Mobile Communications (KECE425)

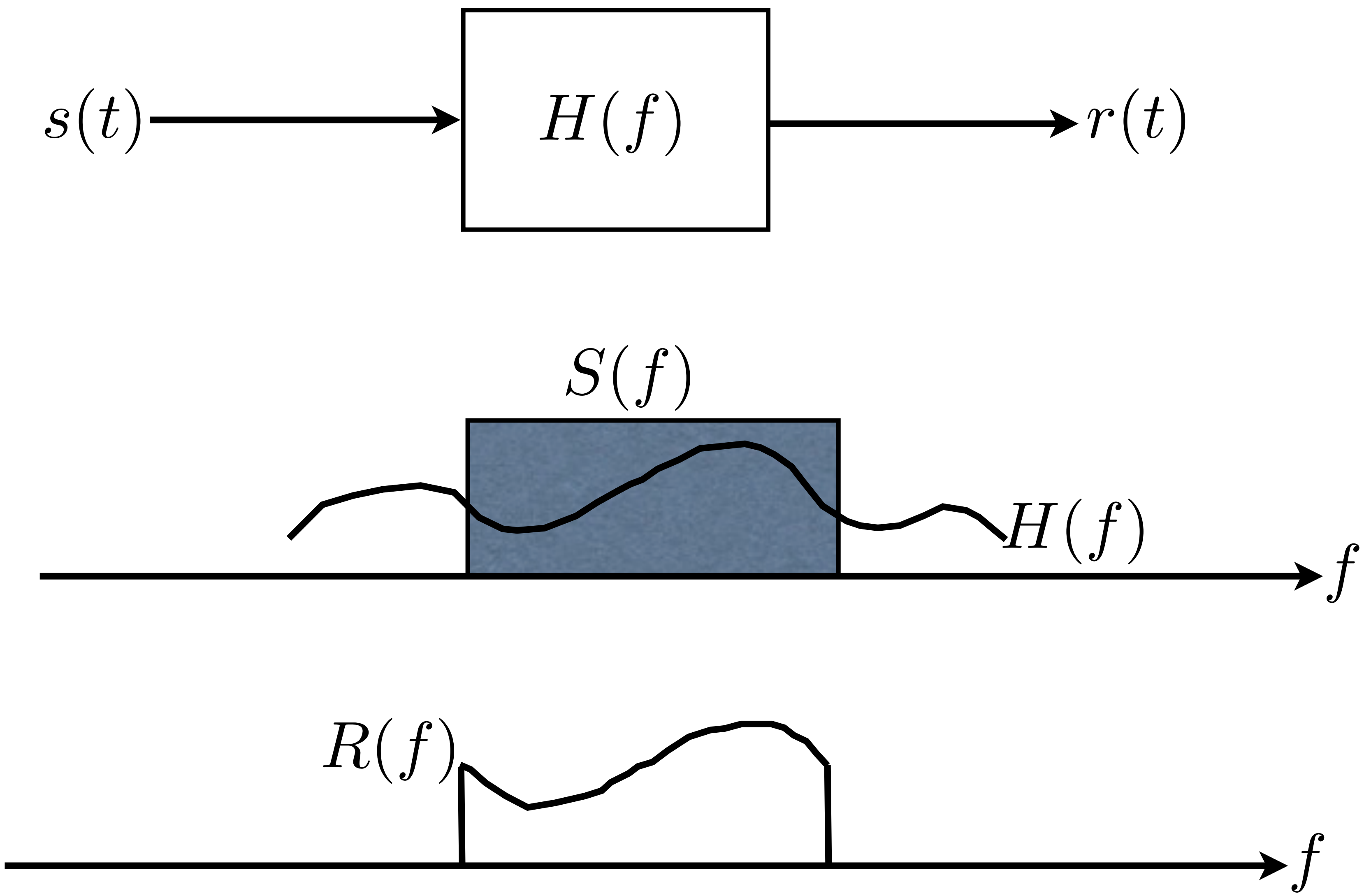
Lecture Note 14

4-16-2014

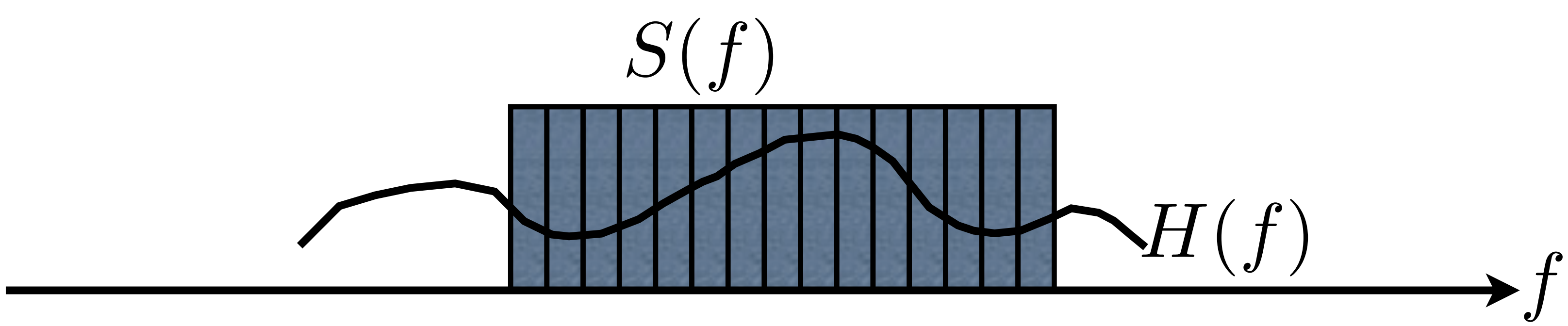
Prof. Young-Chai Ko

Frequency Selective Fading Channels

- Frequency selective channel $\Longrightarrow$ Inter-symbol interference (ISI)

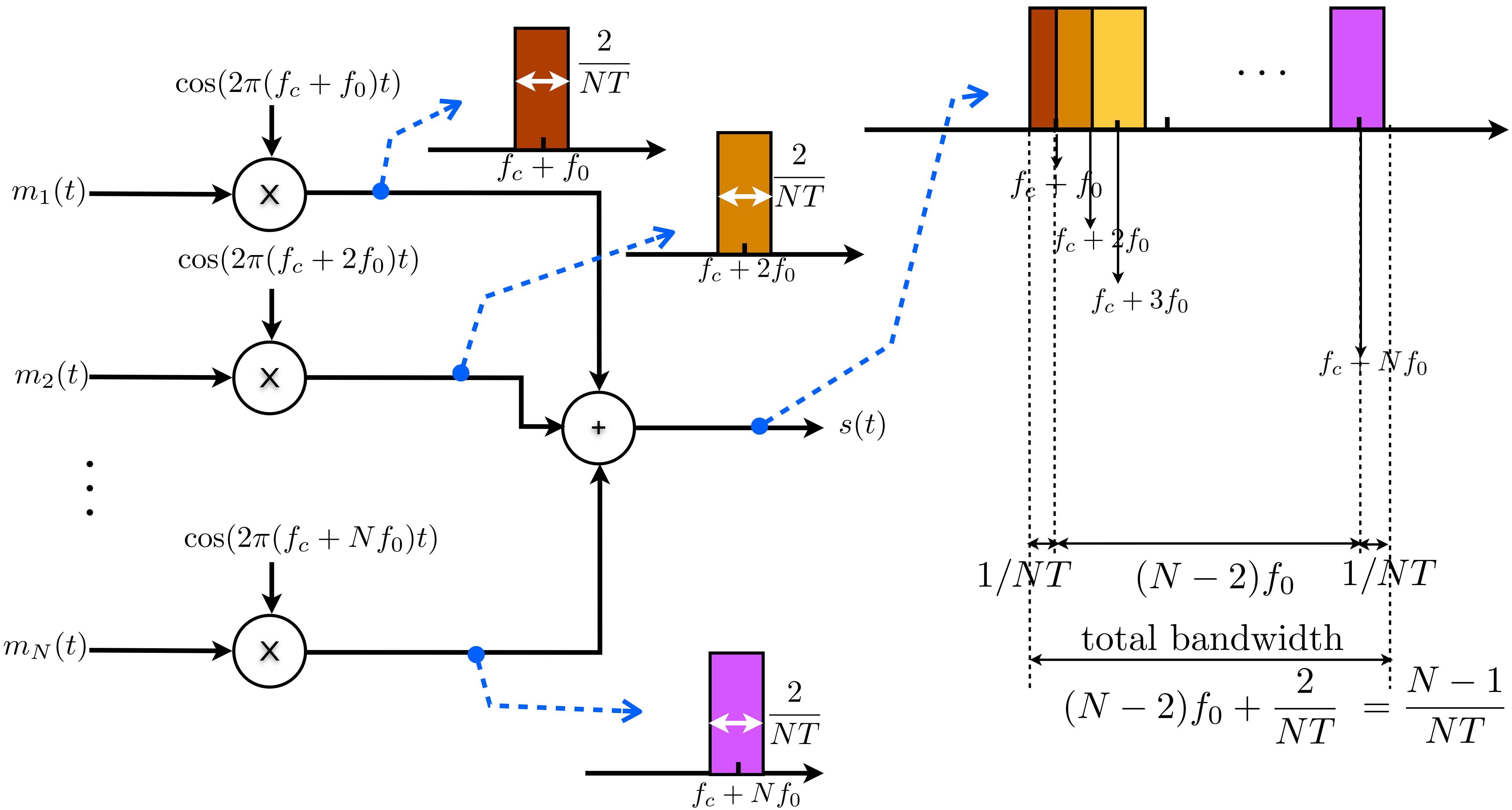


Multi-Carrier Transmission



- Each sub-block is transmitted so that it experiences the flat fading channels, that is, ISI-free channel.

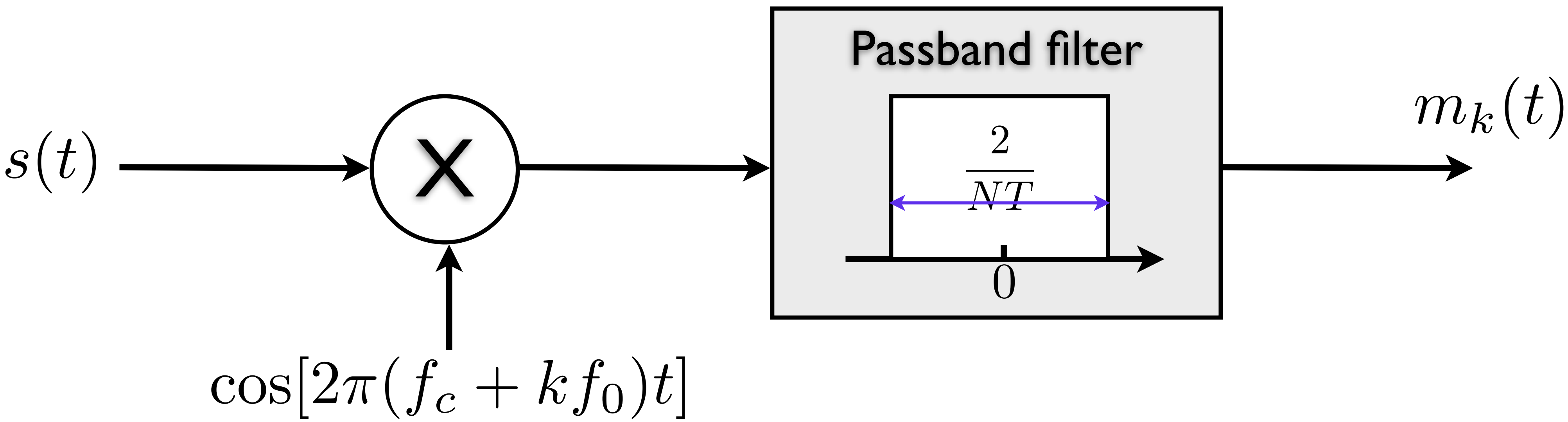
- Orthogonality condition for the receiver
  - If we set $f_0 = \frac{1}{NT}$, then we have the orthogonality among each branch of the transmitter in the multi-carrier transmission.



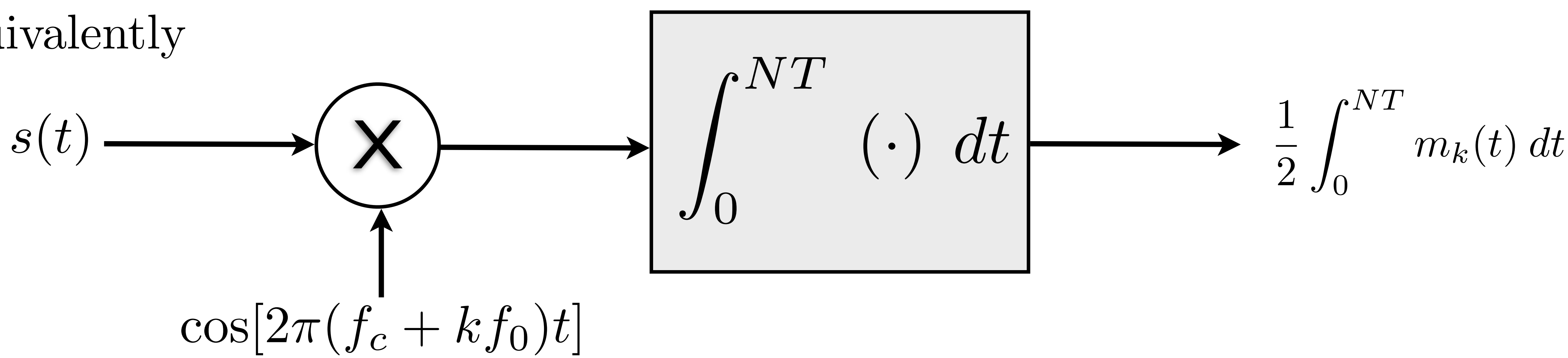
- Transmit signal waveform

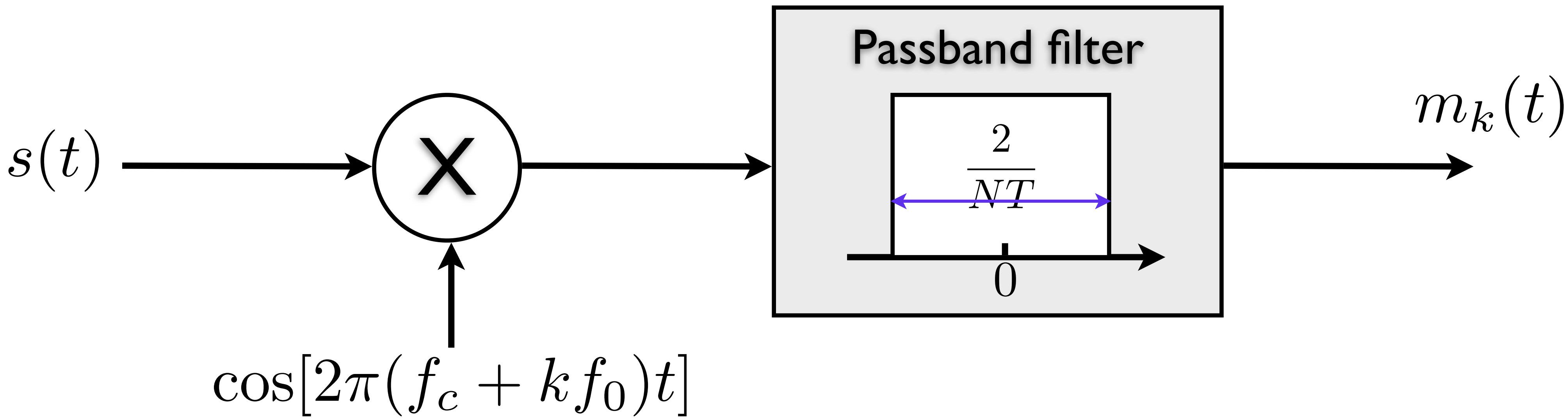
$$s(t) = \sum_{k=1}^{N} m_k(t) \cos[2\pi(f_c + kf_0)t]$$

- Assume that there is no fading and no noise.



or equivalently





$$s(t) = \sum_{j=1}^{N} m_j(t)\cos(2\pi(f_c + jf_0)t)$$

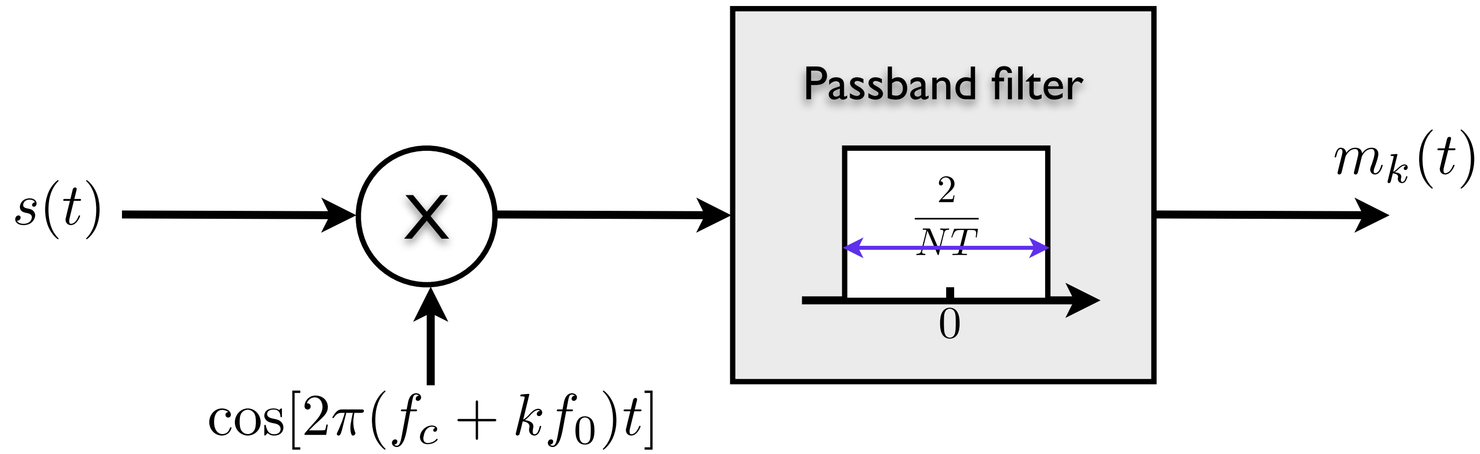
$$\begin{aligned} s(t)\cos(2\pi(f_c + kf_0)t) &= \cos(2\pi(f_c + kf_0)t) \times \sum_{j=1}^{N} m_j(t)\cos(2\pi(f_c + jf_0)t) \\ &= \sum_{j=1}^{N} m_j(t)\,\boxed{\cos(2\pi(f_c + kf_0)t) \times \cos(2\pi(f_c + jf_0)t)} \end{aligned}$$

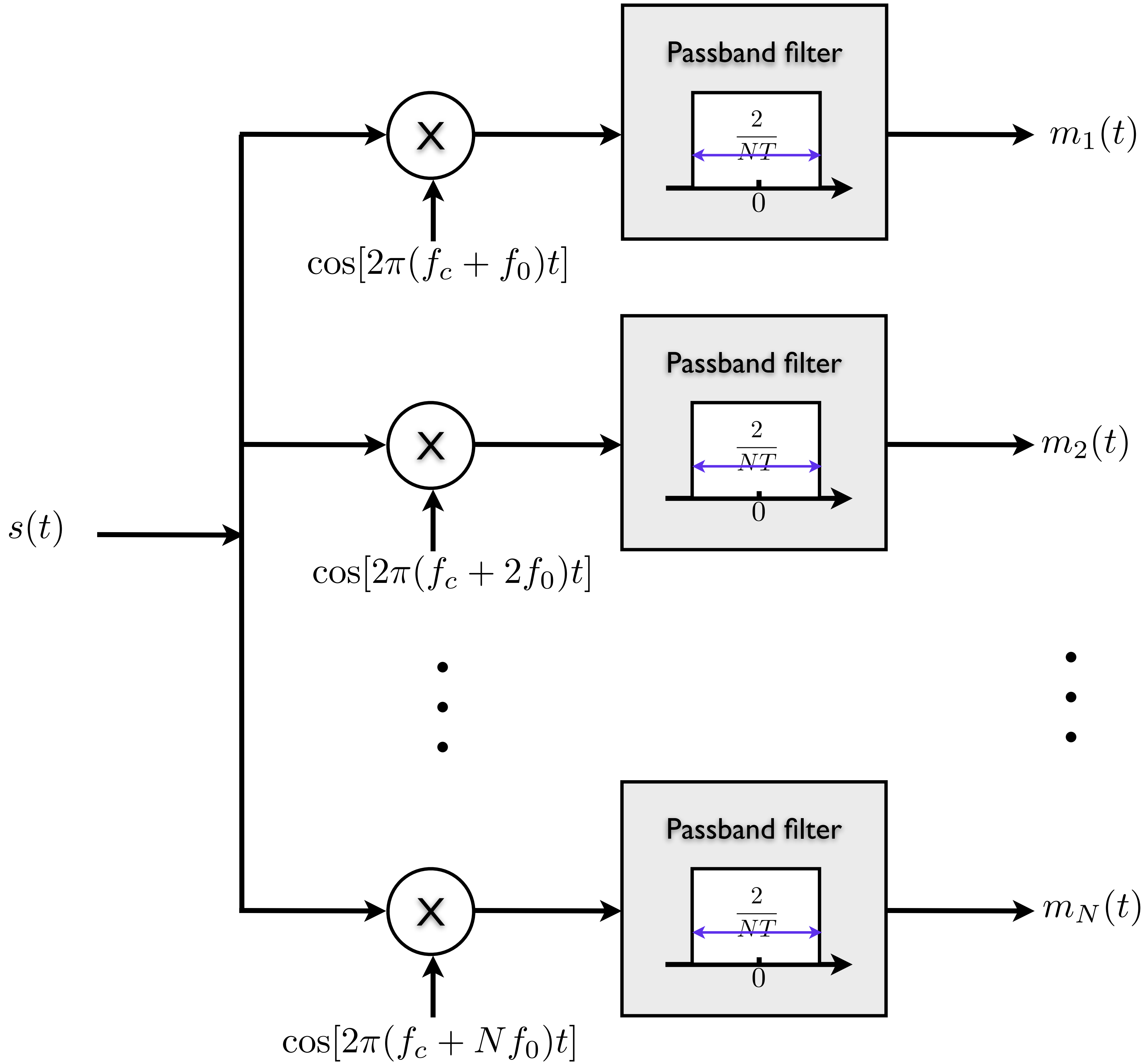
$$\boxed{\cos(2\pi(f_c + kf_0)t) \times \cos(2\pi(f_c + jf_0)t) = \frac{1}{2}\left[\cos\left(2\pi(2f_c + (k+j)f_0)t\right) + \cos(2\pi(k-j)f_0)t)\right]}$$

$$s(t)\cos(2\pi(f_c+kf_0)t) = \frac{1}{2}\sum_{j=1}^{N} m_j(t)\left[\cos\left(2\pi(2f_c+(k+j)f_0)t\right)+\cos(2\pi(k-j)f_0)t)\right]$$

$$= \frac{m_k(t)}{2}+\frac{m_k(t)}{2}\cos(2\pi(2f_c+2kf_0)t)$$

$$+\frac{1}{2}\sum_{j=1,j\neq k}^{N} m_j(t)\left[\cos\left(2\pi(2f_c+(k+j)f_0)t\right)+\cos(2\pi(k-j)f_0)t)\right]$$

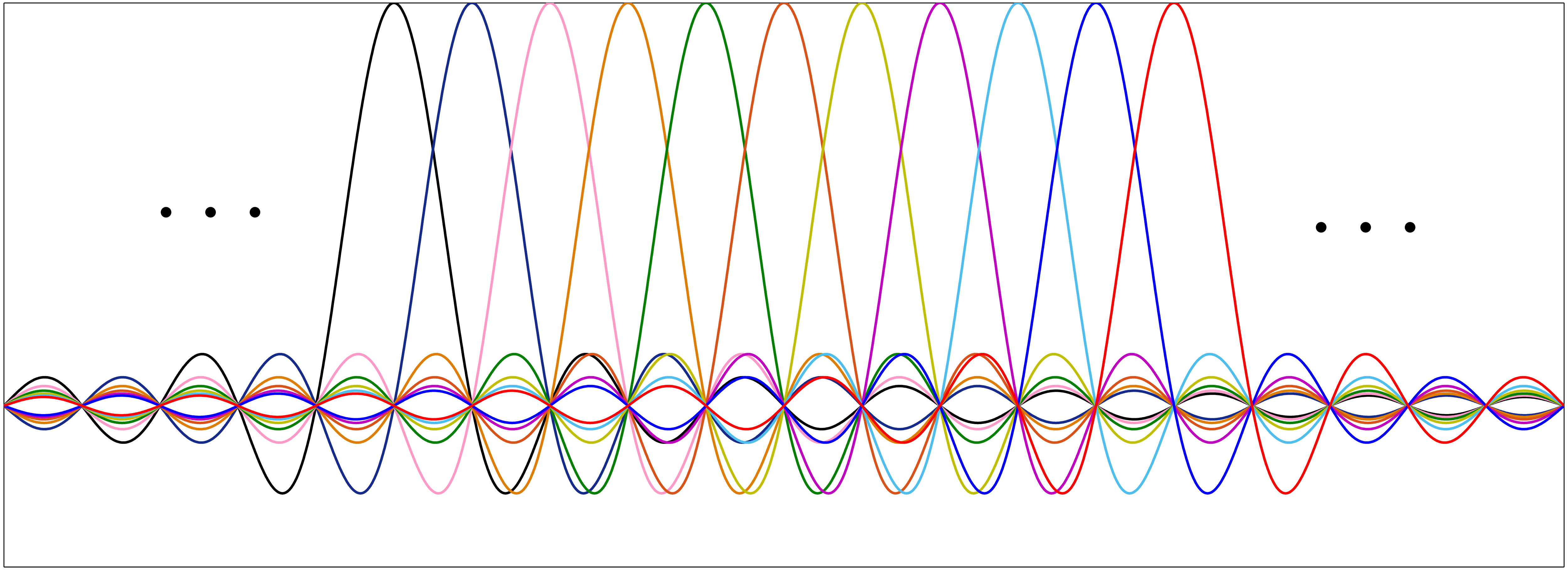




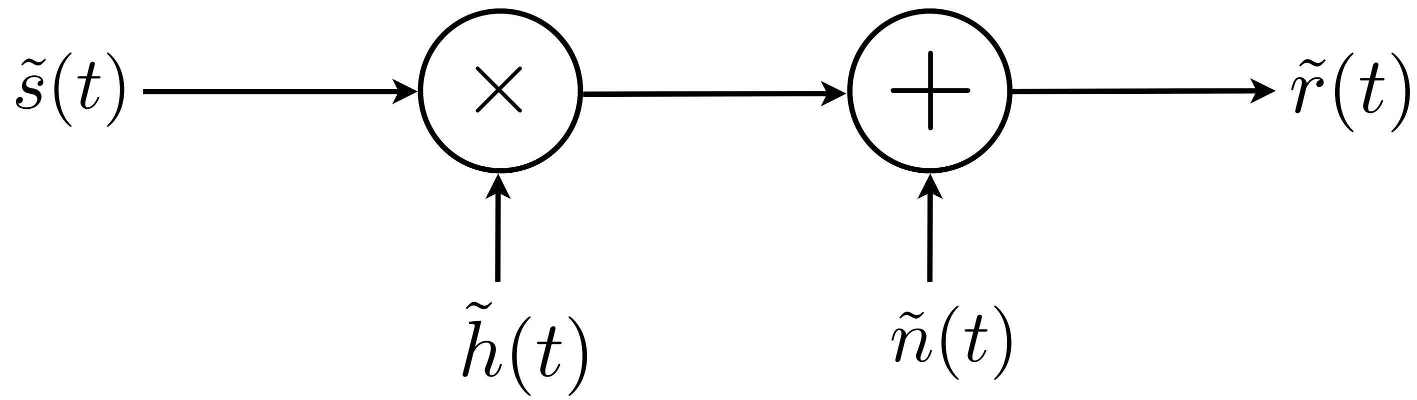
$s(t)$
Passband filter
$\frac{2}{NT}$
0
$m_1(t)$
$\cos[2\pi(f_c + f_0)t]$
Passband filter
$\frac{2}{NT}$
0
$m_2(t)$
$\cos[2\pi(f_c + 2f_0)t]$
Passband filter
$\frac{2}{NT}$
0
$m_N(t)$
$\cos[2\pi(f_c + Nf_0)t]$

- In reality, the information bearing signal $m_k(t)$ is a signal with a certain pulse which satisfies the Nyquist criterion, for example, rectangular pulse, root-raised cosine pulse, and etc.

- In that case, the amplitude spectrum of $m_k(t)$ is sinc type.



- Flat fading channel model



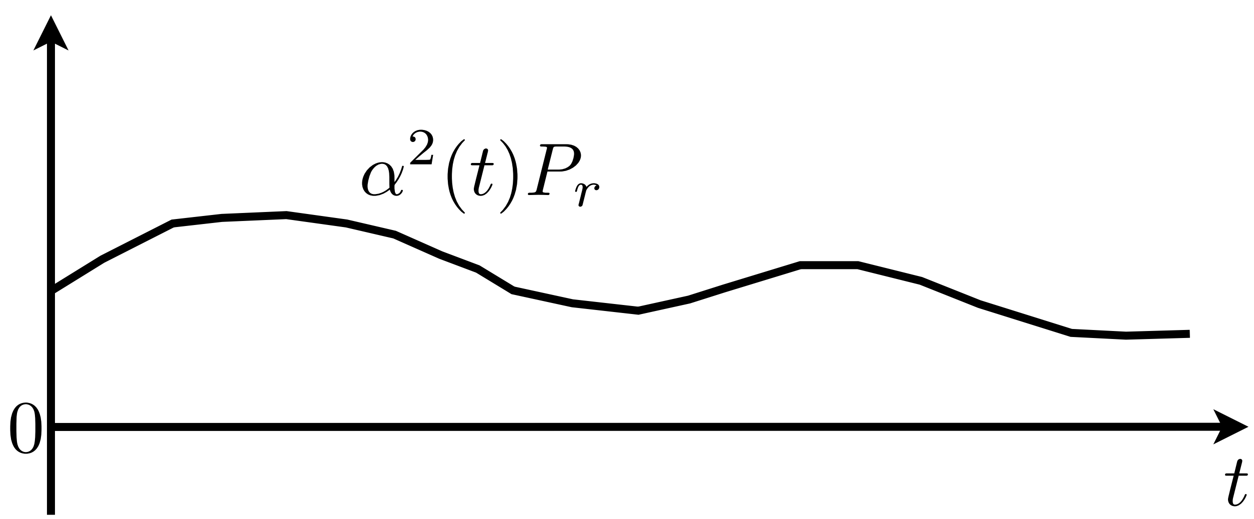
$$\tilde{h}(t) = \alpha(t)e^{j\phi(t)}$$

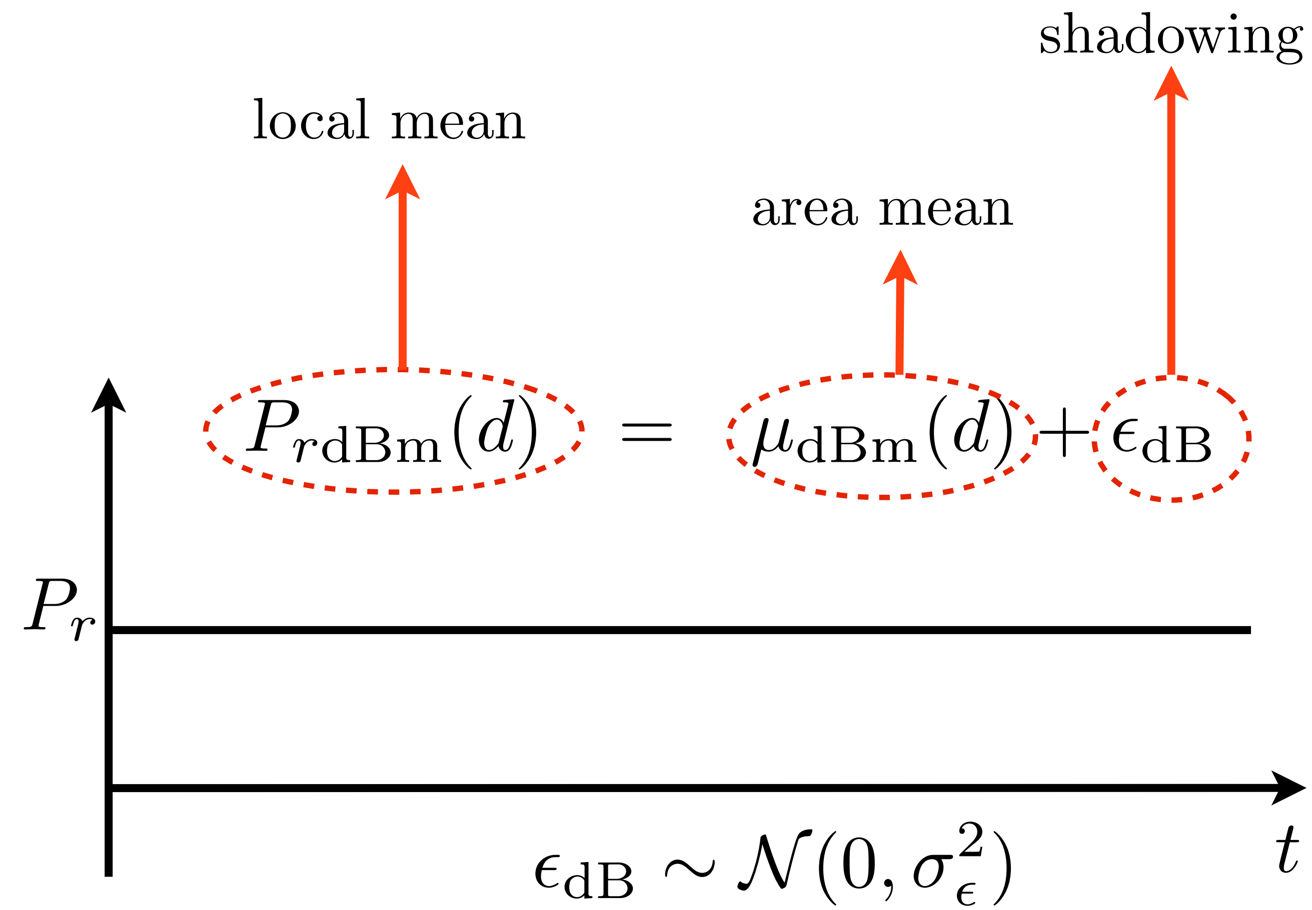
- Distribution of the amplitude $\alpha(t)$:
  - Rayleigh fading channel
  - Ricean fading channel
  - Nakagami-m fading channel

- Received signal

$$r(t) = \alpha(t) e^{j\phi(t)} s(t) + n(t)$$

- Received signal power





- Instantaneous power:

$$\alpha^2(t)P_r \quad \text{or it is often written as } \alpha^2(t)E[s^2(t)]$$

- (Instantaneous) Signal-to-Noise ratio, γ:

$$\gamma = \frac{\alpha^2(t) E_s}{N_0}$$

- Average Signal-to-Noise ratio, $\bar{\gamma}$:

$$E[\gamma] = E\left[\frac{\alpha^2(t) E_s}{N_0}\right] = \frac{E[\alpha^2(t)] E_s}{N_0} = \frac{\Omega_t E_s}{N_0}$$

CDF of Exponential RV

- For Rayleigh channel, $\alpha^2(t)$ follows the exponential distribution:

$$p_{\alpha^2}(x) = \frac{1}{\Omega_p} e^{-x/\Omega_p} \qquad x > 0$$

- Then the CDF of $\alpha^2(t)$ can e written as

$$\begin{aligned} P_{\alpha^2}(x) &= \Pr[\alpha^2 < x] = \frac{1}{\Omega_p} \int_0^x e^{-t/\Omega_p}\, dt \\ &= 1 - e^{-x/\Omega_p} \end{aligned}$$

CDF of Instantaneous SNR

- (Instantaneous) Signal-to-Noise ratio, γ:

$$\gamma = \frac{\alpha^2(t) E_s}{N_0}$$

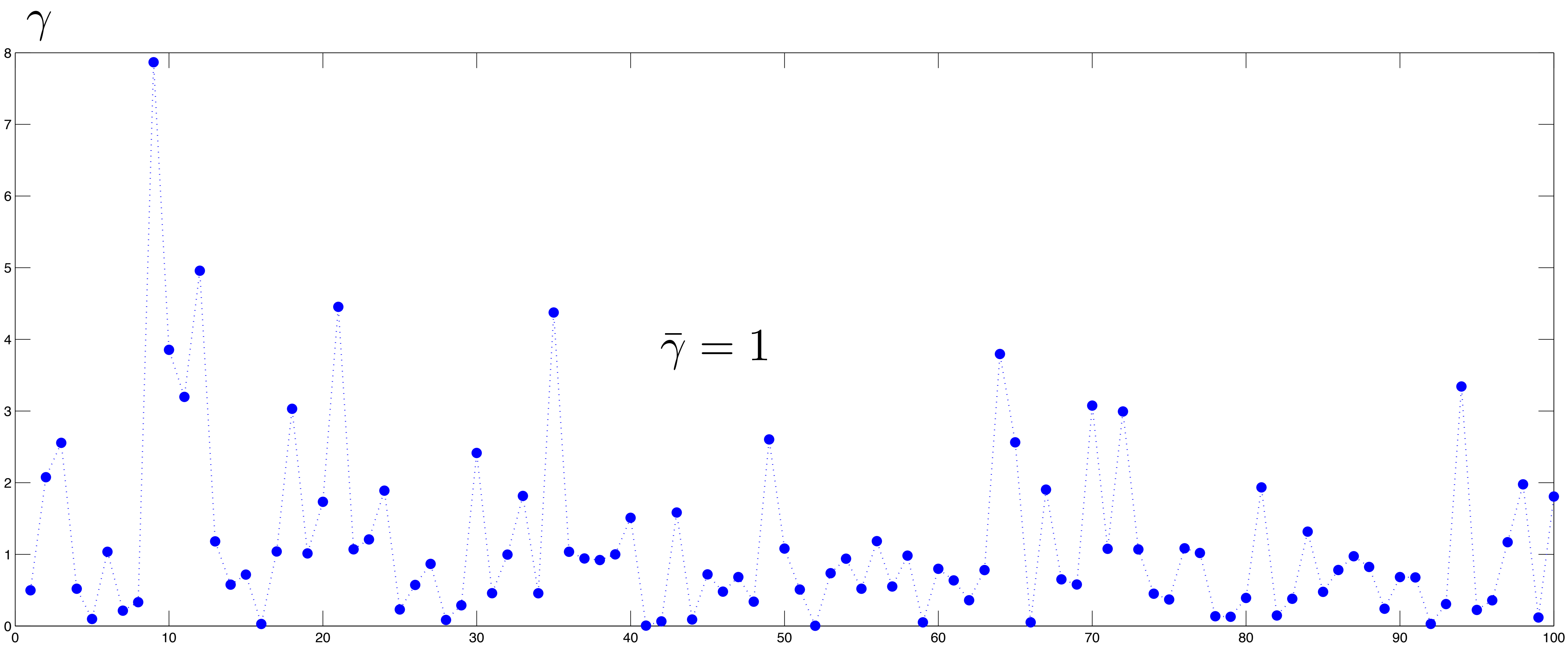
- PDF of γ:

$$p_\gamma(x) = \frac{1}{\bar{\gamma}} e^{-x/\bar{\gamma}},$$

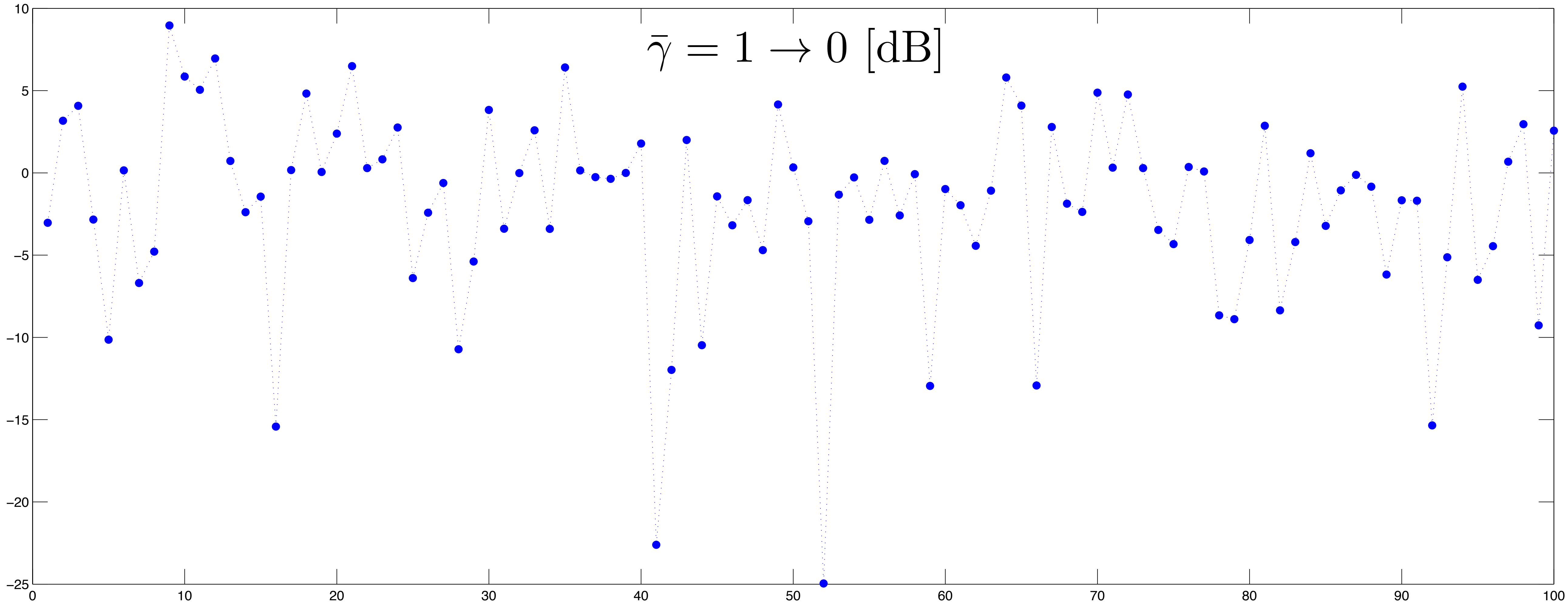
- CDF of γ:

$$P_\gamma(x) = 1 - e^{-x/\bar{\gamma}},$$

- Example of exponential random samples for $\bar{\gamma} = 1$.



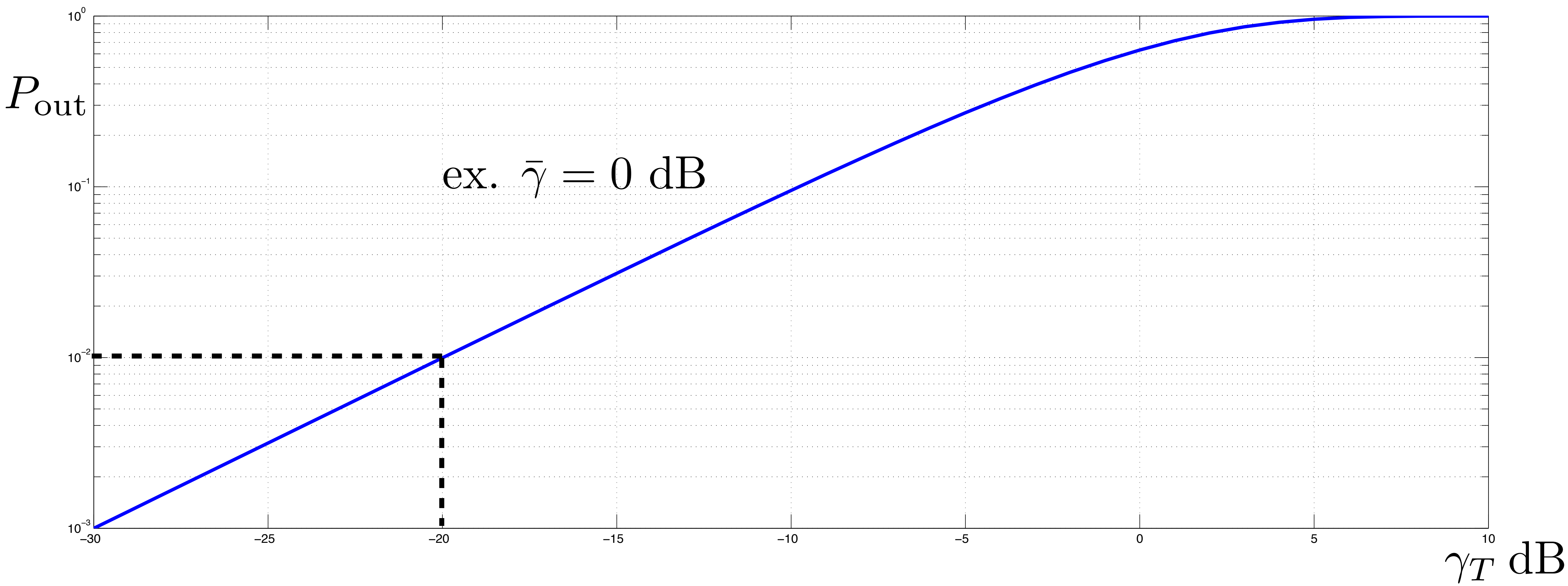
γ [dB]



Outage Probability

- Outage probability, P_{out}:

$$P_{\text{out}} = \Pr[\gamma \leq \gamma_T] = 1 - e^{-\gamma_T/\bar{\gamma}}$$



- Outage probability for $\bar{\gamma} = 0, 2, 4, 6, 8, 10$ dB over Rayleigh channel.