meaning that the question itself must both be well understood and seem significant but that the solution must be unknown" (Sfard, 2001)

Sfard(2001). Balancing Unbalanceable: The NCTM Standards in Light of Theories of Le arning Mathematics. Reston, VA: NCTM.

What is a **problematic task**?

- "the word **problematic** can refer to a situation in which students are helped to see that their present knowledge is as yet insufficient to satisfy certain genuine needs or to answer questions that emerge from what is already known" (Sfard, 2001)

Sfard(2001). Balancing Unbalanceable: The NCTM Standards in Light of Theories of Learning Mathematics. Reston, VA: NCTM.

A framework for introductory calculus in Israel

General goal:

To get acquainted with families of functions.

Problem:

The known methods of investigating functions (build a table, draw a graph) do not seem to work for more complex functions (often not even with computers!).

Subgoal:

To find a new, more reliable, analytical tool for investigating functions.

A promising phenomenon:

Most graphs turn out to be almost linear when watched "closely" (small neighborhood of a point is greatly enlarged)—the existence of a linear approximation.

Questions to be answered:

1. Is the phenomenon above common enough to deserve attention?
2. If the answer to question 1 is yes, how shall we find linear approximations?
3. When we know how to find linear approximations, how shall we use them to investigate functions?

Sfard(2001). Balancing Unbalanceable: The NCTM Standards in Light of Theories of Learning Mathematics. Reston, VA: NCTM.

RUBRICS
(EVALUATION STANDARDS)

In making **evaluation standards**

- Holistic rubric
- Analytical rubric

Holistic Rubric

Generally Holistic rubric

(Malone, 1980)

- 0 (Noncommencement): The student is unable to begin the problem or hands in work that is meaningless.
- 1 (Approach): The student approaches the problem with meaningful work, indicating some understanding of the problem, but an early impasse is reached.
- 2 (Substance): Sufficient detail demonstrates that the student has proceeded toward a rational solution, but major errors or misinterpretations obstruct the correct solution process.
- 3 (Result): The problem is very nearly solved; minor errors produce an invalid final solution.
- 4 (Completion): An approach method is applied to yield a valid solution.

Malone et al.(1980). Measuring problem-solving ability. Reston, VA: NCTM.

At a certain restaurant, two salads and three burgers cost $8.85. One salad and four burgers cost $9.30. What is the cost of one salad?

- 0: No mathematically meaningful work is present.
- 1: The answer is correct, but no setup or explanation of any work is shown or at least one correct step was made toward a solution
- 2: The setup is correct and at least one other correct and useful step is taken, but the solution contains at least one conceptual error or…
- 3: Method or setup suggests that a correct solution is possible. The strategy used is appropriate, and the explanation is included. The solution might contain a minor arithmetic mistake, but it has no conceptual errors.
- 4: Obtains a correct solution, regardless of method. Sufficient evidence or explanation of work must be shown.

Malone et al.(1980). Measuring problem-solving ability. Reston, VA: NCTM.

Problem 1

Imagine that one of your students shows you the following strategy for subtracting whole numbers.

$$\begin{array}{r} 37 \\ -\ 19 \\ \hline -\ 2 \\ \underline{20} \\ 18 \end{array}$$

How do you think the student would use this strategy in the problem below?

$$\begin{array}{r} 423 \\ -167 \end{array}$$

Holistic rubric for Problem 1

Score	Criteria
4	Correct generalization of the strategy and correct answer
2	Correct generalization of the strategy but computational error
0	Incorrect generalization of strategy

Problem 1 Holistic Rubric with data

Score	Criteria
4	Correct generalization of the strategy and correct answer
2	Correct generalization of the strategy but computational error
0	Incorrect generalization of strategy

Correct and best	4	423 −167 -4 -40 300 256
Correct, not best	4	423 −167 -44 300 256 / 423 −167 -4 260 256
Correct, but computational error	2	423 −167 -4 -40 300 266 / 423 −167 -5 -40 300 255
Sign errors	0	423 −167 -4 40 300 336
Other incorrect generalization	0	423 −167 -2 168 256
No idea	0	I have no idea
No response	0	

Problem 2

Write two division story problems that illustrate $24 \div 6$. The problems should be based on two different views or models of division.

Holistic rubric for Problem 2

Score	Criteria
8	4 points for each correct problem as long as they use different models (partitive, measurement, and inverse operator models)
4	Only one view/model
0	No correct model

Problem 2 Holistic rubric with data

Category	Score	Example
Two different models	8	1. **Two correct problems** (partitive, measurement, and inverse operator models) e.g.) Problem 1: In the team marathon, 6 members of the team run equal distance over the 24 mile course. How far does each team member run? (*Sharing*) Problem 2: Bob has to take a pill every six hours. How many pills will Bob take in a 24 hour period? (*Measurement*)
	8	2. **Two correct problems from discrete (set) model and continuous (measurement) model** e.g.) Problem 1: In the team marathon, 6 members of the team run equal distance over the 24 mile course. How far does each team member run? (*Set model*) Problem 2: Jim drove 24 miles from here to his home for 4 hours. What was his average distance per hour?
	6	3. If attempt at sharing division does **not mention explicitly that sharing is equal**. e.g.) Problem 1: Sue baked 24 cookies. She gave them to 6 of her friends. How many cookies did each friend get? Problem 2: Joe had 24 balloons. He put them into groups of 6. How many groups of balloons were there?

Problem 2 Holistic rubric with data

Only One correct model	4	**4. Two different stories only in the same model** e.g.1) Two different stories only in sharing model Problem 1: At dinner molly's mother made a 24-piece pie. She was making dinner for 6 people. If everyone gets an equal amount of pie, how many slices does each person get? (*Sharing*) Problem 2: There are 6 different kinds of dogs at the kennel. Three are 24 total dogs. How many dogs of each kind are there at the kennel? (*Sharing*) e.g.2) Two different stories only in set model Problem 1: At dinner molly's mother made a 24-piece pie. She was making dinner for 6 people. If everyone gets an equal amount of pie, how many slices does each person get? (*Sharing*) Problem 2: There are 6 different kinds of dogs at the kennel. Three are 24 total dogs. How many dogs of each kind are there at the kennel? (*Sharing*)
	4	**5. One correct answer and the other wrong one** e.g.) Problem1: We have 24 cookies and six students. How many cookies does each student receive? (*Sharing division*) Problem 2: You spend six hours of the 24 hour day watching TV. How many hours are you doing other things? (*Wrong*)
	4	**6. Only one correct story problem** e.g.) Problem 1: There are 24 apples. Every 6th apple is green. How many green apples are there total? (*Measurement*)
	2	**7. One partially correct story problem** e.g.) Problem 1: Susan made 24 cookies for her 6 children. How many cookies will each child get? Problem 2: Mandy brought some cars over for her friends to play with. Mandy gave her 6 friends 4 cars each. How many cars did Mandy bring over?
Incorrect answer	0	**8. A wrong story problem** e.g.) Problem 1: There are 24 people on the train. 6% are Asian. How many Asians are on the train? (*Wrong*)

Revised Holistic rubric for Problem 2

Score	Criteria
8	4 points for each correct problem as long as they use different models (partitive, measurement, and inverse operator models)
6	If attempt at sharing division does not mention explicitly that sharing is equal (Subtract 2 points)
4	Only one correct view/model
2	One partially correct story problem
0	No correct model

Problem 3

Below is one student's solution to finding a sum.

$$\begin{array}{r} 258 \\ +389 \\ \hline 17 \\ 130 \\ 500 \\ \hline 647 \end{array}$$

How could you use this student's method to help your class understand the standard addition algorithm?

Holistic rubric for Problem 3

Score	Criteria
10	**A model explanation:** The explanation describes clearly a. what the "standard algorithm" entails (for 258 + 359 or some other multi-digit sum), b. what the student's solution entails (using place value or expanded form & finding partial sums) c. how the steps in (a) and (b) are related and some reasonable suggestion for helping the student see this relation. To get full credit the response must also mathematical vocabulary such as expanded form or place value, correctly.
8	**A good explanation:** The explanation has almost all of the characteristics of a model solution. However, one part is missing or there is some small error in use of language.
6	**A fair explanation addressing both standard and non-standard algorithm:** The explanation indicates some understanding of both the given student's solution and the standard algorithm, but does not explain how they are related. For instance, the response might show another instance of multi-digit addition calculated with the two methods, but there is little or no use of technical vocabulary and little or no explanation of how the two algorithms are related.
4	**A fair explanation addressing only one of the algorithms:** Some progress is made toward a solution, but it does not address both algorithms. For instance the response might have a fairly detailed explanation of the algorithm shown, but not say much about the standard algorithm or its relation to the illustrated algorithm. Ex: *This strategy uses the expanded form of the numbers to add them. For example,(200 + 50 + 8) + (300 + 80 + 9), 8 + 9 = 17, 50 + 80 = 130, 200 + 300 = 500 -> 17 + 130 + 500 = 647.* *This strategy helps students understand place value and could be useful to explain regrouping in the standard algorithm.*
2	**A poor explanation:** There is some entry into the problem, as evidenced by at least one correct relevant mathematics statement. But there is no chain of reasoning related to the question asked. Ex 1. *You could use this algorithm to show place value.* Ex 2: *This method is useful because it does addition much like the standard algorithm except it avoids the carrying/rebundling, which can be difficult for children to understand. This method also helps to show why we carry, which could help a student gain a better understanding of the standard algorithm.*
0	**A completely incorrect explanation or No explanation:**

Third Miscellaneous Item

- Develop a holistic rubrics of TWO problem that you selected in the mid-term exam. Please make sure that there is no between score
- See the examples
- By **November 6th**

References

- Malone, J. A., Douglas, G. A., Kissane, B. V., & Mortlock, R. S. (1980). Measuring problem-solving ability. In S. Krulik & R. E. Reys (Eds.), *Problem solving in school mathematics* (pp. 204—215). Reston, VA: NCTM.
- Sfard, A. (2001). Balancing Unbalanceable: The NCTM Standards in Light of Theories of Learning Mathematics. In J. Kilpatrick, Martin, G., & Schifter, D. (Eds.), *A Research Companion for NCTM Standar ds* (pp. 353-392). Reston, VA: NCTM.