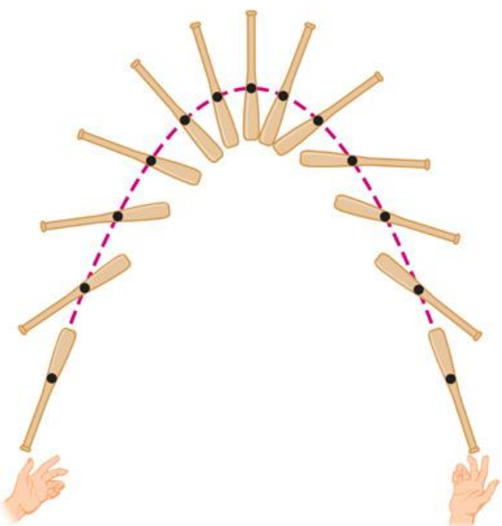
Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Ch. 8 Systems of particles and extended objects

Center of mass

(a)



(b)

조금 더 복잡한 물체의 운동

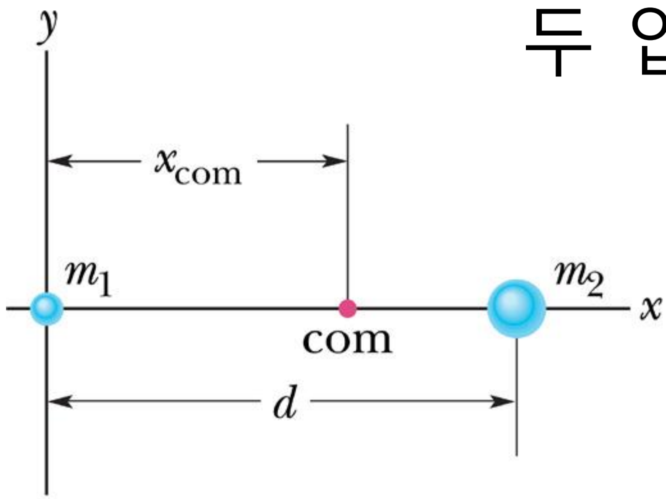
(1)물체의 크기를 무시할 수 없거나
(2)여러 입자들로 이루어진 물리계

가장 간단하게 기술할 수 있는 운동
(질량중심의 운동)

조금 더 복잡한 운동
(질량중심에 대해 움직이는 상대운동 및 회전)

center of mass
(1) 모든 질량이 그 점에 모여 있고
(2) 외부 힘이 모두 그 점에 작용하는 것처럼 움직이는 점.

두 입자로 이루어진 물리계의 질량중심



(a)

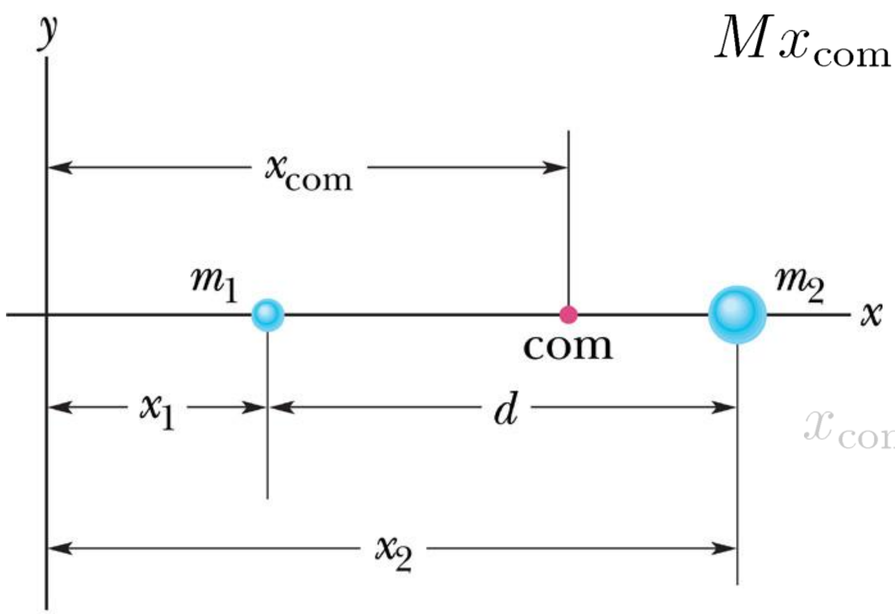
한 물체를 원점에 두었을 때

$$x_{\rm com} = \frac{m_2}{m_1 + m_2} d$$

임의의 점이 원점일 때

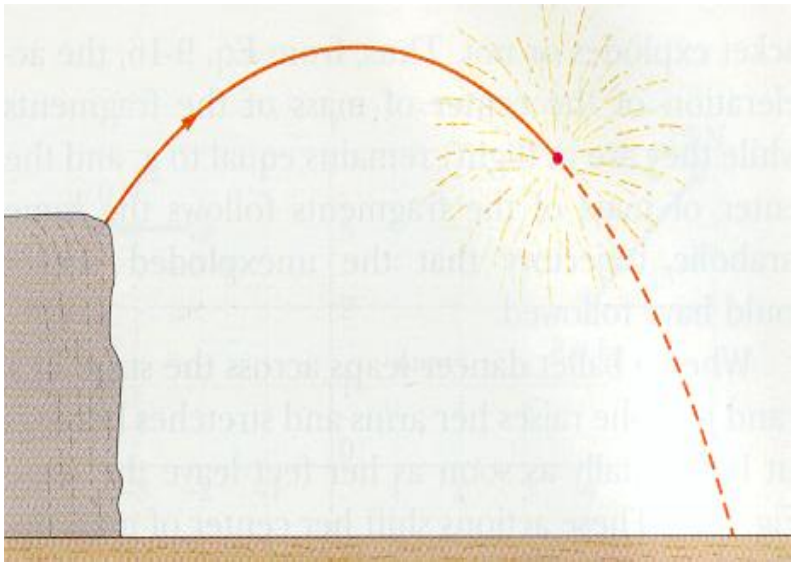
$$M x_{\rm com} = m_1 x_1 + m_2 x_2, \quad M = m_1 + m_2$$

$$x_{\rm com} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

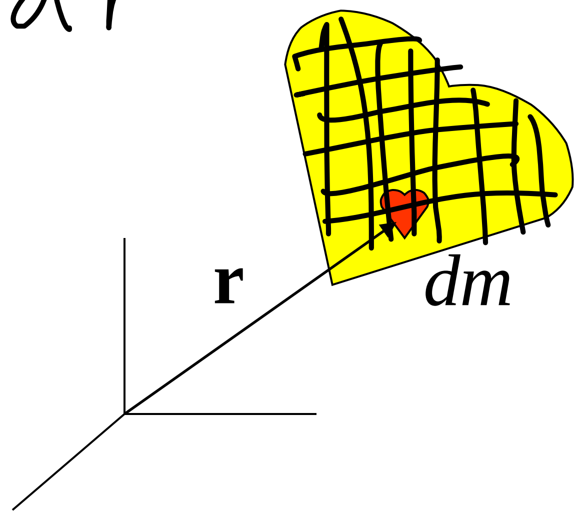


(b)

$$x_{\rm com} = \frac{m_1 x_1 + m_2 x_2 + \cdots + m_n x_n}{M} = \frac{1}{M} \sum_{i=1}^{n} m_i x_i$$



$$dV = d^3r$$



$$M\mathbf{r}_{\text{com}} = \sum_i m_i \mathbf{r}_i$$

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i,$$

$$y_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i y_i,$$

$$z_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i z_i.$$

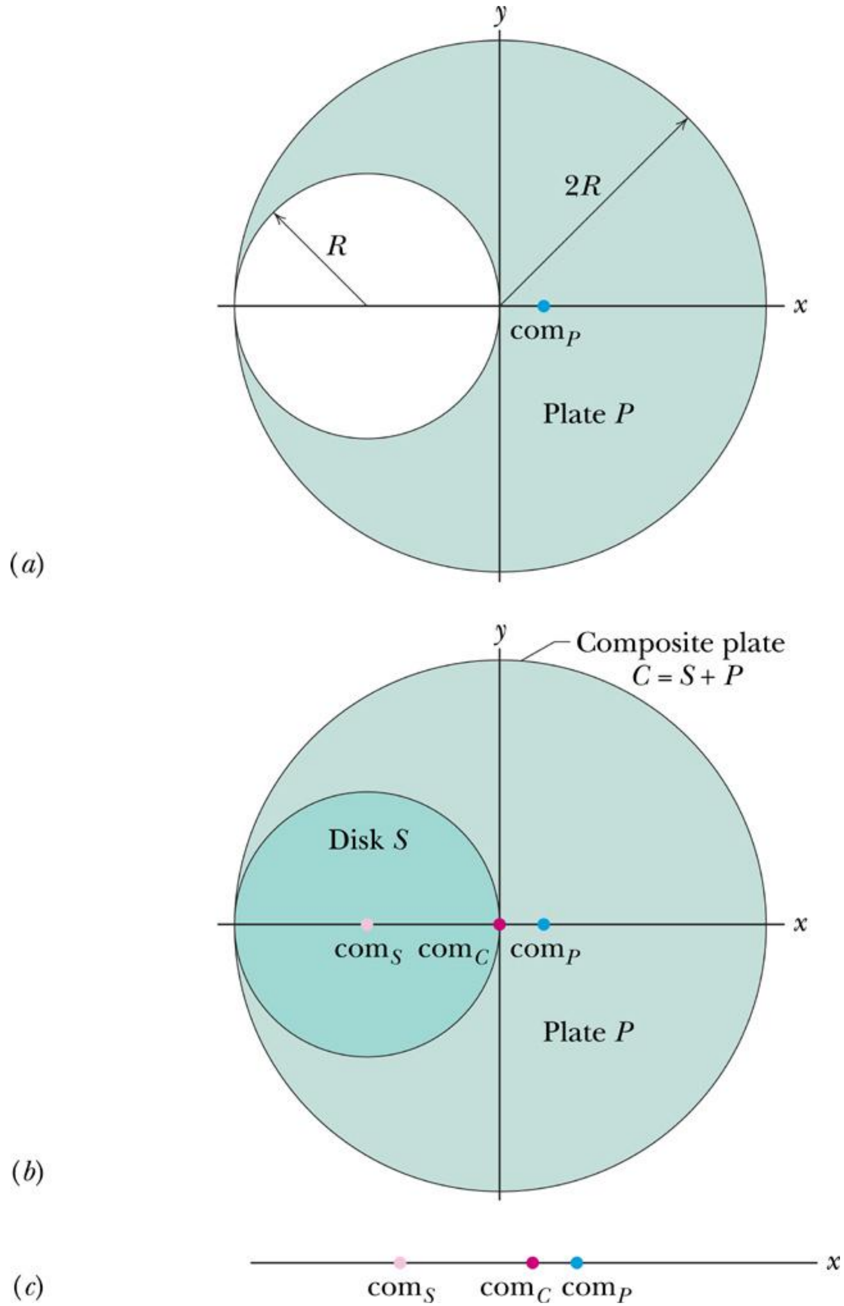
밀도가 일정한 경우: $\rho = \dfrac{dm}{dV} = \dfrac{M}{V}$

$$\mathbf{r}_{cm} = \frac{1}{M}\int \mathbf{r}\,dm$$

$$= \frac{1}{M}\int \mathbf{r}\left(\frac{M}{V}\right) d^3\mathbf{r} = \frac{1}{V}\int \mathbf{r}\,d^3\mathbf{r}$$

$\rho(\vec{r})$

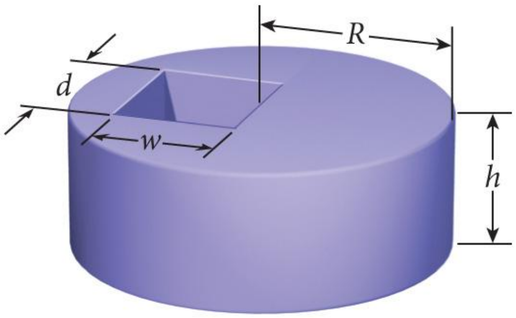
Sample problem

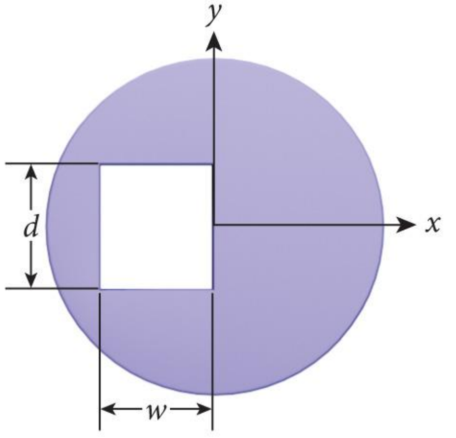


$$x_{S+P} = \frac{m_S x_S + m_P x_P}{m_S + m_P} = 0$$

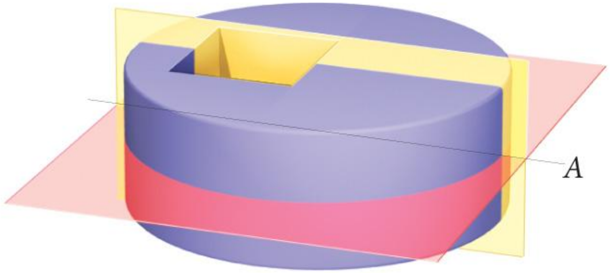
$$x_P = -x_S \frac{m_S}{m_P} = -x_S \frac{1}{3} = \frac{1}{3} R$$

S.P. 8.5 Cm of a disk with a hole



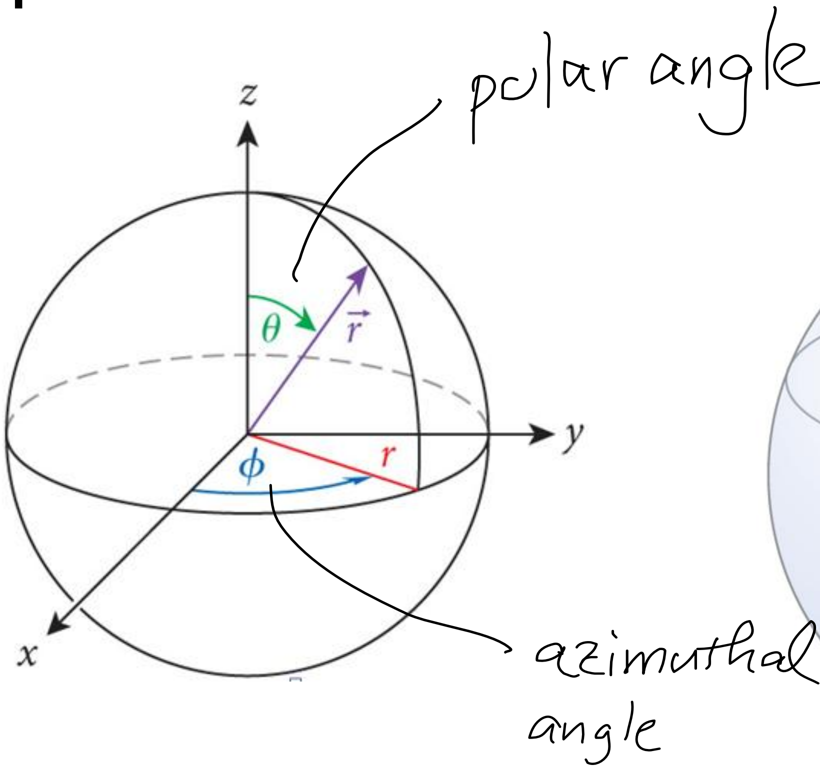


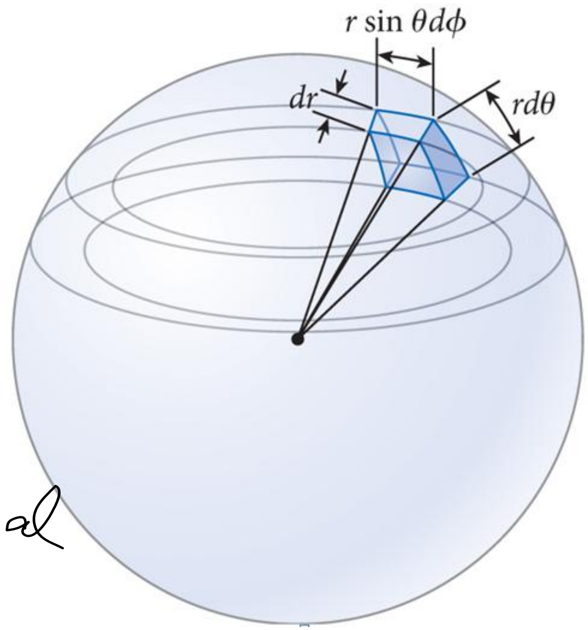
(a)



(b)

Spherical coordinates



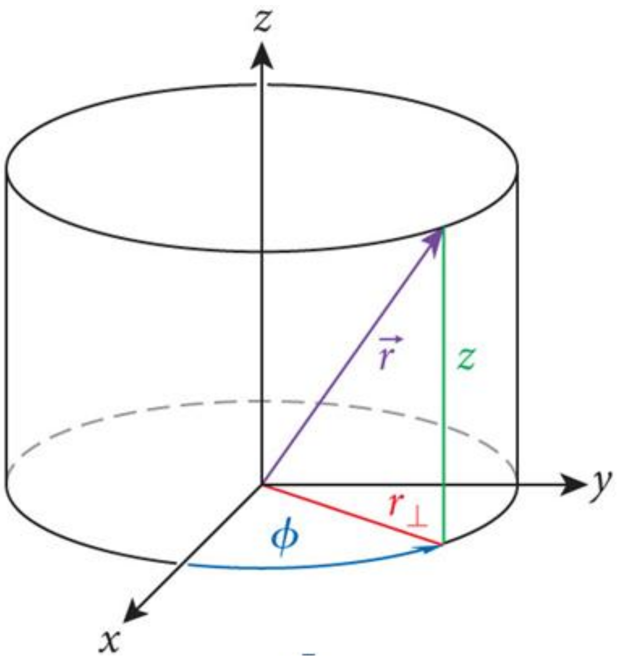


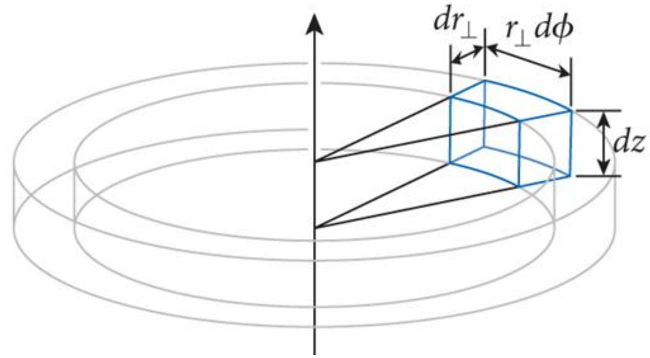
$$
\begin{aligned}
x &= r\sin\theta\cos\phi, \\
y &= r\sin\theta\sin\phi, \\
z &= r\cos\theta.
\end{aligned}
$$

Volume element

$$dV = r^2 dr \sin\theta d\theta d\phi$$

Cylindrical coordinates



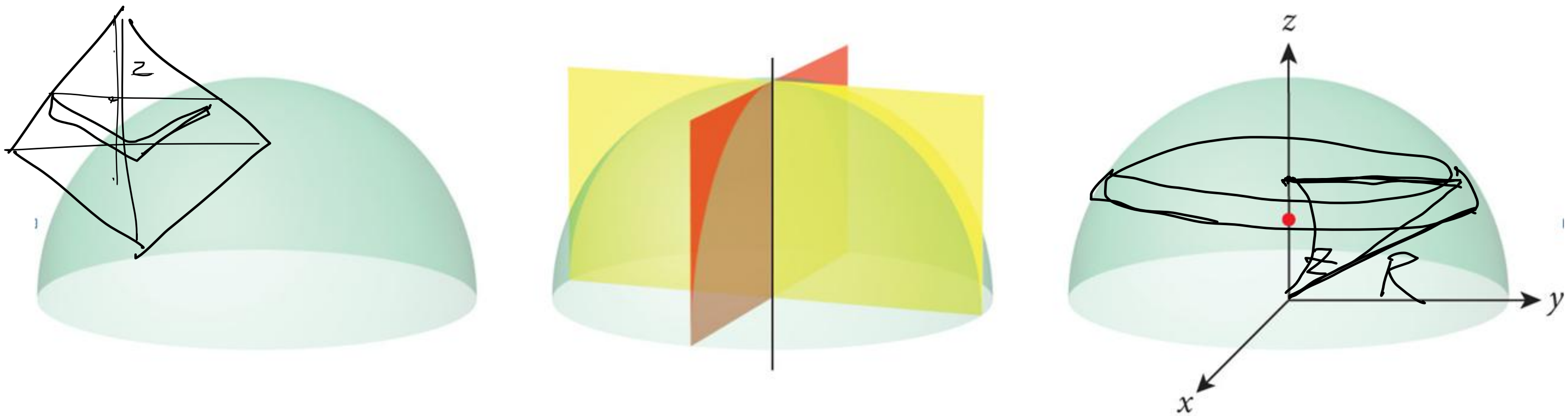


$$
\begin{aligned}
x &= r_\perp \cos\phi, \\
y &= r_\perp \sin\phi, \\
z &= z.
\end{aligned}
$$

Volume element

$$dV = r_\perp dr_\perp d\phi dz$$

Ex. 8.5: CM for a half-sphere



(a) (b) (c)

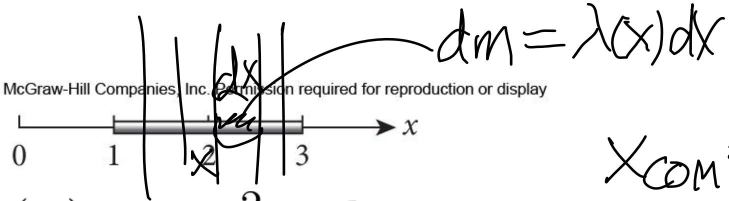
$$z_{CM} = \frac{1}{M} \int z\,dm$$

$$= \frac{1}{V} \int_0^R z\,\pi (R^2 - z^2)\,dz = \frac{\pi}{V} \int_0^R (zR^2 - z^3)\,dz$$

$$= \frac{\pi}{V}\left(\frac{R^4}{4}\right) = \frac{R\,(\pi R^3)}{4\,\frac{2}{3}(\pi R^3)} = \frac{3}{8} R$$

$$z_{\mathrm{CM}} = \frac{3}{8} R$$

SP 8.3: CM of a long, thin rod



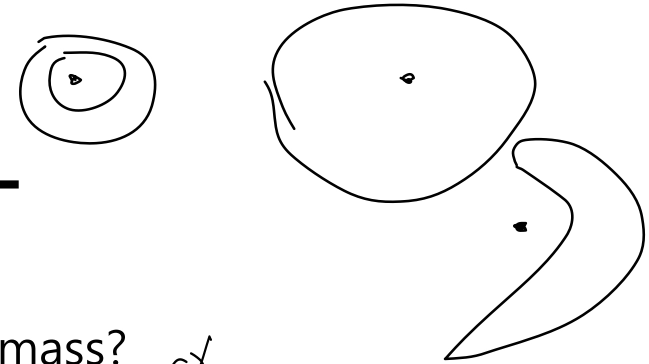
$$\lambda(x) = ax^2 + b$$

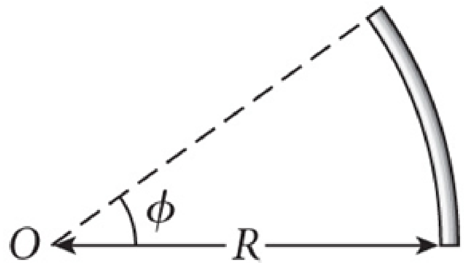
$$dm = \lambda(x)\,dx$$

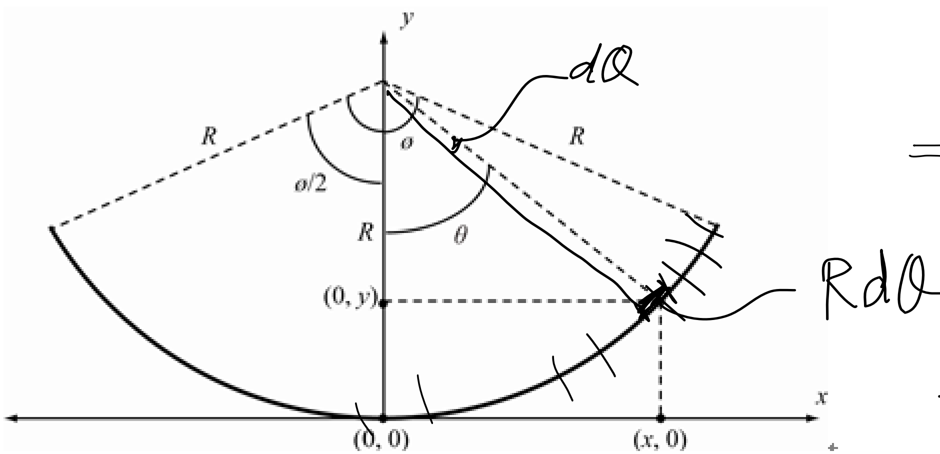
$$x_{COM} = \frac{1}{M}\int x\,dm = \frac{1}{M}\int_1^3 x\,\lambda(x)\,dx = \frac{1}{M}\int_1^3 (ax^3 + bx)\,dx = \frac{1}{M}(20a + 4b) = \frac{20a + 4b}{\frac{26}{3}a + 2b}$$

$$M = \int dm = \int \lambda(x)\,dx = \int_1^3 (ax^2 + b) = \frac{26}{3}a + 2b$$

Prob. 8.21







$$y_{\mathrm{CM}} = R - \frac{R\sin(\phi/2)}{\phi/2}$$

Center of mass?

$$y_{com} = \frac{2}{M}\int_0^{\frac{\phi}{2}} \lambda R\, d\theta\, R(1-\cos\theta)$$

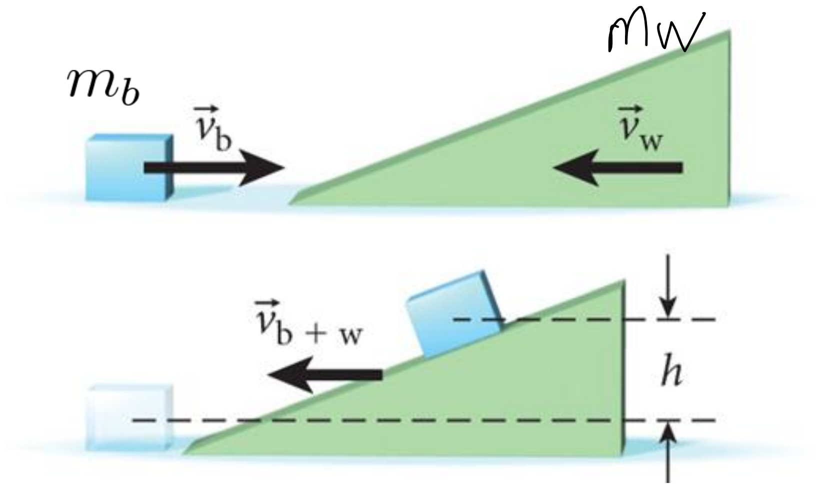
$$= \frac{2\lambda R^2}{M}\int_0^{\frac{\phi}{2}} (1-\cos\theta)\, d\theta$$

$$= \frac{2\lambda R^2}{M}\left(\frac{\phi}{2} - \sin\frac{\phi}{2}\right)$$

$$M = \lambda R \phi$$

$$y_{com} = \frac{2R}{\phi}\left(\frac{\phi}{2} - \sin\frac{\phi}{2}\right) = R\left(1 - \frac{\sin\frac{\phi}{2}}{\phi/2}\right)$$

Prob. 8.36



m_w?

$$m_b v_b + m_w v_w = (m_b + m_w) v_{b+w}$$

$$\frac{1}{2} m_b v_b^2 + \frac{1}{2} m_w v_w^2 = \frac{1}{2}(m_b + m_w) v_{b+w}^2 + m_b g h$$

입자계에 대한 Newton의 법칙

$$\mathbf{F}_{\text{net}} = M\mathbf{a}_{\text{com}}$$

$$F_{\text{net},x} = Ma_{\text{com},x},\ F_{\text{net},y} = Ma_{\text{com},y},\ F_{\text{net},z} = Ma_{\text{com},z}.$$

증명:

$$M\mathbf{r}_{\text{com}} = m_1\mathbf{r}_1 + m_2\mathbf{r}_2 + \cdots + m_n\mathbf{r}_n$$

$$M\mathbf{v}_{\text{com}} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_n\mathbf{v}_n$$

$$\begin{aligned} M\mathbf{a}_{\text{com}} &= m_1\mathbf{a}_1 + m_2\mathbf{a}_2 + \cdots + m_n\mathbf{a}_n \\ &= \mathbf{F}_1 + \mathbf{F}_2 + \cdots + \mathbf{F}_n = \mathbf{F}_{\text{net}} \end{aligned}$$

선운동량의 정의:

$$\mathbf{p} \equiv m\mathbf{v}$$

따라서 뉴턴의 방정식은

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt}$$

가 된다.

$$\mathbf{F}_{\text{net}} = \frac{d\mathbf{p}}{dt} = \frac{d}{dt}(m\mathbf{v}) = m\frac{d\mathbf{v}}{dt} = m\mathbf{a}$$

입자계의 linear momentum

여러 입자들이 있는 경우 선운동량은

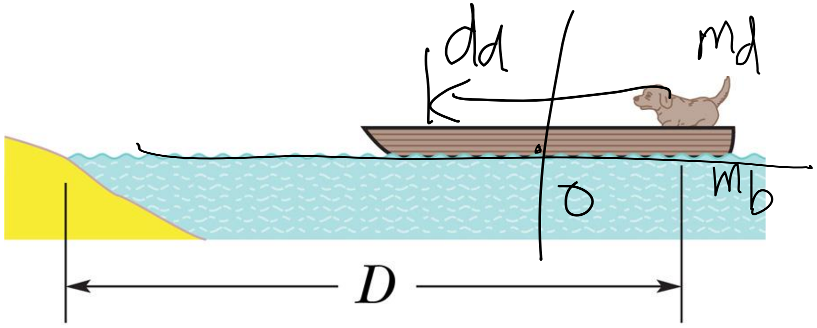
$$\begin{aligned} \mathbf{P} &= \mathbf{p}_1 + \mathbf{p}_2 + \cdots + \mathbf{p}_n \\ &= m_1\mathbf{v}_1 + m_2\mathbf{v}_2 + \cdots + m_n\mathbf{v}_n = M\mathbf{v}_{\mathrm{com}} \end{aligned}$$

이 됨을 알 수 있다. 따라서 위 식을 시간에 대해 미분하면

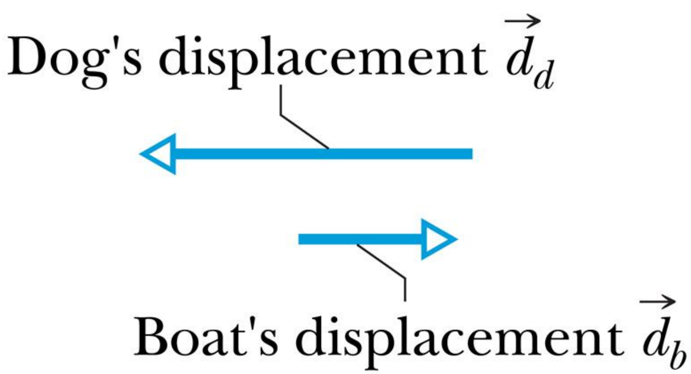
$$\frac{d\mathbf{P}}{dt} = M\frac{d\mathbf{v}_{\mathrm{com}}}{dt} = M\mathbf{a}_{\mathrm{com}}$$

을 얻는다.

Problem : 개와 배



(a)



(b)

$$m_d d_d + m_b d_b = 0$$

$$|d_b| = \frac{m_d}{m_b}|d_d|$$

$$|d_d| - |d_b| = \frac{m_b - m_d}{m_b}|d_d|$$

$$D - \frac{m_b - m_d}{m_b}|d_d|$$