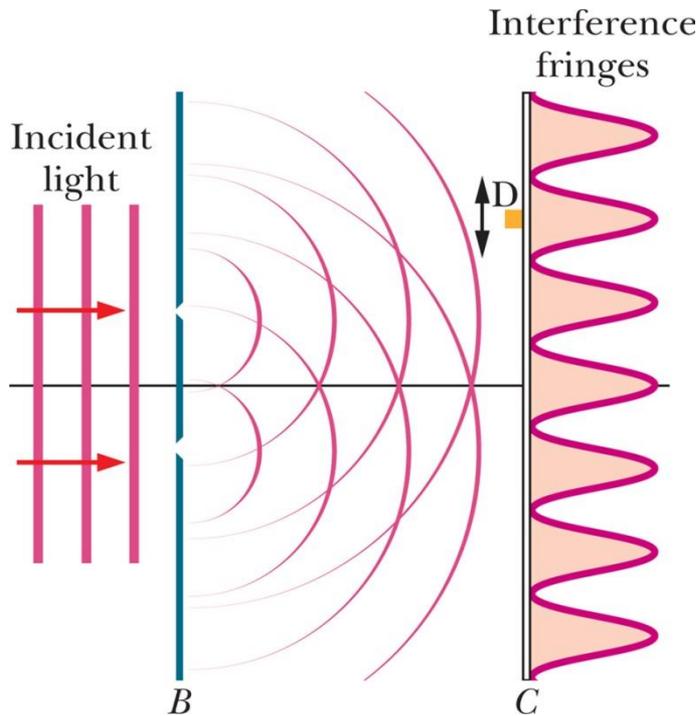


Copyright statement

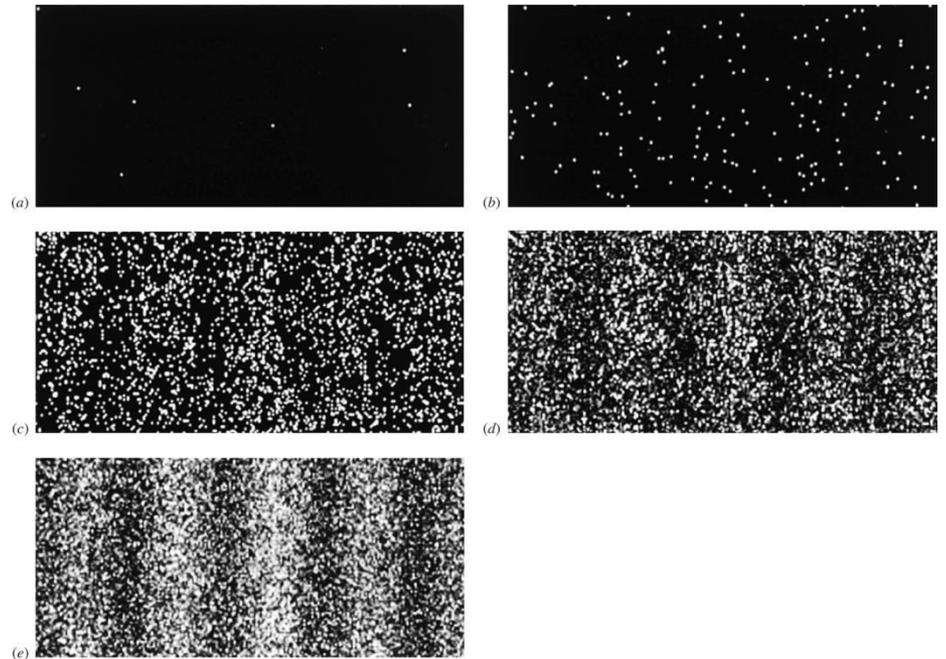
- The images and the pictures in this lecture are provided by the CDs accompanied by the books
 1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
 2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

빛의 파동적 성질

(1) 기본적인 실험



(2) 단일 광자 실험



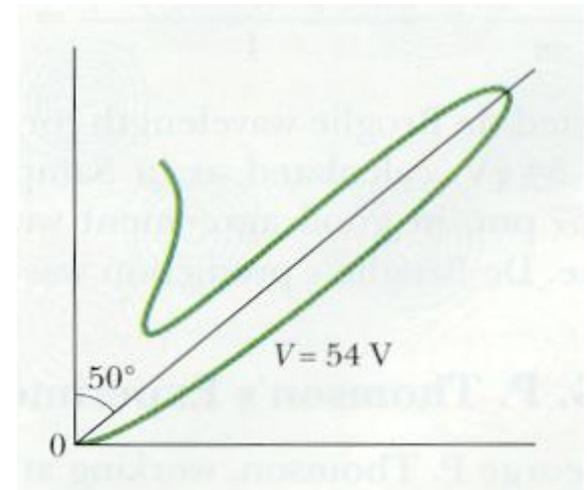
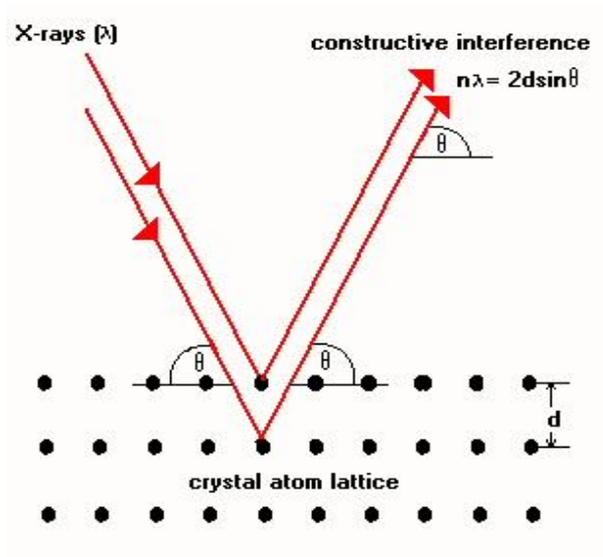
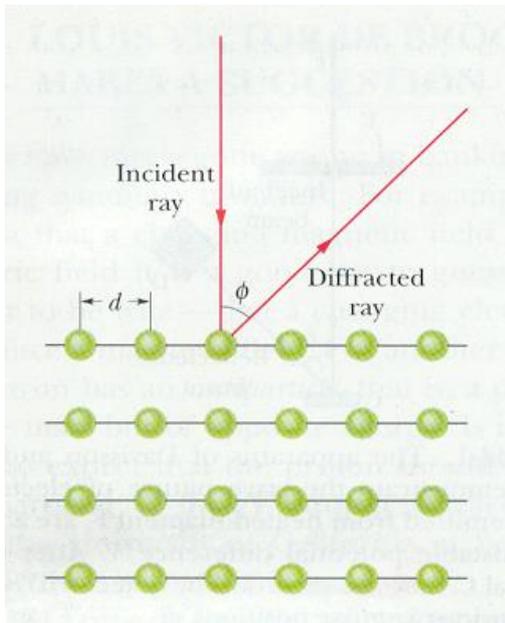
광자를 발견할 확률

$$P \propto I \propto E^2$$

전자의 파동적 성질

Davisson과 Germer의 실험

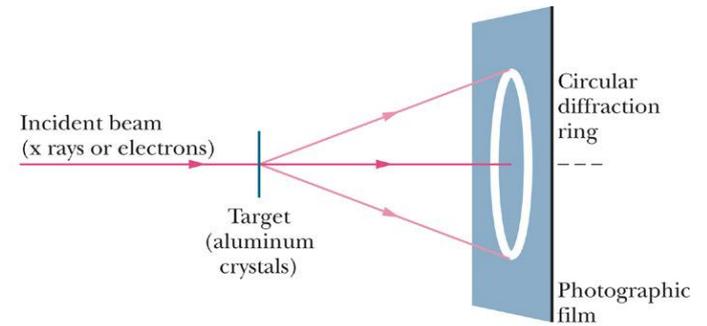
$$p = \frac{h}{\lambda}$$



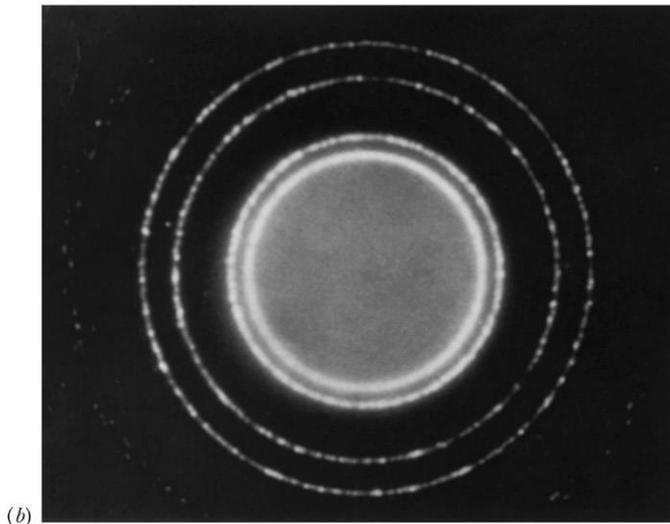
Baseball: $v = 35\text{m/s}$, $m = 150\text{g}$

$$\lambda = \frac{h}{p} = \frac{h}{mv} = 1.26 \times 10^{-34} \text{ m}$$

전자의 파동적 성질 2 $p = \frac{h}{\lambda}$

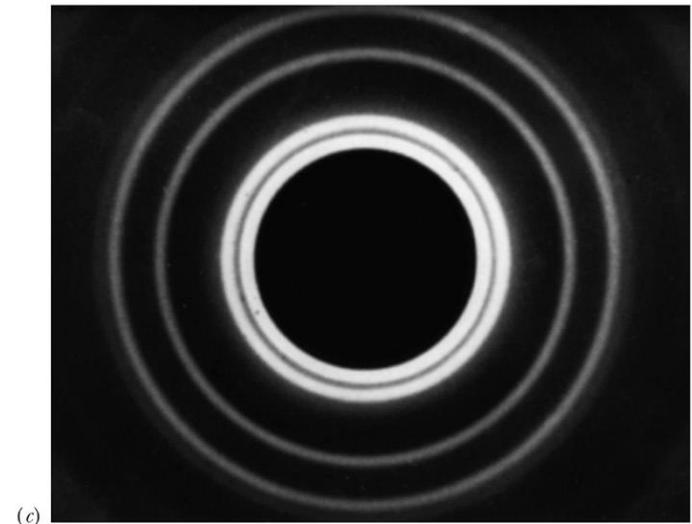


(a)



(b)

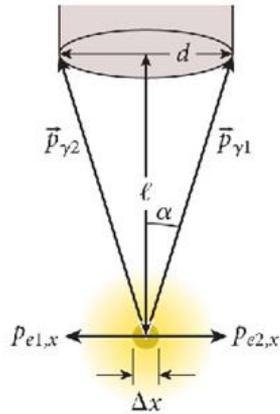
엑스선



(c)

전자

Uncertainty principle



$$\Delta x = \frac{\lambda}{2 \sin \alpha} \approx \frac{\lambda l}{d}$$

$$\Delta p_x = 2 |p_{\gamma 1,x}| = \frac{2h \sin \alpha}{\lambda} \approx \frac{h}{\Delta x}$$

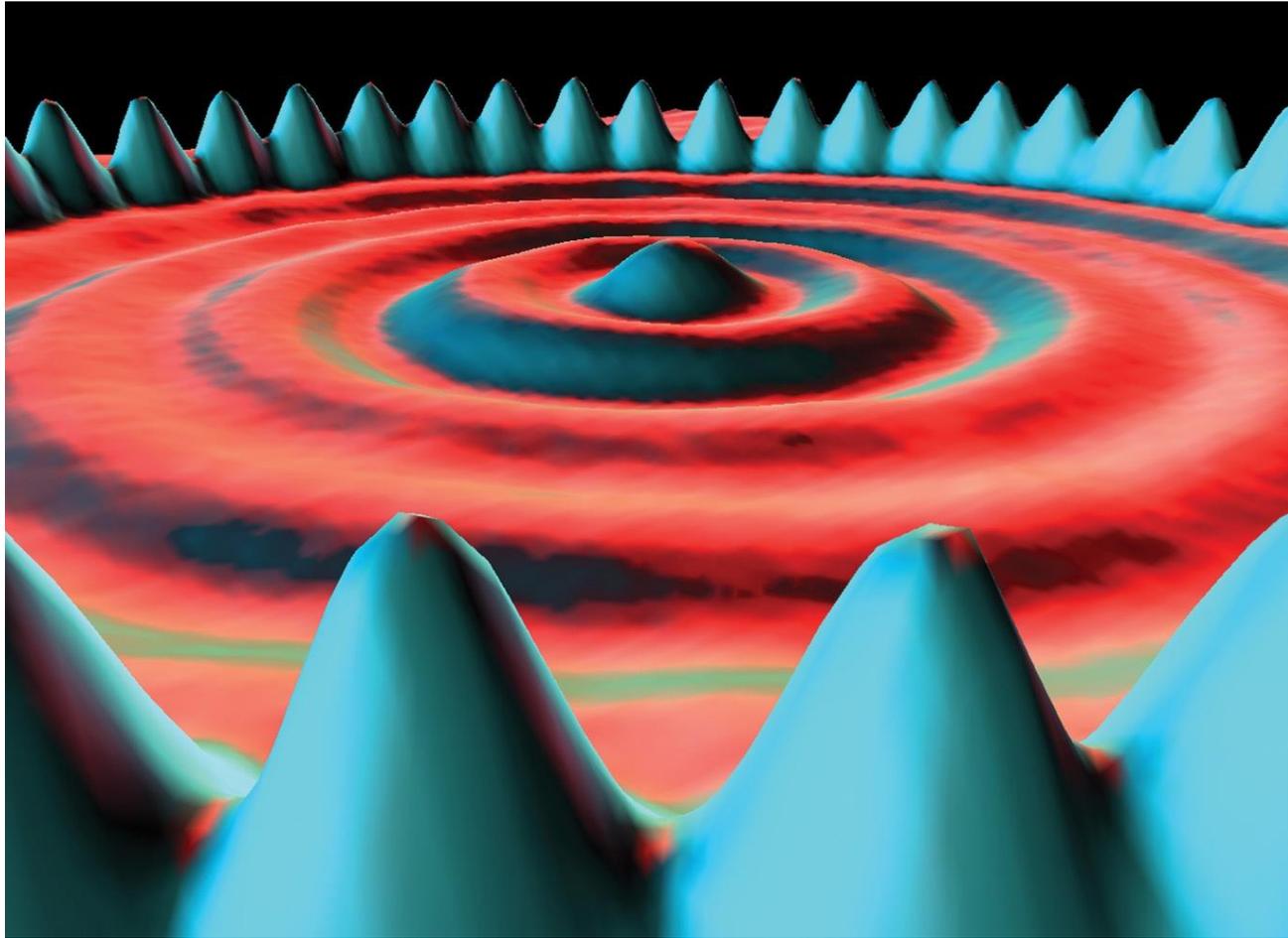
Energy-time uncertainty

$$\Delta E = \Delta \frac{p^2}{2m} = \frac{p\Delta p}{m} = v\Delta p$$

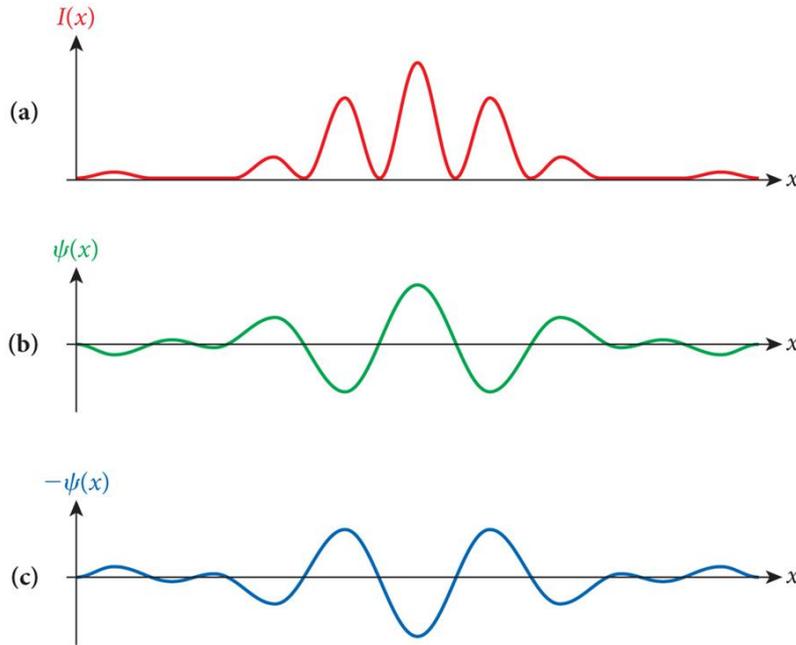
$$\Delta t = \frac{\Delta x}{v}$$

$$\Delta E\Delta t = \Delta p\Delta x \geq \frac{\hbar}{2}$$

Chapter 37 Quantum mechanics



Wave function



$$\rho(x)dx = |\psi(x)|^2 dx$$

Schrodinger equation

$$\Psi(x, y, z, t) = \psi(x, y, z)e^{-i\omega t}$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} [E - U(x)]\psi = 0$$

$$2\pi/\lambda = k$$

Free particle

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2m}{h^2} \frac{mv^2}{2}\psi = 0$$

$$\frac{d^2\psi}{dx^2} + \left(2\pi\frac{p}{h}\right)^2\psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0$$

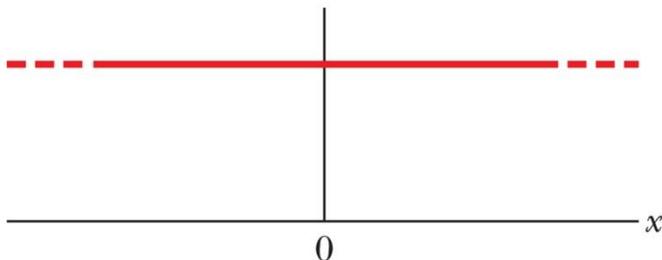
$$\psi = Ae^{ikx} + Be^{-ikx}$$

$$\Psi(x, t) = \psi(x)e^{-i\omega t} = Ae^{i(kx-\omega t)} + Be^{-i(kx+\omega t)}$$

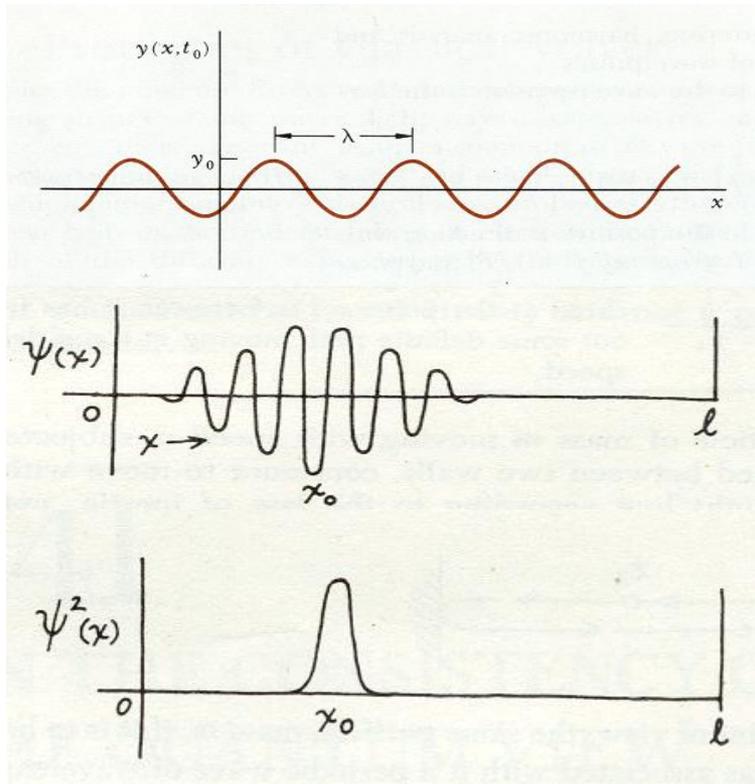
Probability
density $|\psi(x)|^2$

Prob. density

$$|\psi|^2 = |\psi_0 e^{ikx}|^2 = |\psi_0|^2$$



Heisenberg's uncertainty principle



$$\Delta x \cdot \Delta p_x \geq \hbar$$

$$\Delta y \cdot \Delta p_y \geq \hbar$$

$$\Delta z \cdot \Delta p_z \geq \hbar$$

example

Electron: $m_e = 9.11 \times 10^{-31} \text{ kg}$, $v = 2.05 \times 10^6 \text{ m/s}$, $\Delta v = 1.5\%$

$$p_e = m_e v = 1.87 \times 10^{-24} \text{ kg} \cdot \text{m/s}$$

$$\Delta x = \frac{h}{\Delta p} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.015)(1.87 \times 10^{-24} \text{ kg} \cdot \text{m/s})}$$
$$= 2.37 \times 10^{-8} \text{ m} \approx 24 \text{ nm} \Rightarrow 100 \text{ atomic diameter}$$

Golfball: $m = 45 \text{ g}$, $v = 35 \text{ m/s}$, $\Delta v = 1.5\%$

$$\Delta x = \frac{h}{\Delta p} = 3 \times 10^{-32} \text{ m}$$

파동역학

파동 \Leftrightarrow 입자 \Rightarrow 물질파 \Rightarrow 파동역학

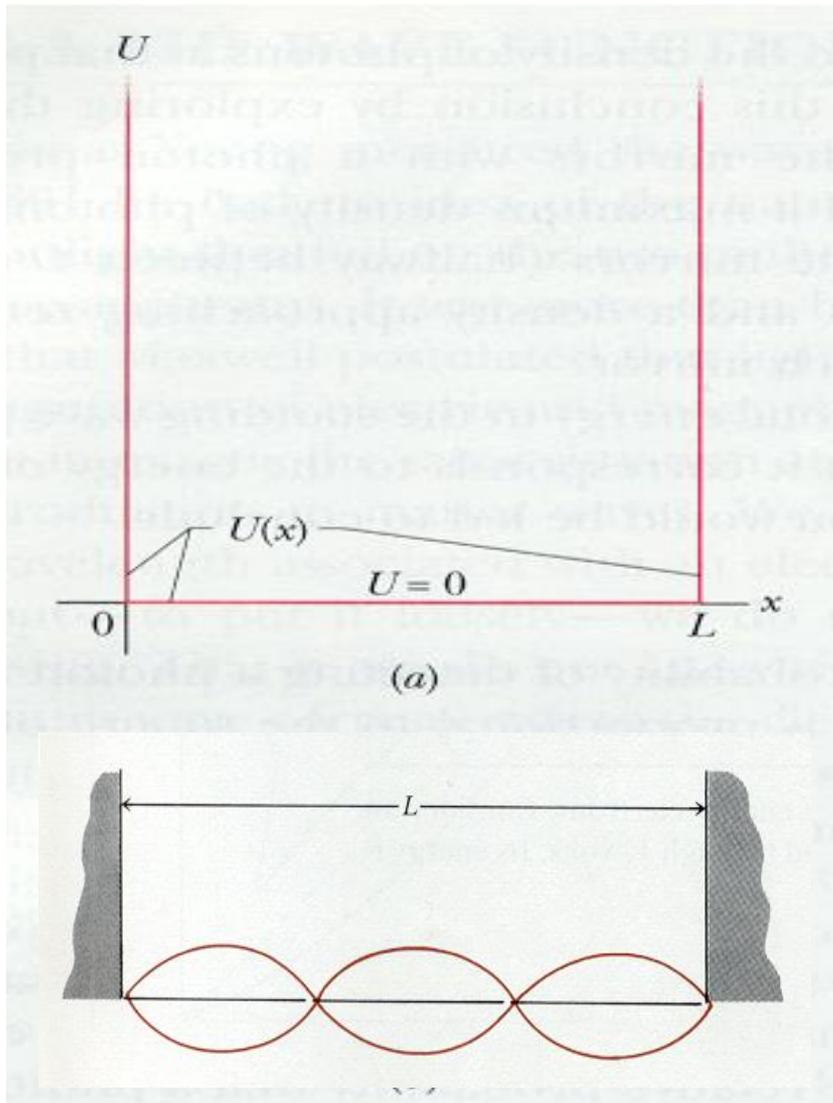
$$\psi(\vec{r}, t) = Ae^{i(\vec{k}\cdot\vec{r}-\omega t)}$$

$$E = hf = \frac{h}{2\pi} (2\pi f) = \hbar\omega$$

$$p = \frac{h}{\lambda} = \frac{h/2\pi}{\lambda/2\pi} = \hbar\left(\frac{2\pi}{\lambda}\right) = \hbar k$$

$$\int \psi^2(x, t) d\mathbf{x} = 1 \Rightarrow |\psi(x, t)|^2 : \textit{probability}$$

1차원 상자



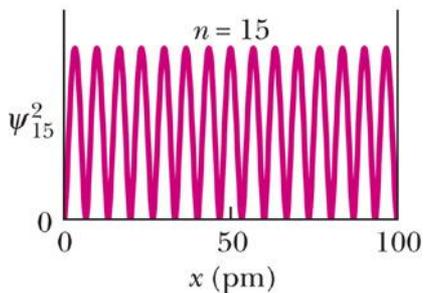
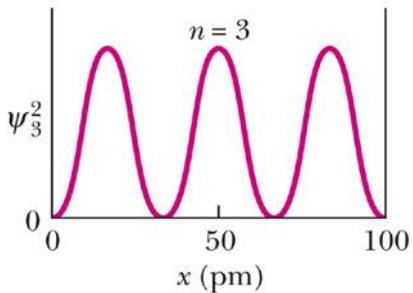
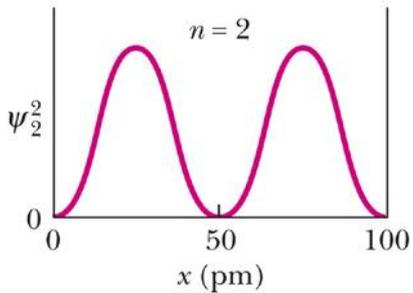
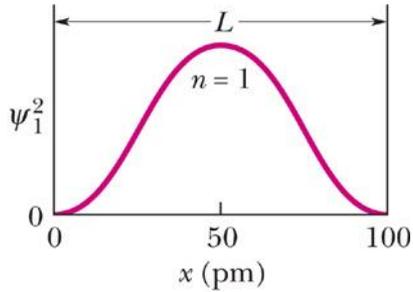
$$n\lambda = 2L$$

$$E = K + U = \frac{p^2}{2m}$$

$$p = \frac{h}{\lambda} = \frac{h}{2L/n}$$

$$\therefore E_n = \frac{h^2}{8mL^2} n^2$$

갇힌 전자의 파동함수



$$\psi_n(x) = A \sin\left(\frac{n\pi}{L}x\right), \quad (n = 1, 2, 3, \dots)$$

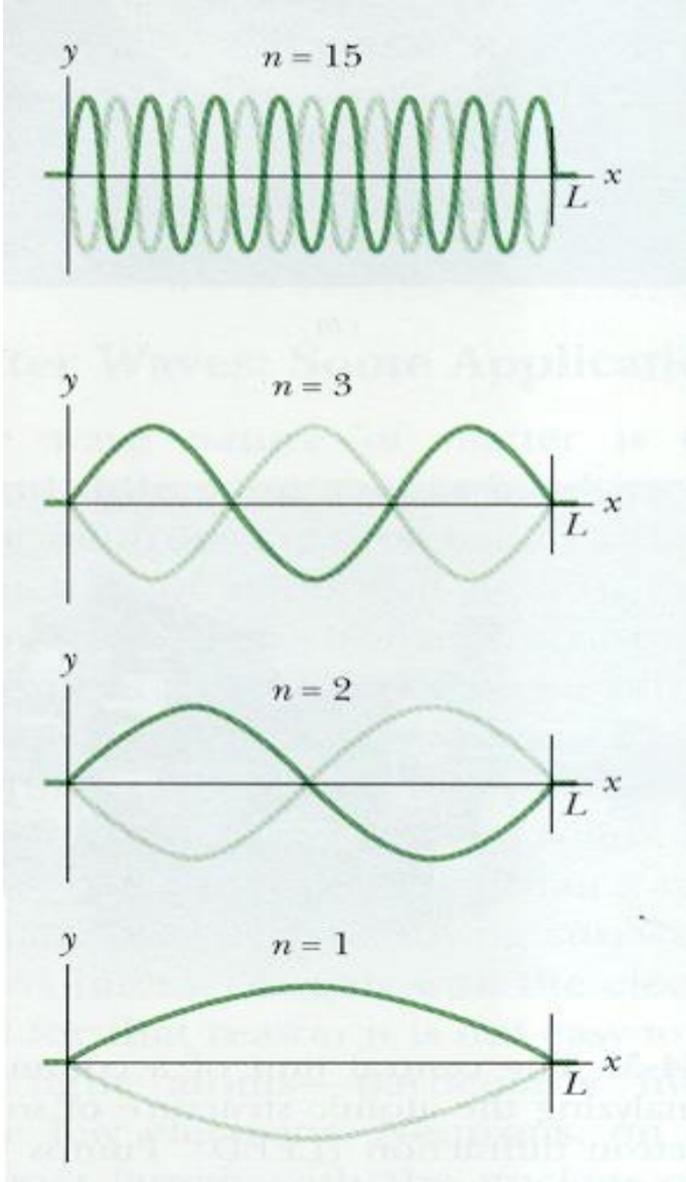
확률 $p(x)dx = |\psi_n(x)|^2 dx$

$$p(x) = A^2 \sin^2\left(\frac{n\pi}{L}x\right), \quad (n = 1, 2, 3, \dots)$$

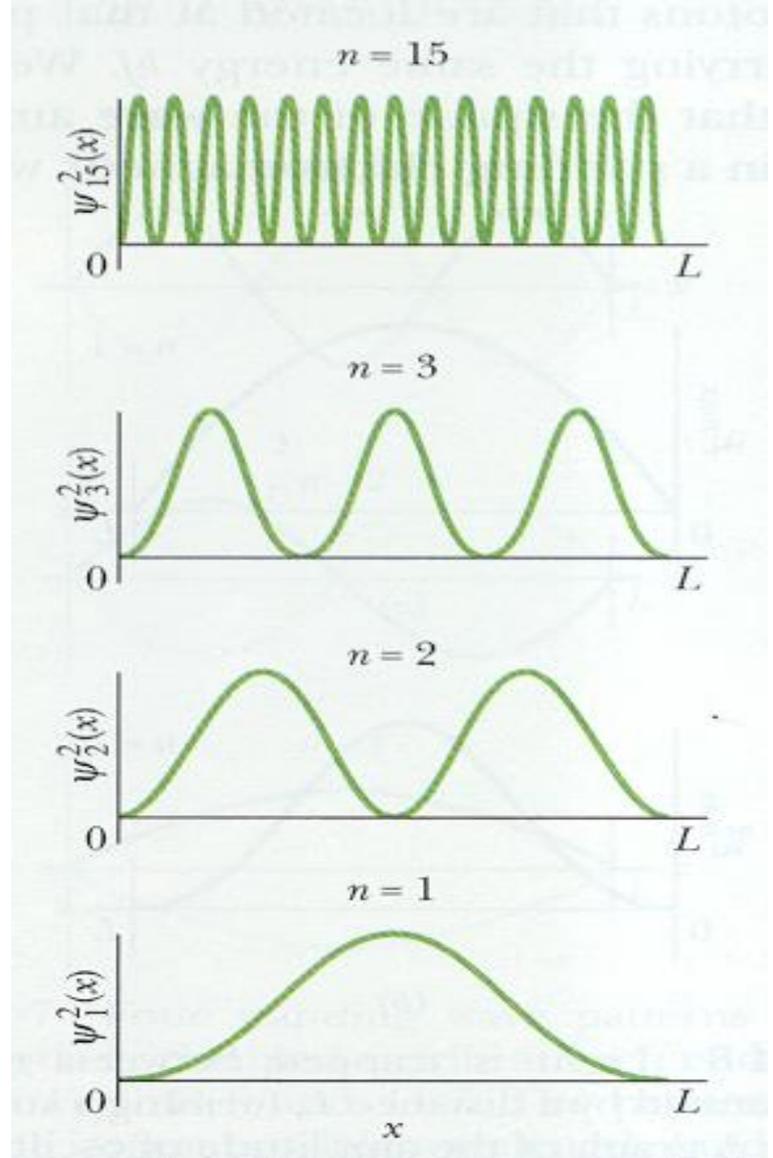
x_1 과 x_2 사이에서 검출할 확률

$$\int_{x_1}^{x_2} p(x)dx = \int_{x_1}^{x_2} A^2 \sin^2 \frac{n\pi x}{L} dx$$

$$\int_{-\infty}^{\infty} p(x)dx = 1 \longrightarrow A = \sqrt{\frac{2}{L}}$$



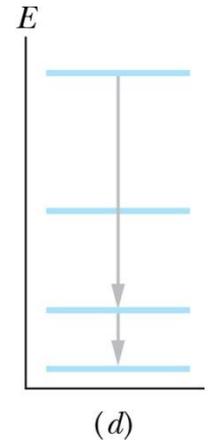
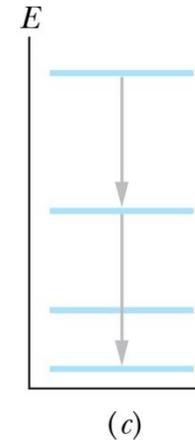
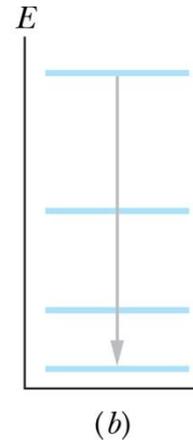
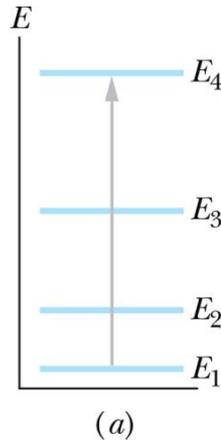
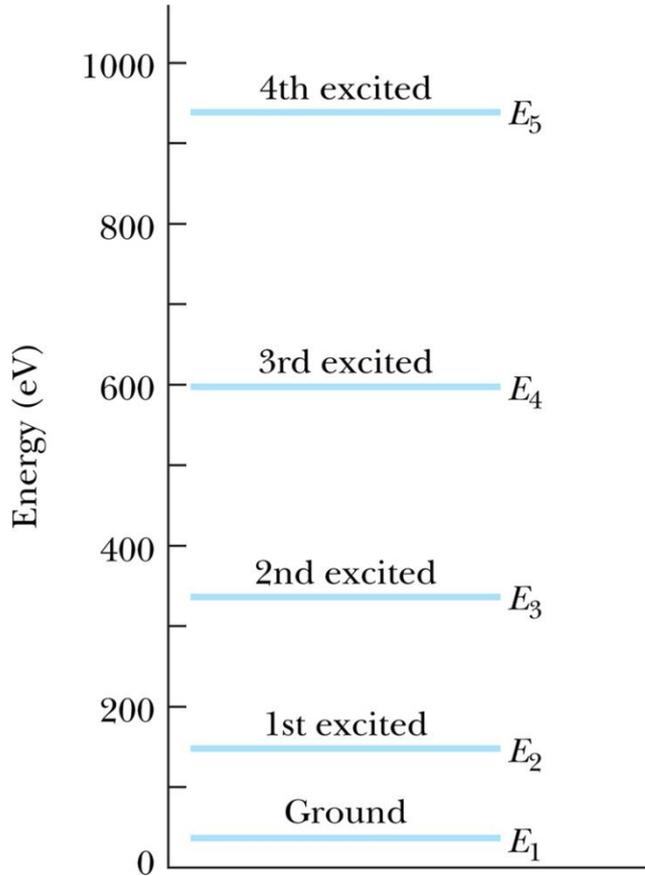
$$\psi(x,t)$$



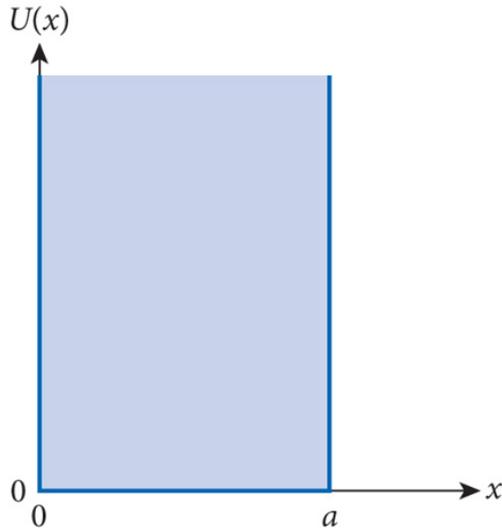
$$|\psi(x,t)|^2 : \textit{probability}$$

에너지의 변화

$$\Delta E = E_{\text{high}} - E_{\text{low}}$$



Infinite potential well



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$$U(x) = \begin{cases} \infty & \text{for } x < 0 \\ 0 & \text{for } 0 \leq x \leq a \\ \infty & \text{for } x > a. \end{cases}$$

$$\psi(0) = \psi(a) = 0$$

$$k = \frac{n\pi}{a}, \quad n = 1, 2, 3, \dots$$

$$E = \frac{\hbar^2 k^2}{2m} = \frac{\hbar^2 n^2 \pi^2}{2ma^2}$$

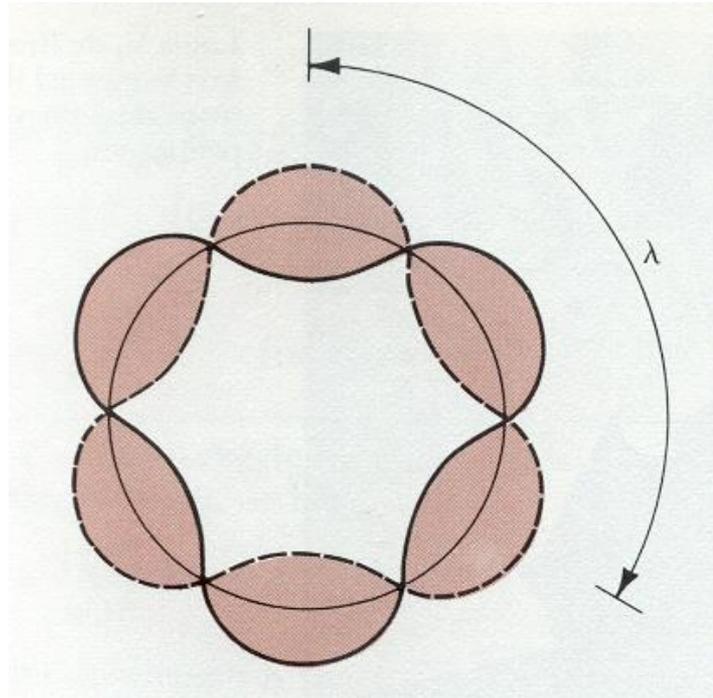
normalization

$$1 = \int_{-\infty}^{\infty} |\psi(x)|^2 dx = |D|^2 \int_0^a \sin^2 \frac{n\pi}{a} x = \frac{|D|^2}{2}$$

$$\therefore \psi(x) = \sqrt{\frac{2}{a}} \sin \frac{n\pi x}{a}$$



Bohr의 수소원자 모형



- (1) 수소원자는 원(타원)궤도를 돈다.
- (2) 이 상태를 정상상태라고 하며 에너지를 잃지 않는다.
- (3) 에너지를 방출하거나 흡수할 때는 한 정상상태에서 다른 정상상태로 갈 때 나타난다.
- (4) 정상상태에서 각운동량은

$$L = n\hbar, \quad (n = 1, 2, 3, \dots)$$
 만을 갖는다.

$$\frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2} = \frac{mv^2}{r}$$

$$mvr = n\hbar$$



$$r = \frac{h^2\epsilon_0}{\pi me^2} n^2 = an^2, \quad (n = 1, 2, 3, \dots)$$

$$a = \frac{h^2\epsilon_0}{\pi me^2} = 5.3 \times 10^{-11} \text{ m}$$

양자화된 에너지



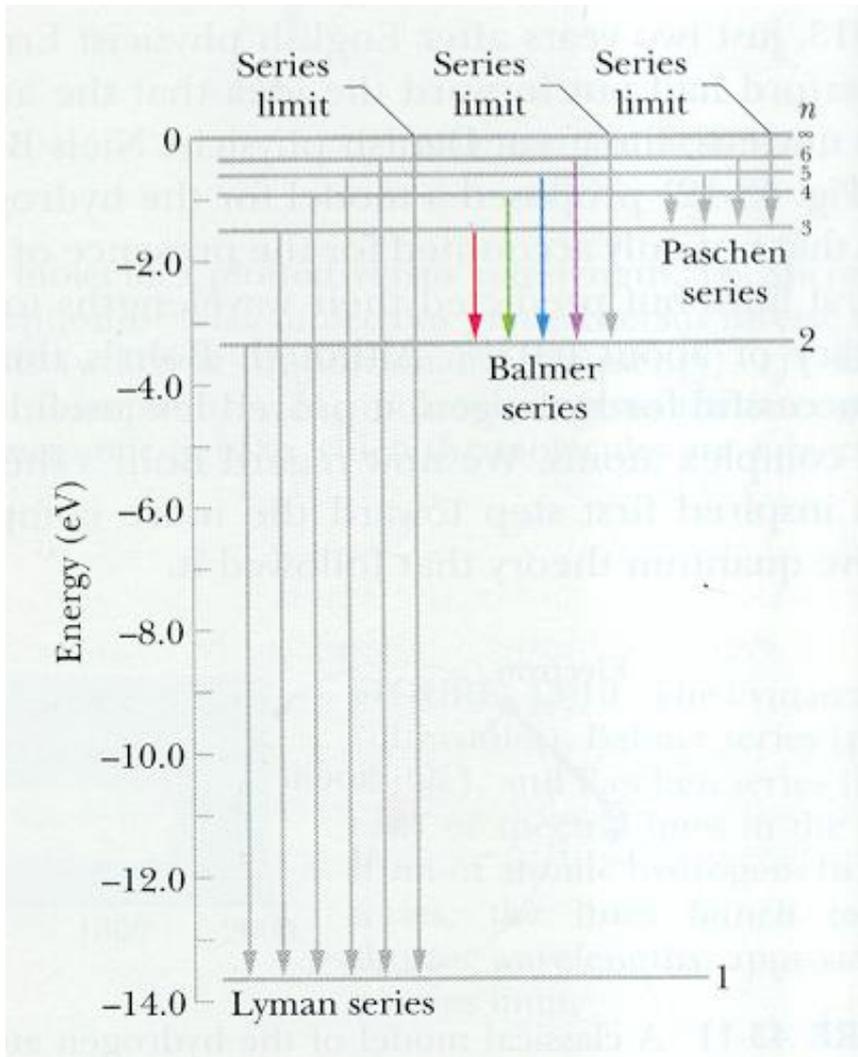
에너지 변화 $hf = \Delta E = E_{\text{high}} - E_{\text{low}}$

$$\frac{1}{\lambda} = -\frac{me^4}{8\epsilon^2 h^3 c} \left(\frac{1}{n_{\text{high}}^2} - \frac{1}{n_{\text{low}}^2} \right) = R \left(\frac{1}{n_{\text{low}}^2} - \frac{1}{n_{\text{high}}^2} \right)$$



Rydberg 상수

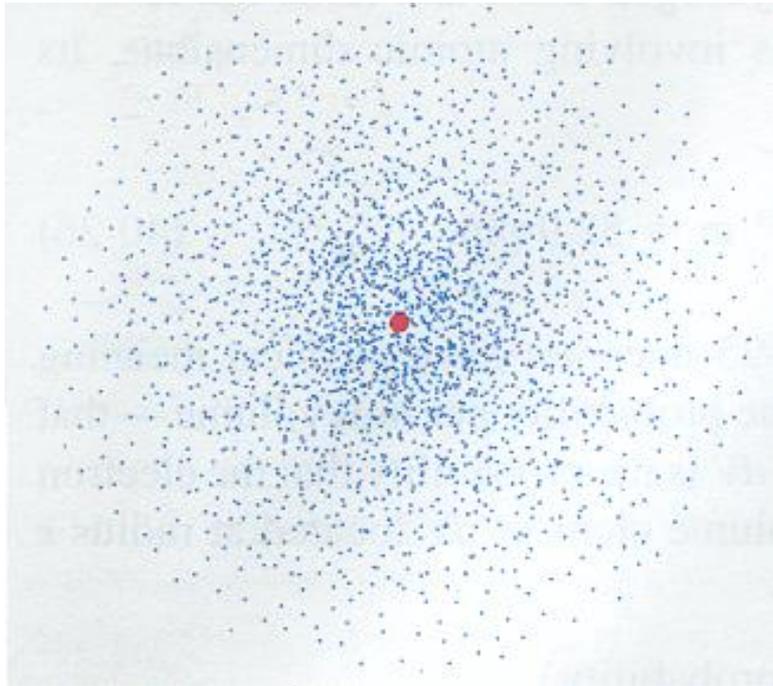
수소원자에 대한 양자역학적 이해



$$E_n = -\frac{13.6}{n^2} \text{ eV}$$

$$hf = E_m - E_n$$

전자분포



$$n = 1$$