Mathematical Physics 1

Assignment 5

Due: April 15 (Tuesday), 2014

Boas Chapter 5

Section 5: 1, 5
Section 6: 4, 6, 11, 18, 23, 25

Boas Chapter 6

Section 3: 1, 12, 14, 16

Arfken

1.

Show that (in 3-D space)

(a) $\sum_i \delta_{ii} = 3,$

(b) $\sum_{ij} \delta_{ij}\varepsilon_{ijk} = 0,$

(c) $\sum_{pq} \varepsilon_{ipq}\varepsilon_{jpq} = 2\delta_{ij},$

(d) $\sum_{ijk} \varepsilon_{ijk}\varepsilon_{ijk} = 6.$

2.

Show that (in 3-D space)

$$\sum_k \varepsilon_{ijk}\varepsilon_{pqk} = \delta_{ip}\delta_{jq} - \delta_{iq}\delta_{jp}.$$

3.

Using the vectors

$$\mathbf{P} = \hat{\mathbf{e}}_x \cos\theta + \hat{\mathbf{e}}_y \sin\theta,$$
$$\mathbf{Q} = \hat{\mathbf{e}}_x \cos\varphi - \hat{\mathbf{e}}_y \sin\varphi,$$
$$\mathbf{R} = \hat{\mathbf{e}}_x \cos\varphi + \hat{\mathbf{e}}_y \sin\varphi,$$

prove the familiar trigonometric identities

$$\sin(\theta + \varphi) = \sin\theta\cos\varphi + \cos\theta\sin\varphi,$$
$$\cos(\theta + \varphi) = \cos\theta\cos\varphi - \sin\theta\sin\varphi.$$

4.

Derive the law of sines (see Fig. 3.4):

$$\frac{\sin\alpha}{|\mathbf{A}|} = \frac{\sin\beta}{|\mathbf{B}|} = \frac{\sin\gamma}{|\mathbf{C}|}.$$

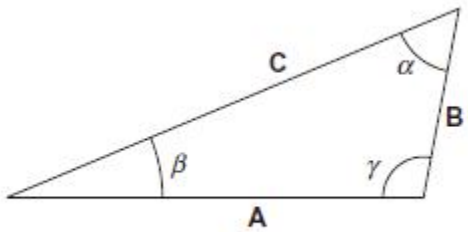

FIGURE 3.4 Plane triangle.

5.

Show that

$$(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C}).$$