KECE321 Communication Systems I

(Haykin Sec. 3.1-Sec. 3.6)

Lecture #9, April 4, 2012

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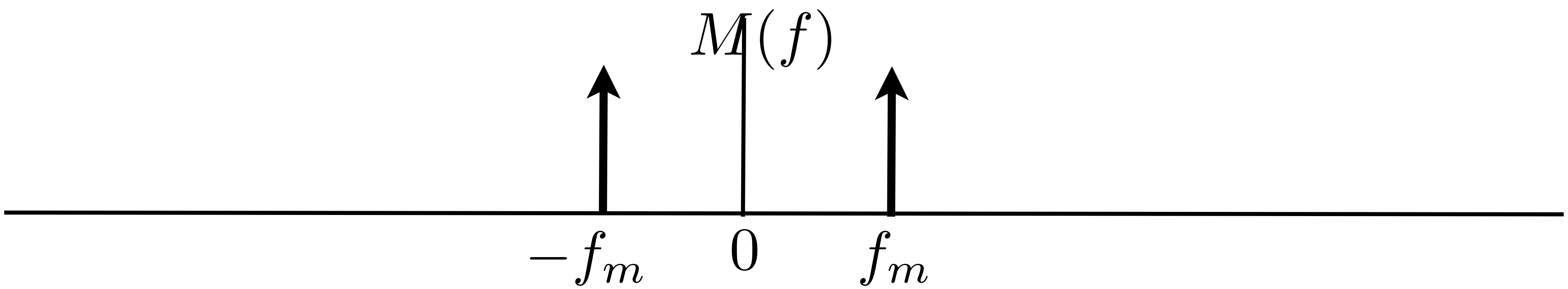
Summary

- Amplitude modulation
  - Single-tone modulation
  - Limitations of AM
  - Modification of AM
    - Double Sideband-Suppressed Carrier Modulation
    - Single-Sideband Modulation
    - Vestigial Sideband Modulation

Single -Tone Modulation

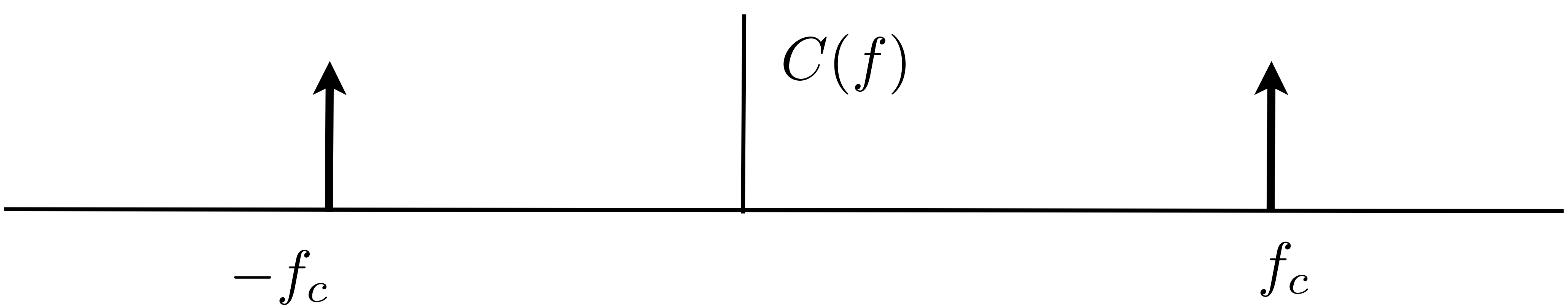
- Consider single-tone modulating wave (message signal)

$$m(t) = A_m \cos(2\pi f_m t)$$



- carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$



- AM wave

$$s(t) = A_c[1 + \mu \cos(2\pi f_m t)] \cos(2\pi f_c t)$$

where $\mu = k_a A_m$

- Maximum envelope value and Minimum envelope value

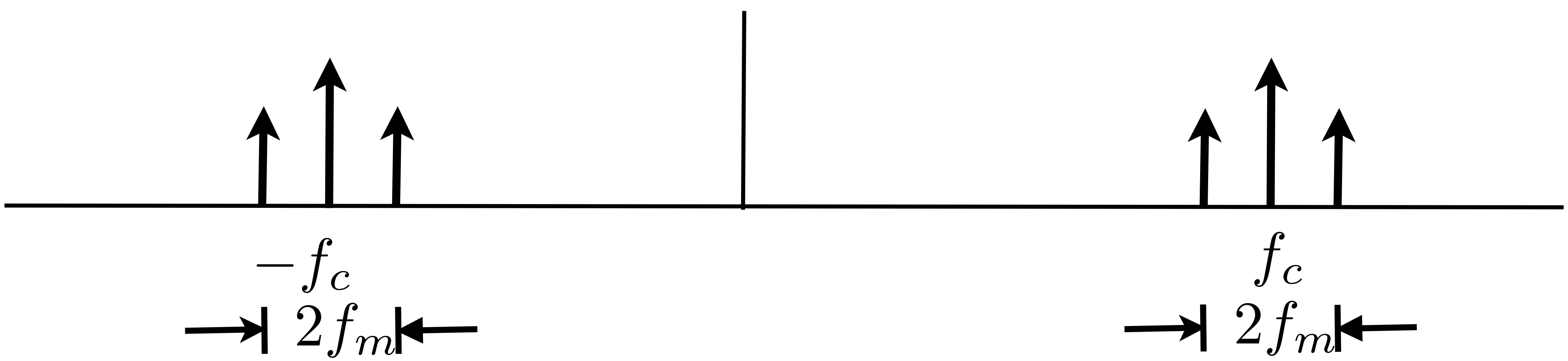
$$A_{\max} = A_c(1 + \mu), \quad A_{\min} = A_c(1 - \mu)$$

- Ratio between the max and min values

$$\frac{A_{\max}}{A_{\min}} = \frac{A_c(1 + \mu)}{A_c(1 - \mu)} \qquad \Longrightarrow \mu = \frac{A_{\max} - A_{\min}}{A_{\max} + A_{\min}}$$

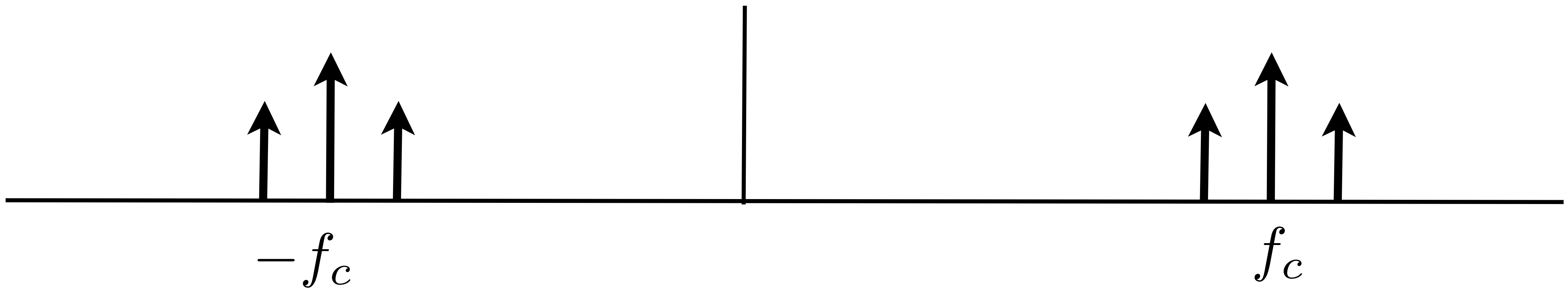
- Fourier transform

$$
\begin{aligned}
s(t) &= A_c \cos(2\pi f_c t) + \frac{1}{2}\mu A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2}\mu A_c \cos[2\pi(f_c - f_m)t] \\
S(f) &= \frac{1}{2}A_c\left[\delta(f - f_c) + \delta(f + f_c)\right] \\
&\quad + \frac{1}{4}\mu A_c\left[\delta(f - f_c - f_m) + \delta(f + f_c + f_m)\right] \\
&\quad + \frac{1}{4}\mu A_c\left[\delta(f - f_c + f_m) + \delta(f + f_c - f_m)\right]
\end{aligned}
$$



- Recall the single-tone modulated signal

$$s(t) = A_c \cos(2\pi f_c t) + \frac{1}{2}A_c\mu \cos[2\pi(f_c + f_m)t)] + \frac{1}{2}A_c\mu \cos[2\pi(f_c - f_m)t]$$



- Power calculation
  - Carrier power = $\frac{1}{2}A_c^2$
  - Upper-side-frequency power = $\frac{1}{8}\mu^2 A_c^2$
  - Lower-side-frequency power = $\frac{1}{8}\mu^2 A_c^2$

- Power ratio
  - Total power = $\frac{1}{2}A_c^2 + \frac{1}{8}\mu^2 A_c^2 + \frac{1}{8}\mu^2 A_c^2 = 0.25\left(2+\mu^2\right)A_c^2$
  - Power portion of carrier signal

$$\frac{\text{Carrier power}}{\text{Total power}} = \frac{2}{2+\mu^2}$$

  - Power portion of message signal

$$\frac{\text{Uppersideband+Lowersideband}}{\text{Total power}} = \frac{\mu^2}{2+\mu^2}$$

  - If $\mu = 1$, only 1/3 out of total power is allocated to the message signal.

Envelope Detector

- The narrowband message signal modulated by AM can be recovered at the receiver by a simple envelope detector circuit

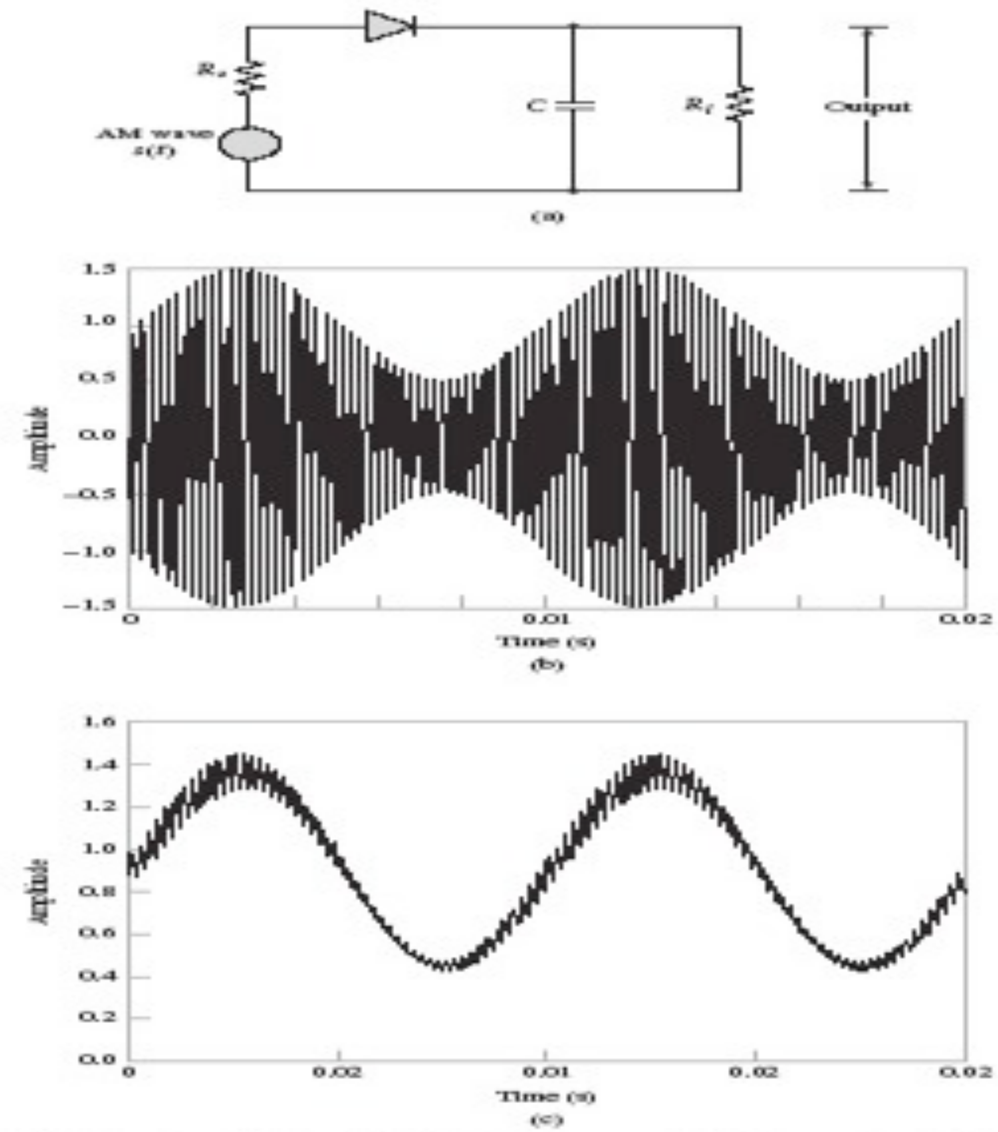


FIGURE 3.9 Envelope detector. (a) Circuit diagram. (b) AM wave input. (c) Envelope detector output

$$\frac{1}{f_c} << R_l C << \frac{1}{W}$$

[Ref: Haykin & Moher, Textbook]

Review on AM

- Amplitude modulation

$$s(t) = A_c[1 + k_a m(t)] \cos(2\pi f_c t)$$

Message bandwidth = W

Transmission bandwidth: $B_T = 2W$

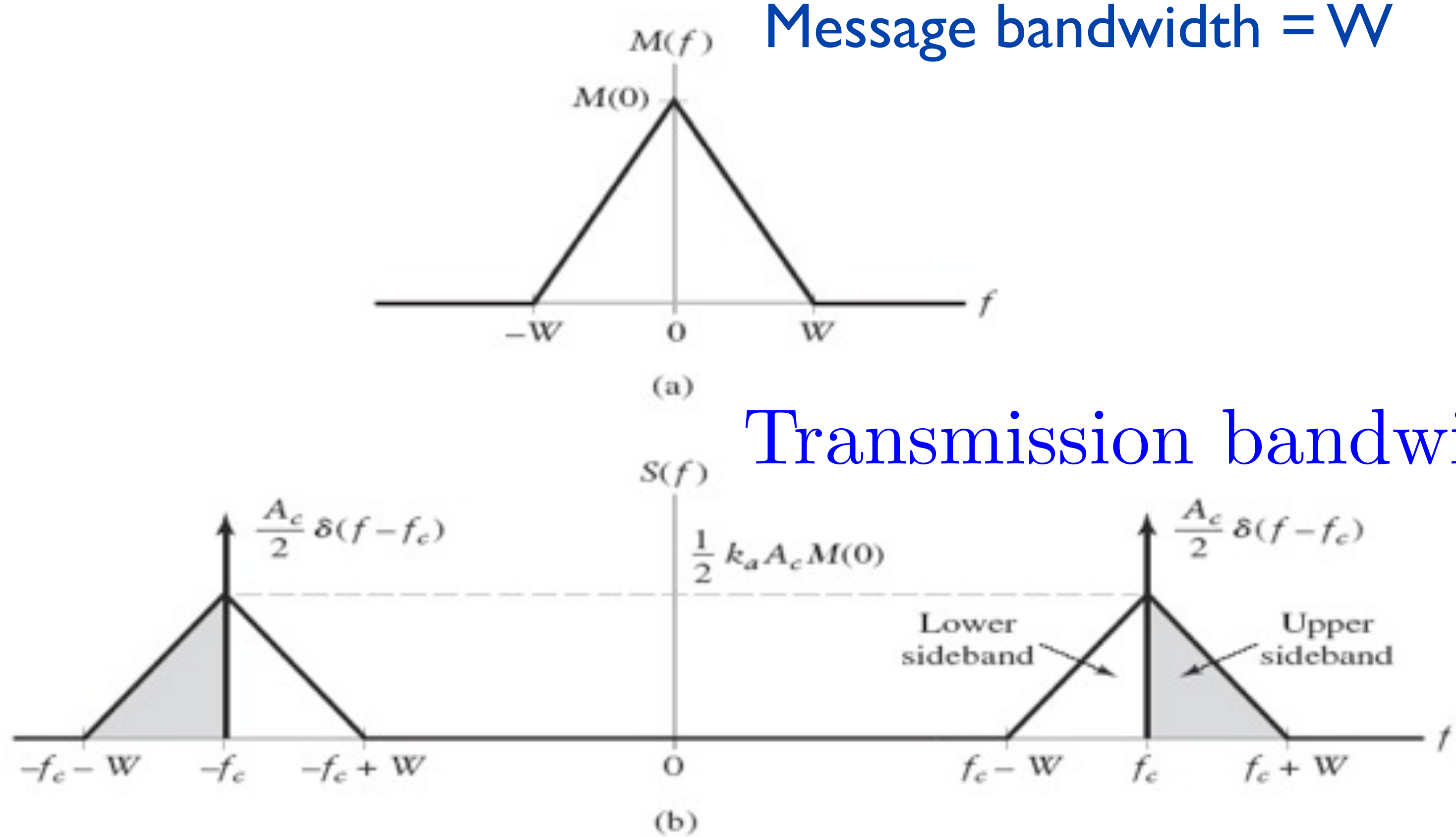


FIGURE 3.2 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of AM wave $s(t)$.

Virtues and Limitations of Amplitude Modulations

- Physical limitation
    - Amplitude modulation is wasteful of transmitted power.
        - The transmission of the carrier wave represents a waste of power.
    - Amplitude modulation is wasteful of channel bandwidth
        - As far as the transmission of information is concerned, only one sideband is necessary, and the communication channel therefore needs to provide only the same bandwidth as the message signal.
        - It requires a transmission bandwidth equal to twice the message bandwidth.

Modifications of Amplitude Modulations

- Three different modifications of amplitude modulation
  - Double sideband-suppressed carrier (DSB-SC) modulation
    - The transmitted wave consists of only the upper and lower sidebands
    - But the channel bandwidth requirement is the same as before.
  - Single sideband (SSB) modulation
    - The modulation wave consists only of the upper sideband or the lower sideband.
    - To translate the spectrum of the modulating signal to a new location in the frequency domain.

- Vestigial sideband (VSB) modulation
  - One sideband is passed almost completely and just a trace of the other sideband is retained.
  - The required channel bandwidth is slightly in excess of the message bandwidth by an amount equal to the width of the vestigial sideband.

Double Sideband-Suppressed Carrier (DSB-SC) Modulation

- Theory
  - DSB-SC (product modulation) consists of the product of the message and the carrier wave

$$s(t) = c(t)m(t) = A_c \cos(2\pi f_c t)m(t)$$

  - Fourier transform of the modulated signal

$$S(f) = \frac{1}{2}A_c\left[M(f - f_c) + M(f + f_c)\right]$$

    - Its only advantage is saving transmitted power, which is important enough when the available transmitted power is at a premium.

Phase Reversal Problem in DSB-SC

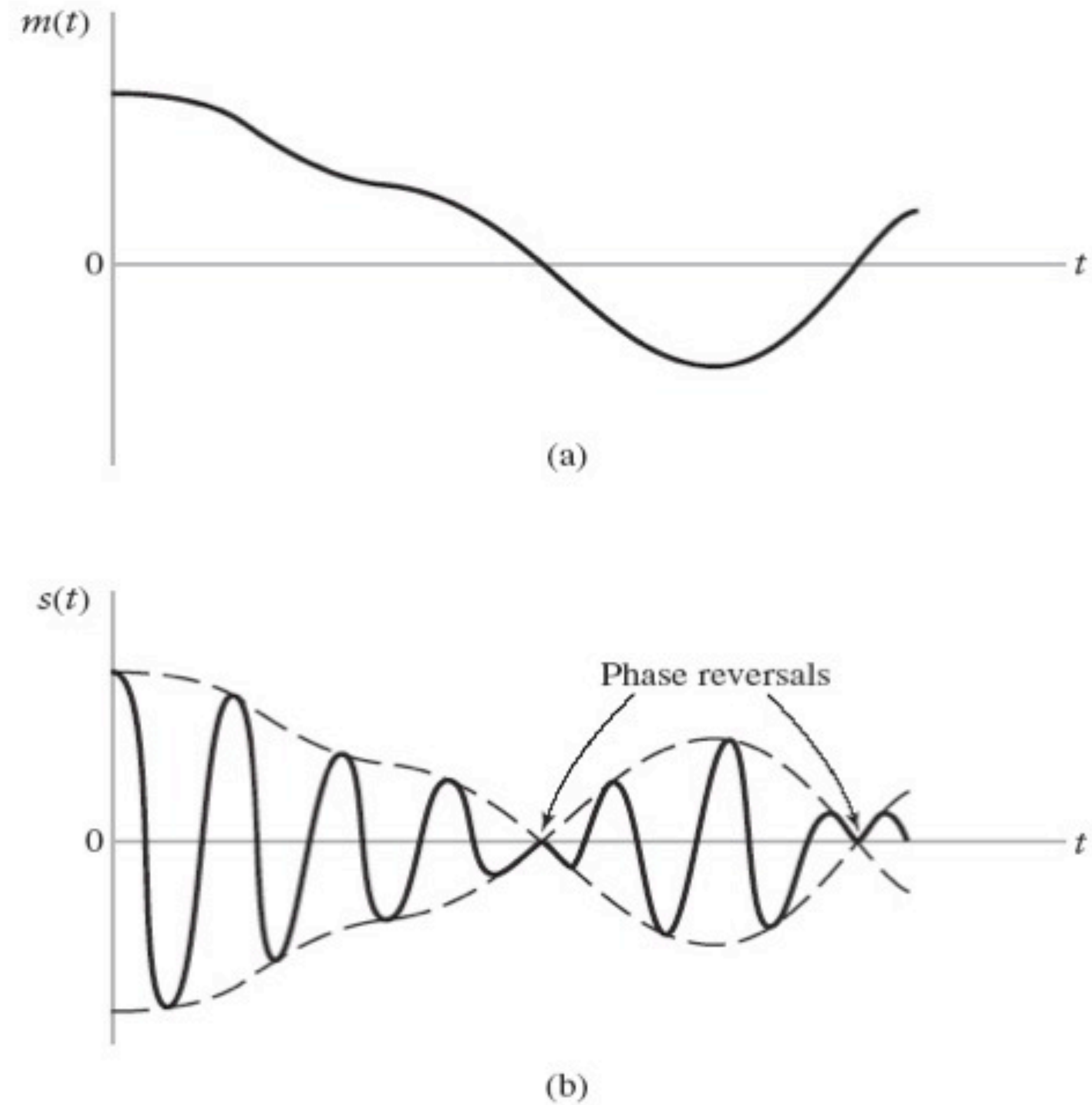


FIGURE 3.10 (*a*) Message signal $m(t)$. (*b*) DSB-SC modulated wave $s(t)$.

Spectrum of the Modulated Signal by DSB-SC

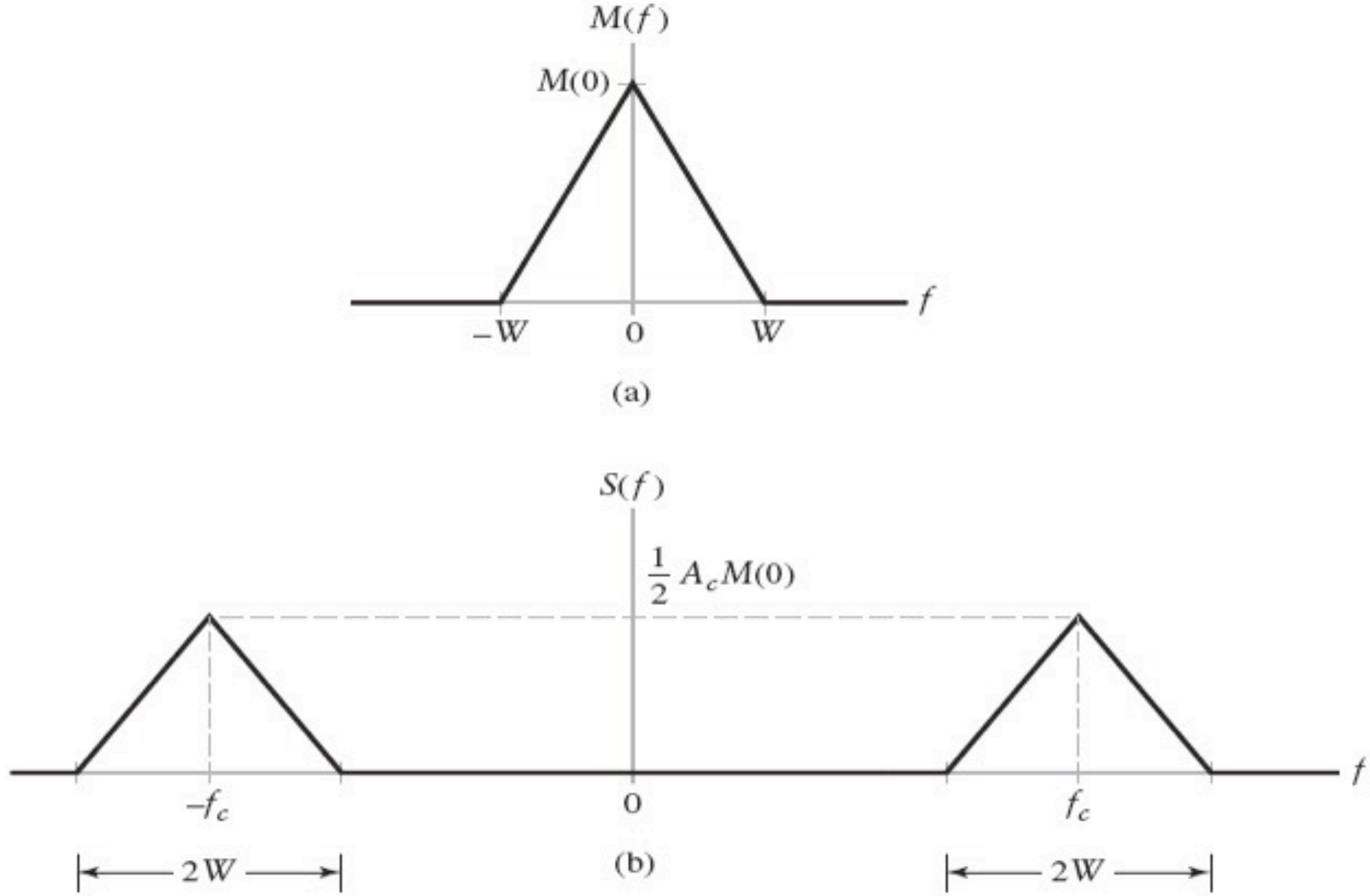


FIGURE 3.11 (*a*) Spectrum of message signal $m(t)$. (*b*) Spectrum of DSB-SC modulated wave $s(t)$.

Sinusoidal DSB-SC Spectrum

- Consider a sinusoidal modulating message signal given as

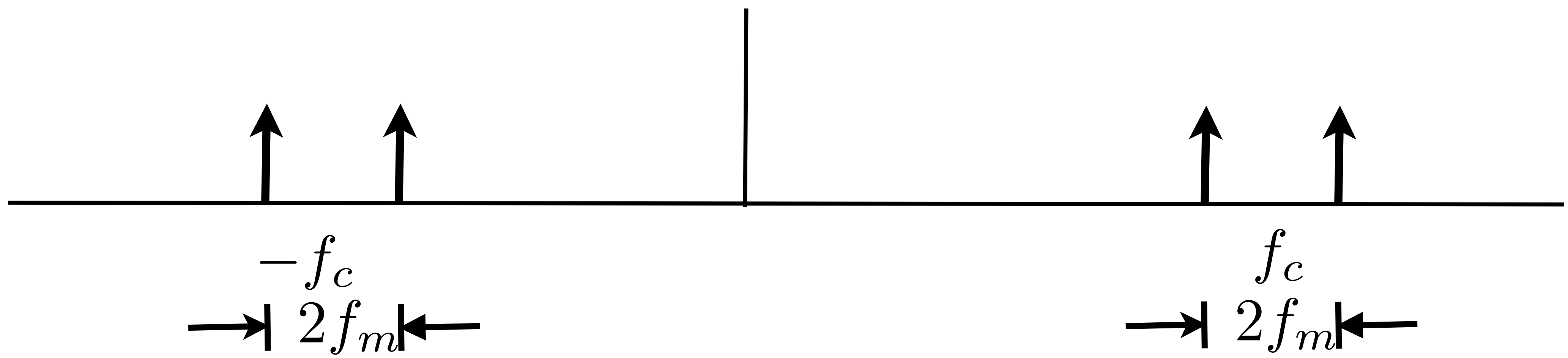
$$m(t) = A_m \cos(2\pi f_m t)$$

- The message spectrum is

$$M(f) = \frac{A_m}{2}\delta(f - f_m) + \frac{A_m}{2}\delta(f + f_m)$$

- The spectrum of DSB-SC modulated wave is

$$\begin{aligned} S(f) \quad &= \quad \frac{A_c A_m}{4}\delta(f - (f_c + f_m)) + \frac{A_c A_m}{4}\delta(f - (f_c - f_m)) \\ &\quad + \frac{A_c A_m}{4}\delta(f + (f_c + f_m)) + \frac{A_c A_m}{4}\delta(f + (f_c - f_m)) \end{aligned}$$


$-f_c$
$2f_m$
f_c
$2f_m$

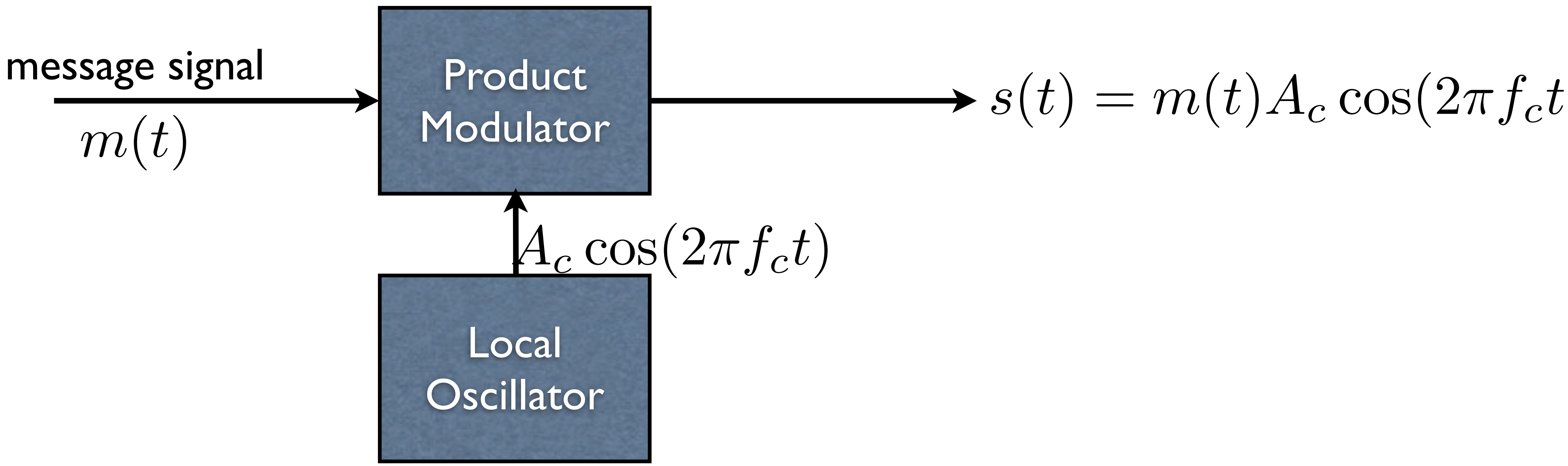
Detection of DSB-SC

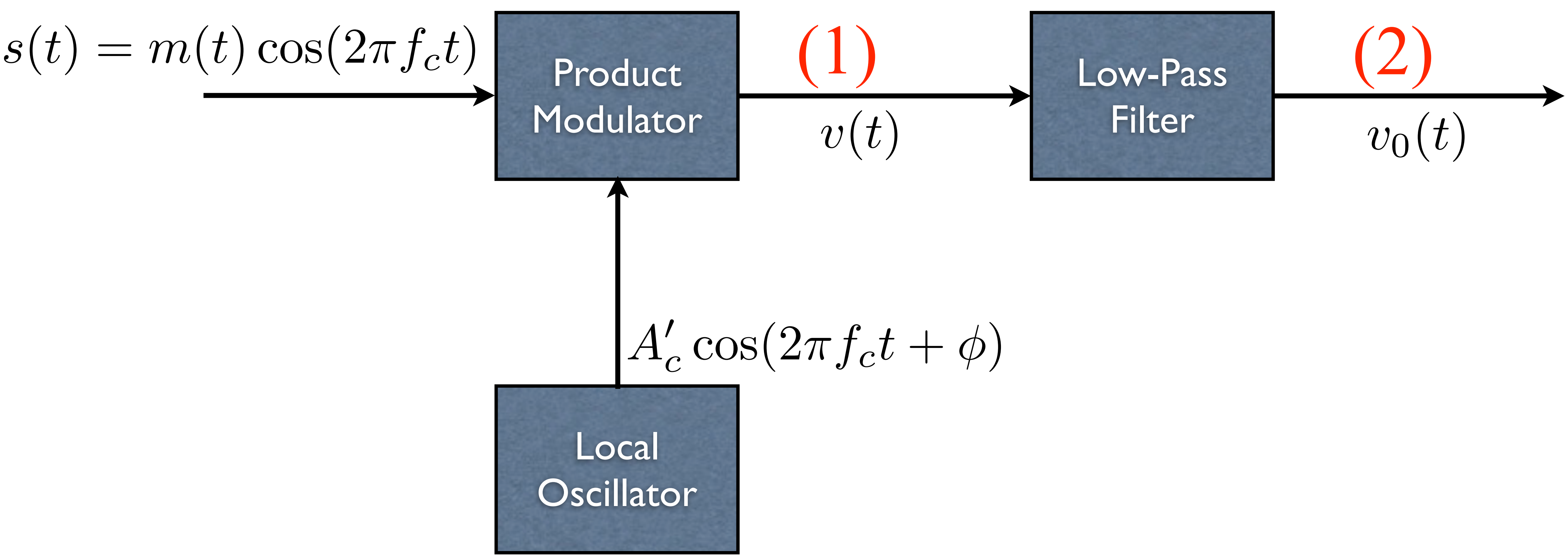
- Envelope detector cannot be used for the detection of the DSB-SC modulated message signal due to the phase reversal problem.

- Instead of envelope detector, coherent detection method is often employed, which is a bit expensive/complicated compared to the envelope detector.

    - Trade-off between the power and the complexity (or cost).

DSB-SC Modulator and Coherent Detector





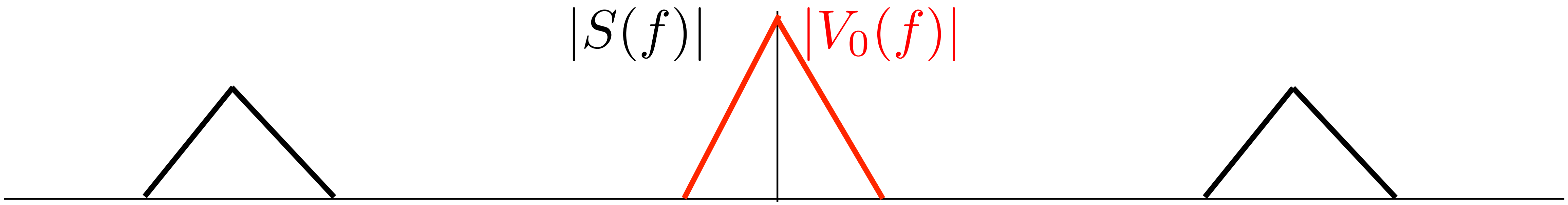
- Signal at the output of the product modulator in the coherent detector

$$\begin{aligned} v(t) &= s(t)A_c' \cos(2\pi f_c t + \phi) \\ &= A_c A_c' \cos(2\pi f_c t)\cos(2\pi f_c t + \phi) \\ &= \frac{A_c A_c'}{2}\cos(4\pi f_c t + \phi)m(t) + \frac{A_c A_c'}{2}\cos(\phi)m(t) \end{aligned}$$

- Signal at the output of the low-pass filter

$$v_0(t) = \frac{A_c A_c'}{2}\cos(\phi)m(t)$$

- The quadrature null effect
  - The zero demodulated signal occurs for $\phi = \pm\pi/2$
  - The phase error ϕ in the local oscillator causes the detector output to be attenuated by a factor equal to $\cos(\phi)$

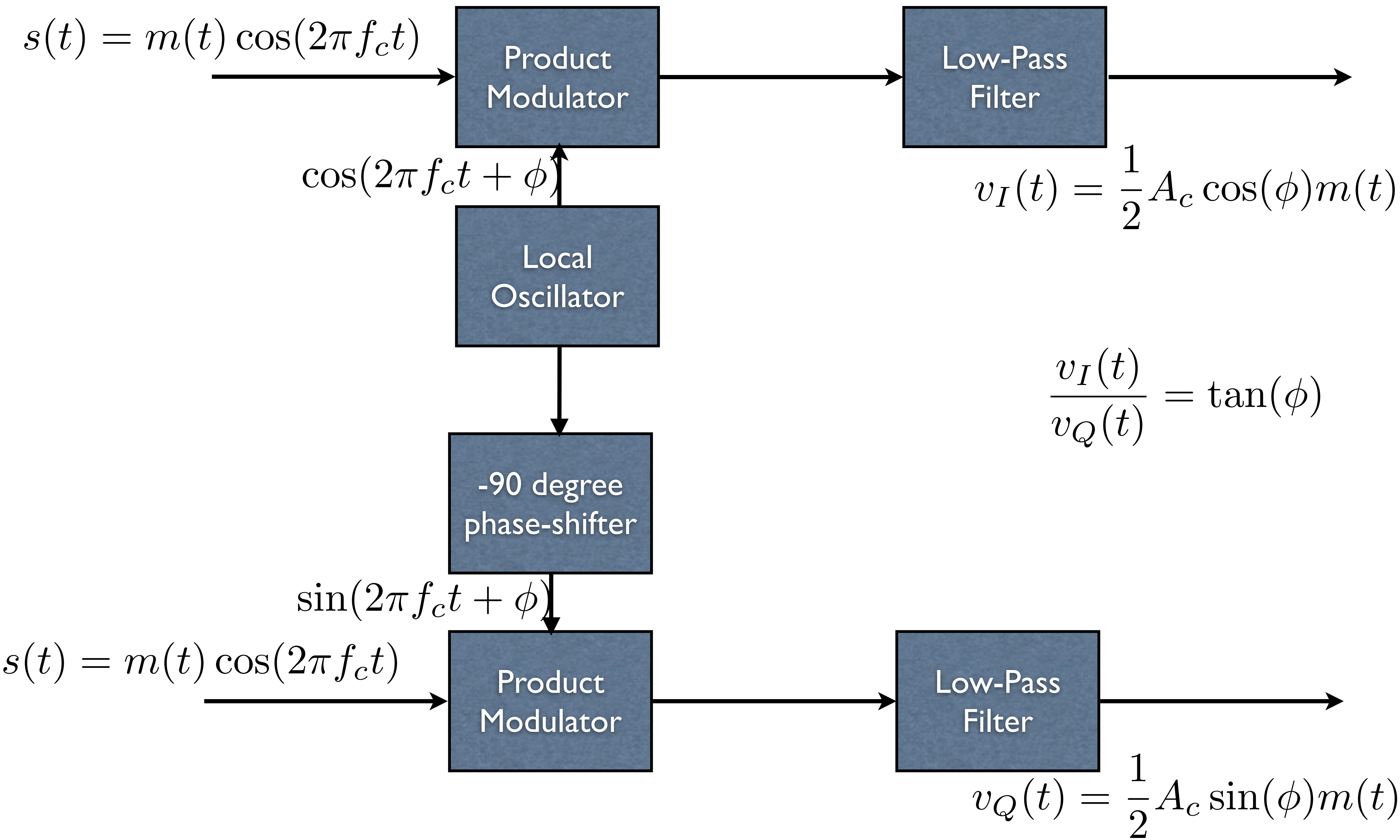


$$|V_0(0)| = \frac{A_c A_c'}{2} M(0)\cos(\phi)$$

Some Remarks on Coherent Detector

- Local oscillator may generate the incorrect frequency due to
  - temperature, aging, and so on.
- Coherent detection of a DSB-SC modulated wave requires
  - the locally generated carrier in the receiver needs to be synchronous in both frequency and phase with the carrier of the transmitter.
  - which is a rather demanding requirement.
- To overcome this problem it will be nice if we can estimate the phase error and the receiver with the estimation of phase error and correction based on the estimated phase error is called "Costas receiver".

In-Phase Coherent-Detector and Quadrature-Phase Coherent Detector



- Tagent function

$$\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + \frac{17}{315}x^7 + \cdots, \qquad \text{for } |x| < \frac{\pi}{2}$$

- For small value of x

$$\tan(x) \approx x$$

Block Diagram of Costas Receiver

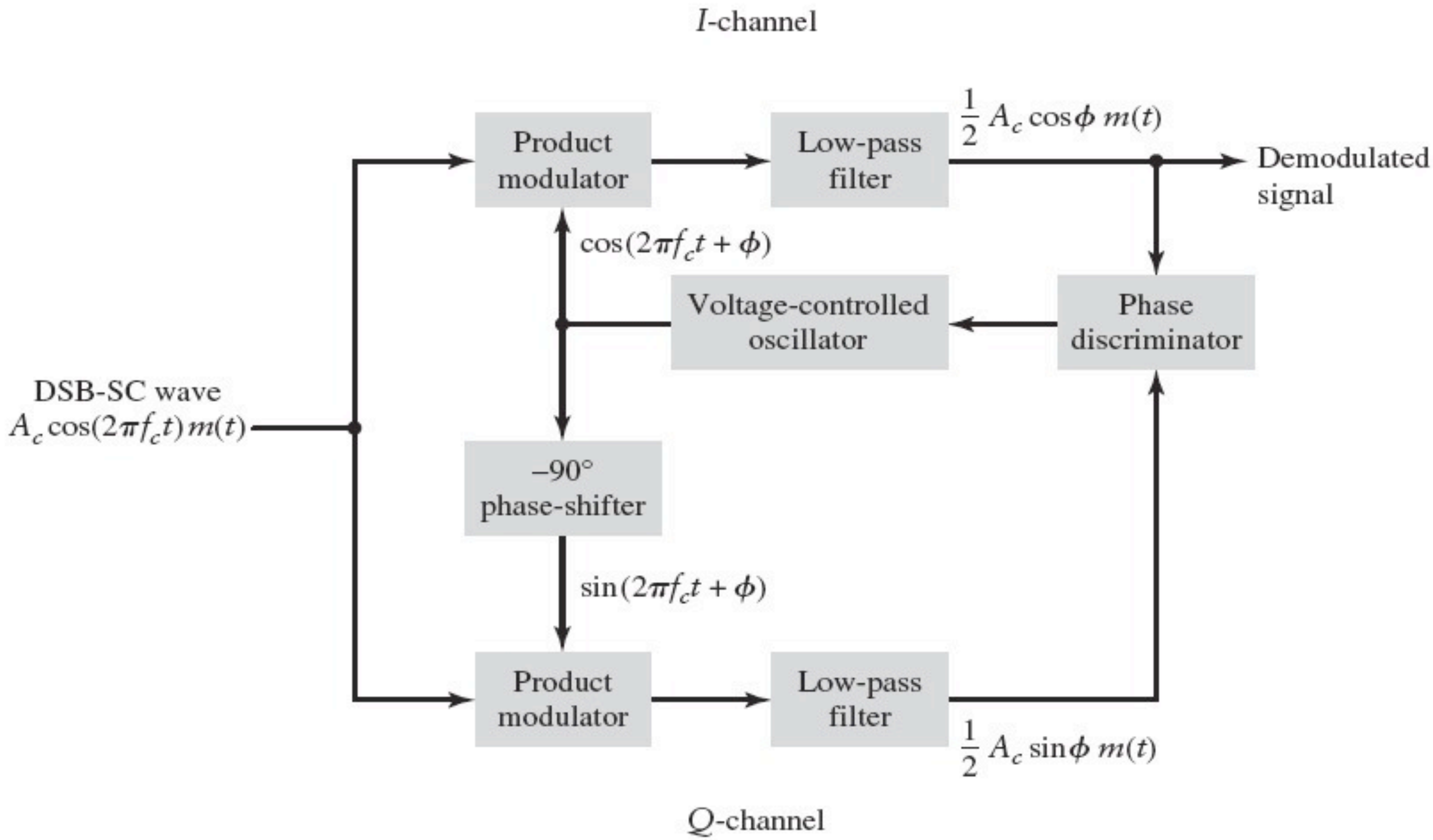


FIGURE 3.16 Costas receiver for the demodulation of a DSB-SC modulated wave.

[Ref: Haykin & Moher, Textbook]

Costas Receiver

- Consists of two coherent detectors supplied with the same input signal
    - Two local oscillators signals that are in phase quadrature with respect to each other.
        - I-Channel: In-phase coherent detector
        - Q-Channel: Quadrature-phase coherent detector
- Phase control in the Costas receiver ceases with modulation
    - Which means that phase-lock would have to be re-established with the reappearance of modulation.

Quadrature Amplitude Modulation (QAM)

- This scheme enables two DSB-SC modulated waves to occupy the same channel bandwidth.
- Bandwidth-conversion system
- This system send a pilot signal outside the passband of the modulated signal to maintain the synchronization

$$s(t) = A_c m_1(t)\cos(2\pi f_c t) + A_c m_2(t)\sin(2\pi f_c t)$$

Block Diagram of QAM Modulator/Demodulator

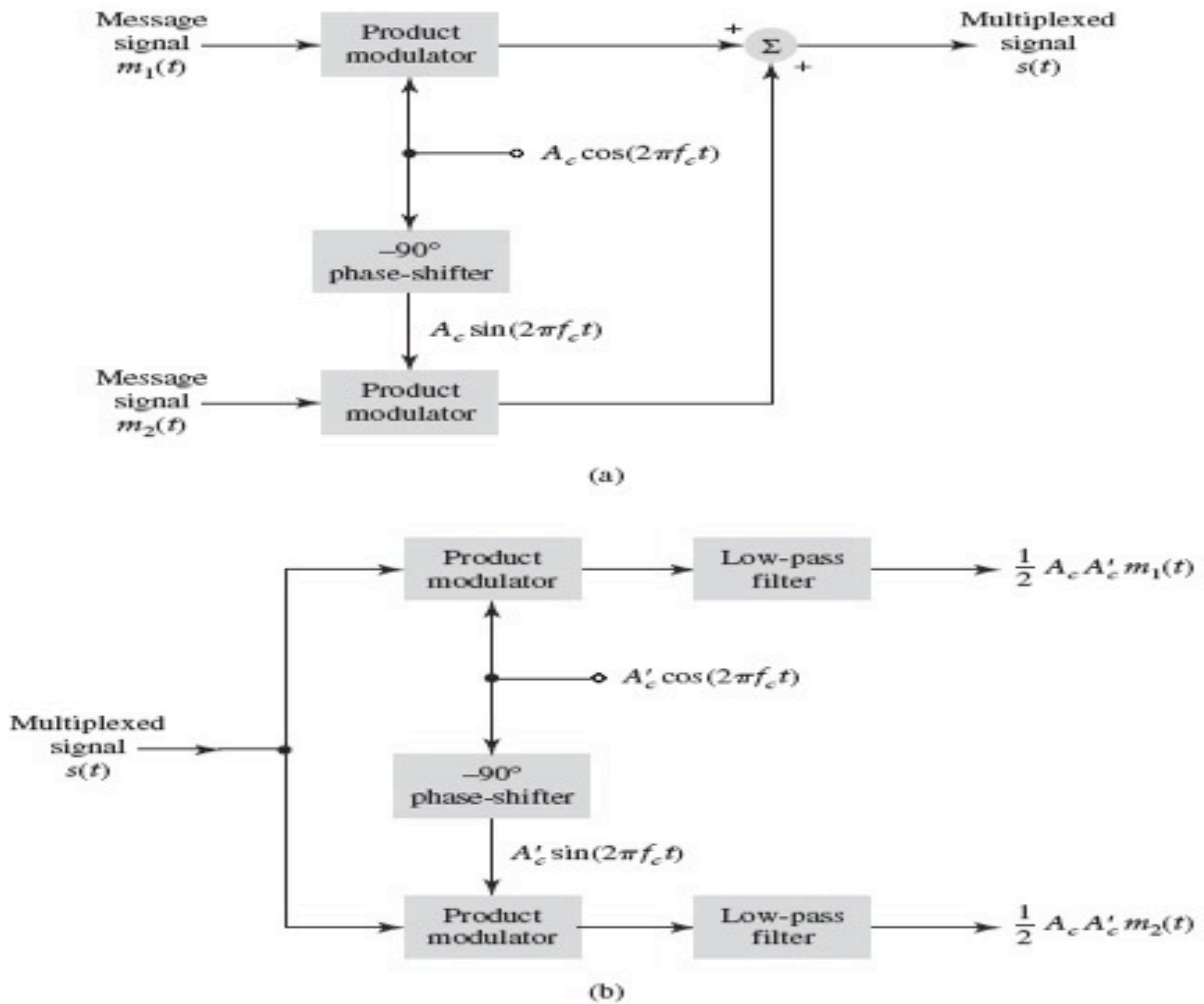


FIGURE 3.17 Quadrature-carrier multiplexing system: (*a*) Transmitter, (*b*) receiver.

[Ref: Haykin & Moher, Textbook]

Hilbert Transform

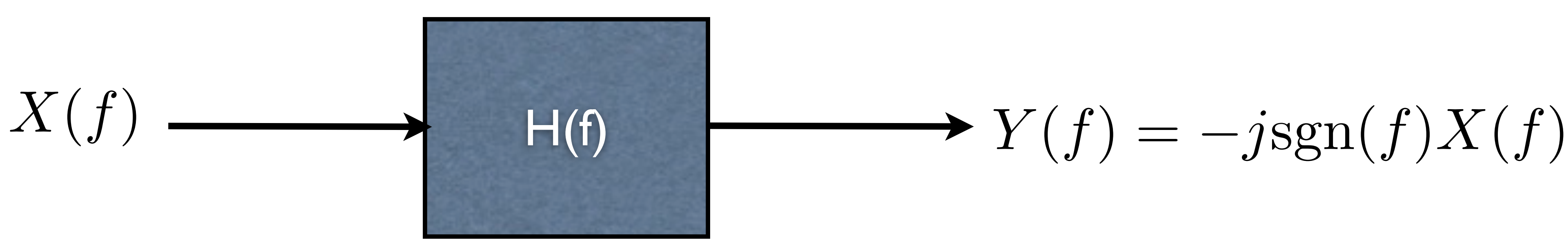
- Consider a filter that simply phase shifts all frequency components of its input by $-\pi/2$ radians, that is, its transfer function is

$$H(f) = -j\text{sgn}f$$

- Note that

$$|H(f)| = 1, \text{ and } \angle H(f) = \begin{cases} -\pi/2 & f > 0, \\ \pi/2 & f < 0 \end{cases}$$

- Input-Hilbert filter-Output signals



- Let us denote

$$\hat{x}(t) = \mathcal{F}^{-1}[Y(f)]$$

- Then

$$\hat{x}(t) = \mathcal{F}^{-1}[-j\mathrm{sgn}(f)X(f)] = h(t) * x(t)$$

- Now let us calculate the inverse transform of h(t).
    - Recall $\mathcal{F}[\mathrm{sgn}(t)] = \dfrac{1}{j\pi f}$, then using the duality property we have

$$\mathcal{F}^{-1}[\mathrm{sgn}(f)] = \frac{1}{j\pi(-f)} = \frac{j}{\pi t}$$

- We get the Fourier transform pair

$$\frac{j}{\pi t} \Longleftrightarrow \mathrm{sgn}(f) \qquad \text{or} \qquad \frac{1}{\pi t} \Longleftrightarrow -j\mathrm{sgn}(f)$$

- Now we obtain the output of the filter

$$\hat{x}(t) = x(t) * h(t) = \int_{-\infty}^{\infty} \frac{x(\lambda)}{\pi(t-\lambda)}\, d\lambda$$

- The function $\hat{x}(t)$ is defined as the Hilbert transform of $x(t)$.

- Remarks

  - The Hilbert transform corresponds to a phase shift of $-\pi/2$.

  - The Hilbert transform of $\hat{x}(t)$

$$\hat{\hat{x}}(t) = -x(t)$$

Properties of Hilbert Transform

1. Energies are equal

$$|\hat{X}(f)|^2 = |-j\text{sgn}(f)|^2|X(f)|^2 = |X(f)|^2$$

2. A signal and its Hilbert transform are orthogonal;

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0 \qquad\qquad \int_{-\infty}^{\infty} X(f)\hat{X}(f)\,df = 0$$

Analytic Signals

- Definition of the analytic signal $x_p(t)$

$$x_p(t) = x(t) + j\hat{x}(t)$$

- Fourier transform of the analytic signal

$$X_p(f) = X(f) + j[-j\text{sgn}(f)X(f)] = X(f)[1 + \text{sgn}(f)]$$

or

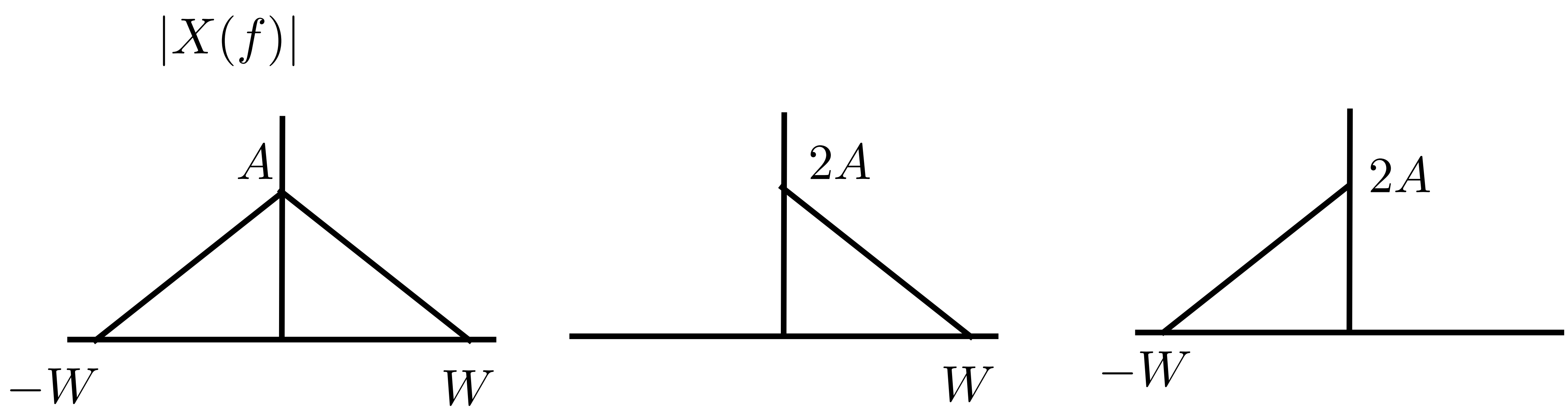
$$X_p(f) = \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

- We can also show that

$$x_q(t) = x(t) - j\hat{x}(t)$$

and its Fourier transform

$$X_q(f) = X(f)\left[1 - \mathrm{sgn}(f)\right]$$
$$= \begin{cases} 0, & f > 0 \\ 2X(f), & f < 0 \end{cases}$$



Single-Sideband (SSB) Modulation

- SSB modulation
  - Suppress one of the two sidebands in the DSB-SC modulated wave prior to transmission
- Method of SSB Modulations
  - Time domain expression
    - SSB signal from DSB-SC is derived using the Hilbert transform.
  - Frequency domain expression
    - SSB signal from DSB-SC is generated Analytic signal is derived using the analytic signal.

Generation of LSB SSB