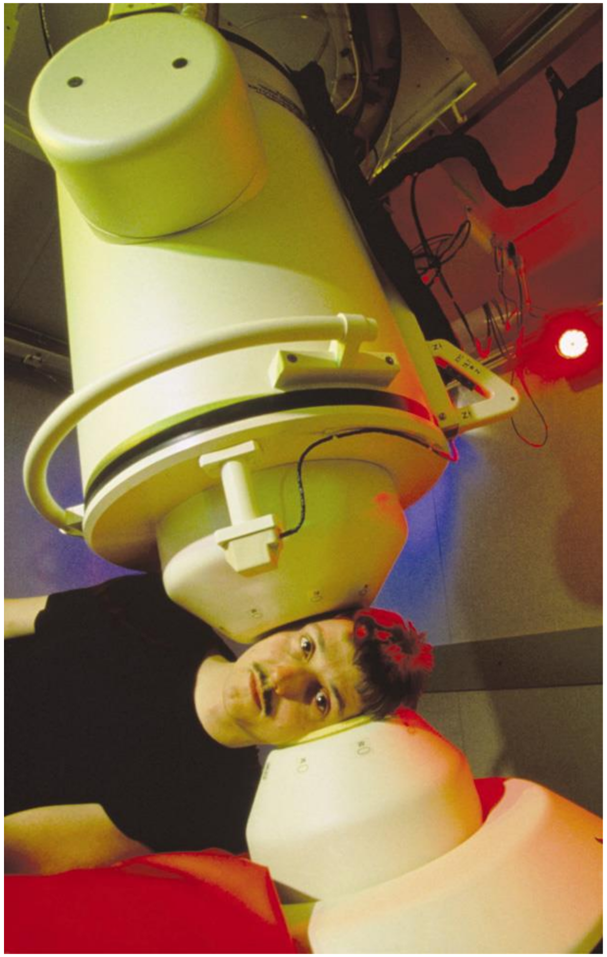
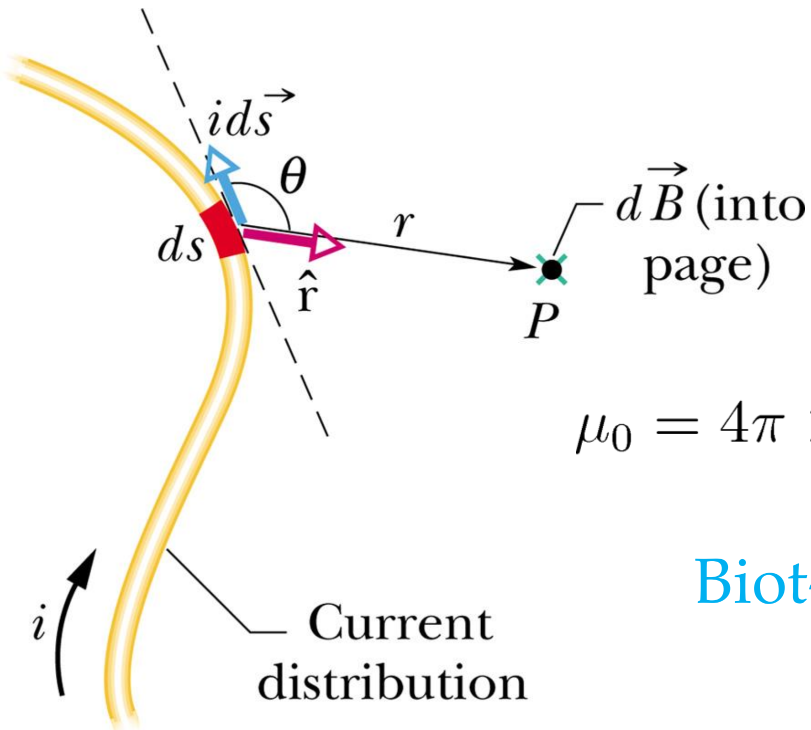
# Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

# Chap. 28 Magnetic fields of moving charges



# Biot-Savart law




$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

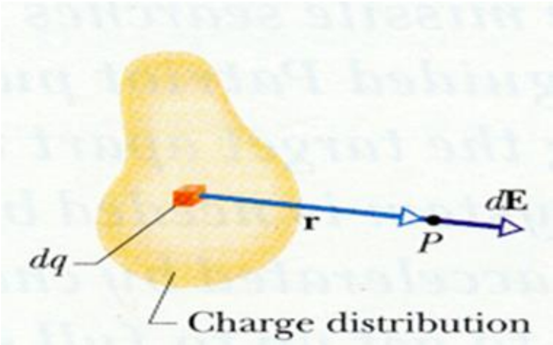
Magnetic permeability

$$\mu_0 = 4\pi\times10^{-7}\ \mathrm{T\cdot m/A} = 1.26\times10^{-6}\ \mathrm{T\cdot m/A}$$

Biot-Savart law

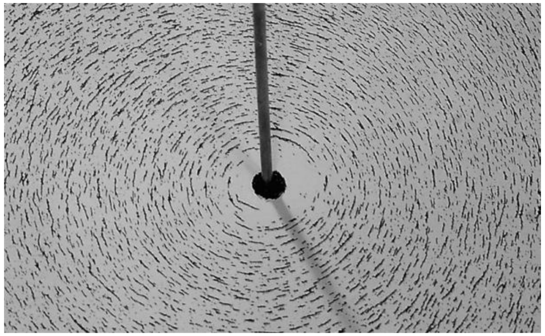
$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\vec{s}\times\vec{r}}{r^3}$$

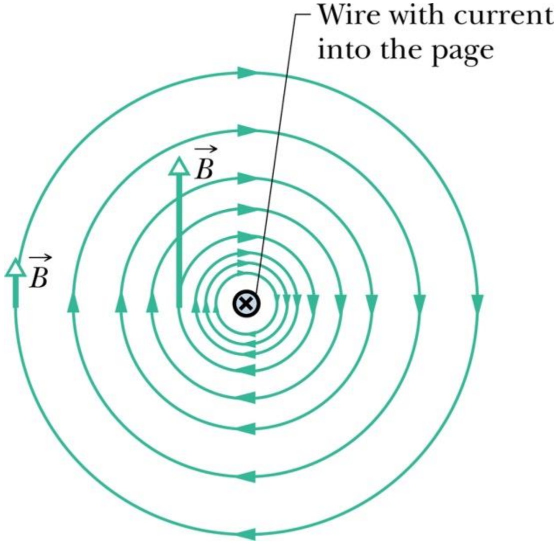
* Electric fields

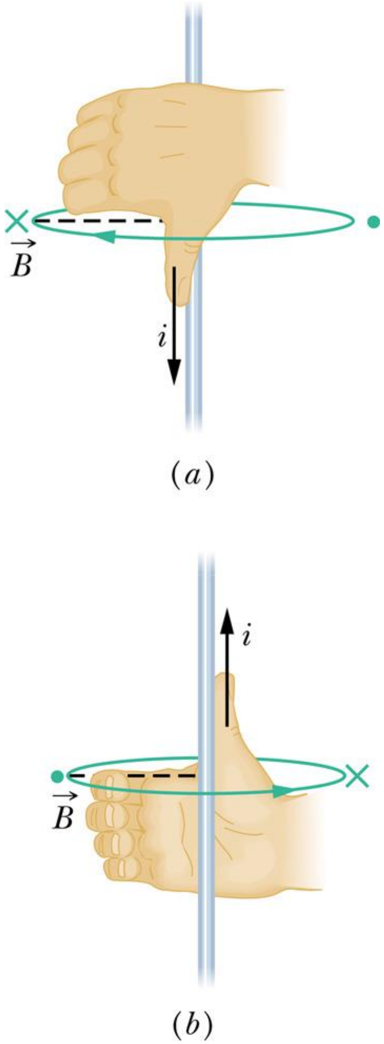


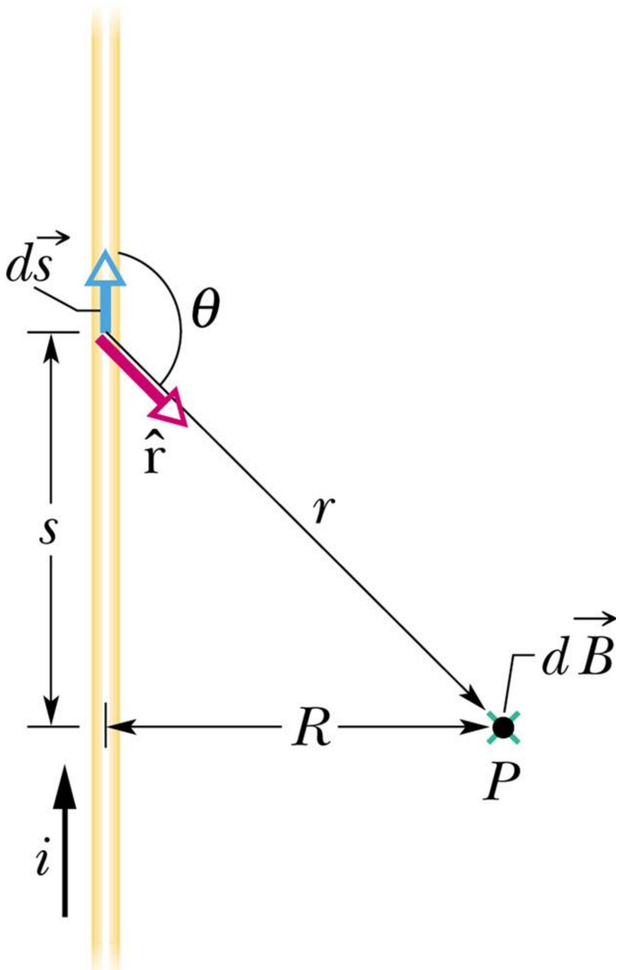

$$d\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{dq\ \vec{r}}{r^3}$$

# Magnetic field from a long, straight wire







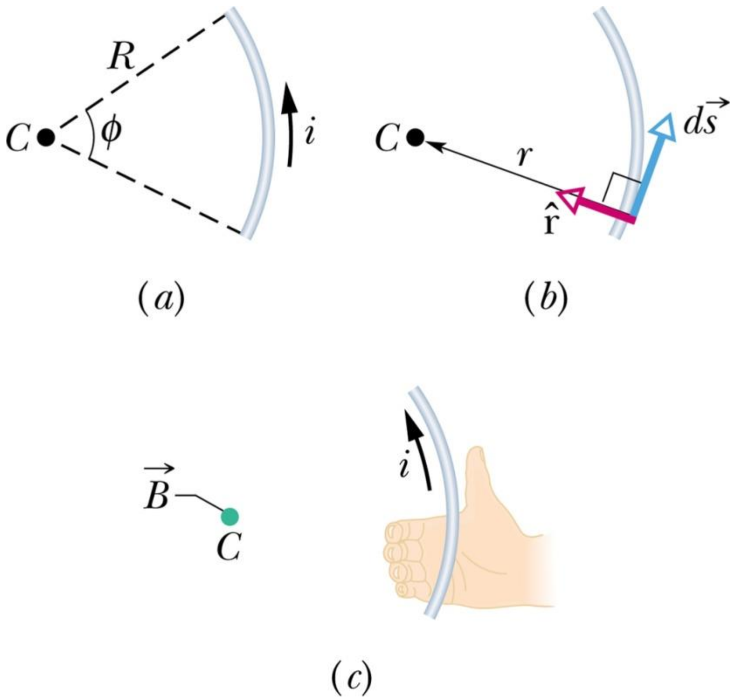

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin\theta}{r^2}$$

$$B = 2\int_0^\infty dB = \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{\sin\theta ds}{r^2}$$

$$= \frac{\mu_0 i}{2\pi}\int_0^\infty \frac{R\,ds}{(s^2+R^2)^{3/2}}$$

$$= \frac{\mu_0 i}{2\pi}\left[\frac{s}{(s^2+R^2)^{1/2}}\right]_0^\infty = \frac{\mu_0 i}{2\pi R}$$

# Magnetic field due to current through an arc



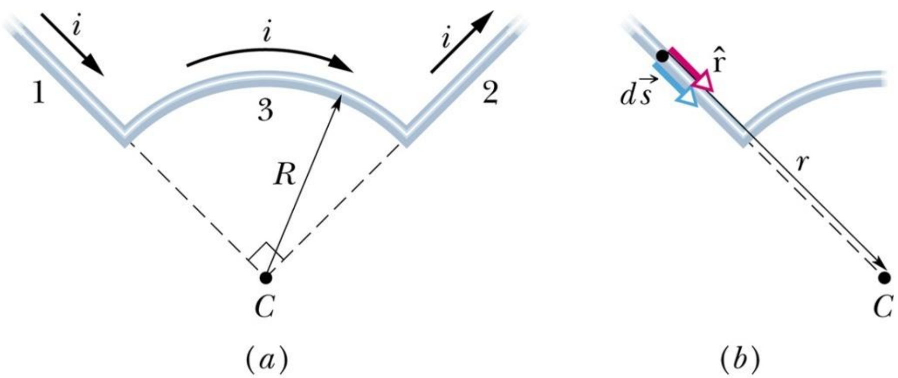

$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{R^2} = \frac{\mu_0}{4\pi}\frac{i\,ds}{R^2}$$

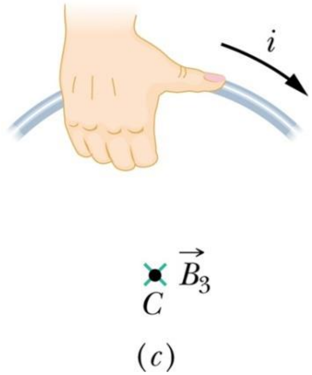
$$B = \int dB = \int_0^{\phi}\frac{\mu_0}{4\pi}\frac{iR\,d\phi}{R^2} = \frac{\mu_0 i}{4\pi R}\int_0^{\phi} d\phi = \frac{\mu_0 i\phi}{4\pi R}$$

원형 도선의 경우 중심에서는

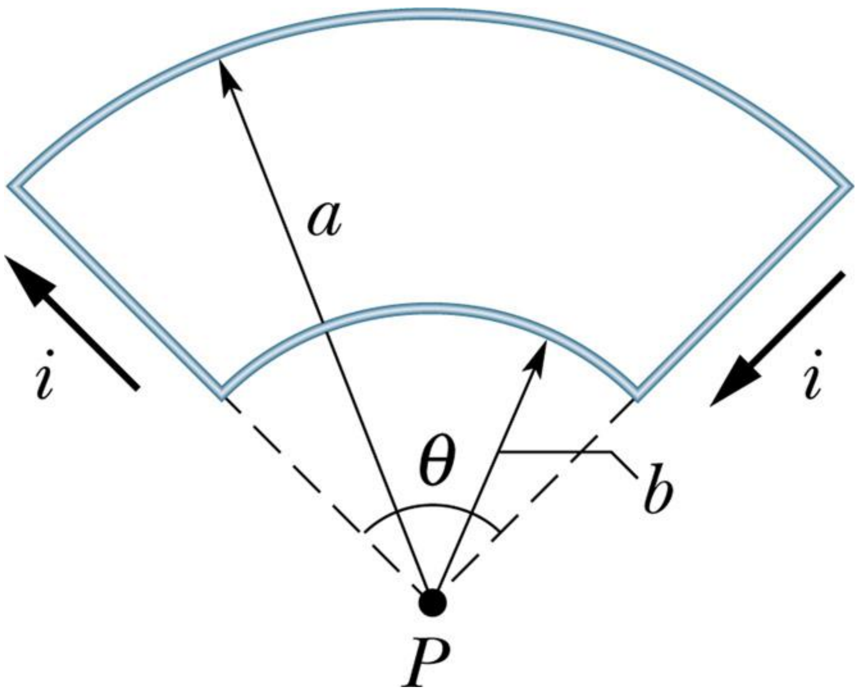
$$B = \frac{\mu_0 i}{2R}$$

# Example

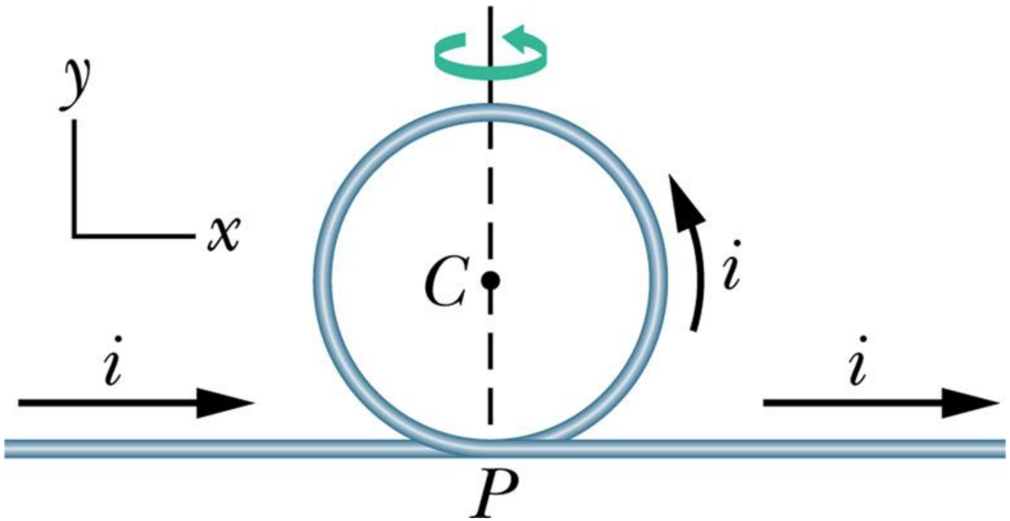


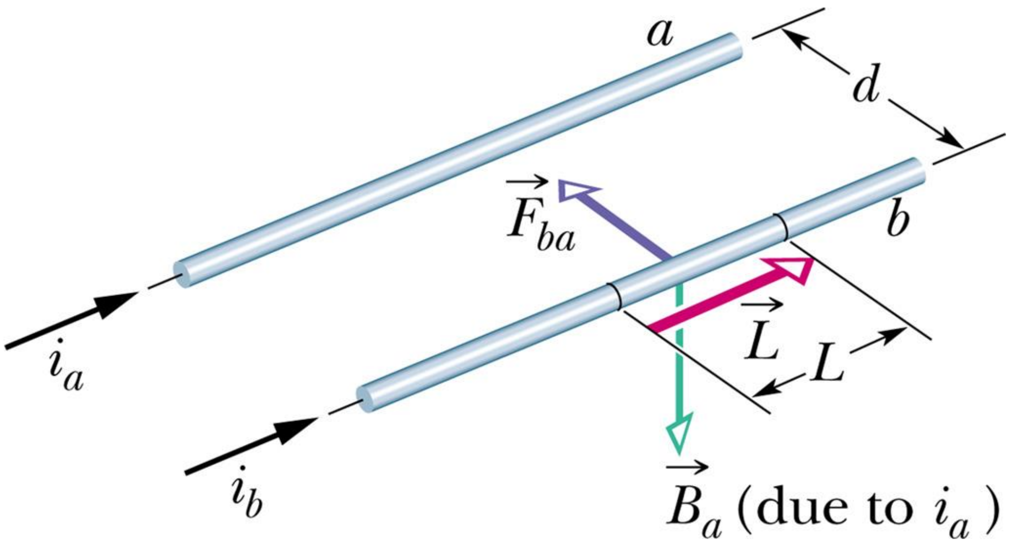
# Problem



# Problem



# Force between two parallel wires

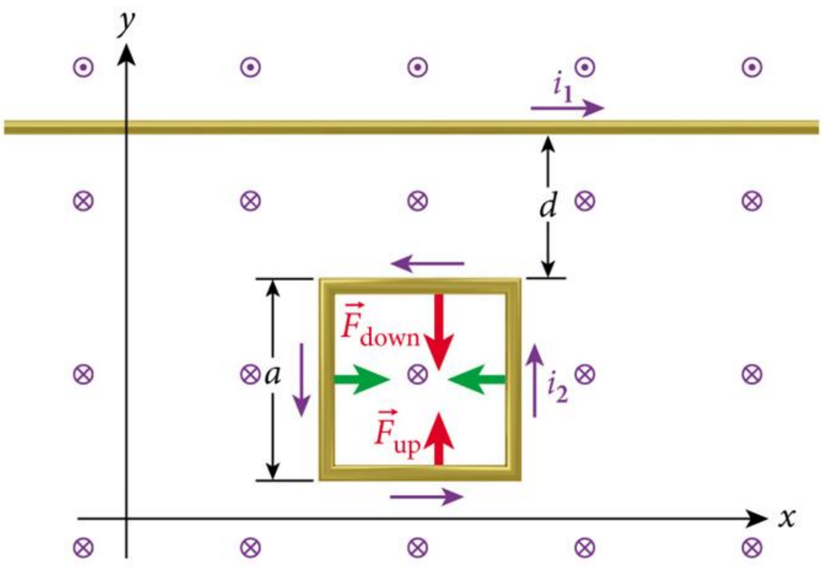



$$B_a = \frac{\mu_0 i_a}{2\pi d}$$

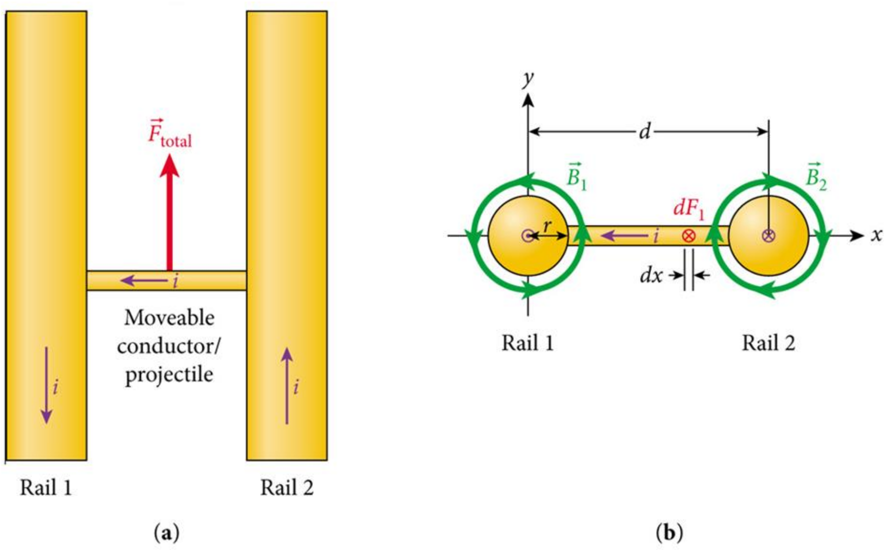
$$\vec{F}_{ba} = i_b \vec{L} \times \vec{B}_a \qquad F_{ba} = i_b B_a \sin 90^\circ = \frac{\mu_0 L i_a i_b}{2\pi d} = F_{ab}$$

전류 방향이 같으면 잡아당기고, 다르면 밀어낸다.

# Example 28.1 Force on a loop

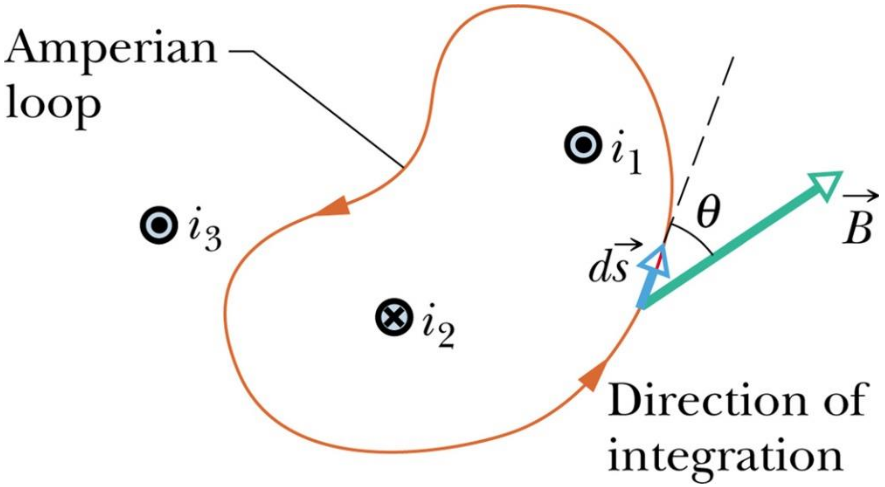


# S.P. 28.1 Electromagnetic rail accelerator



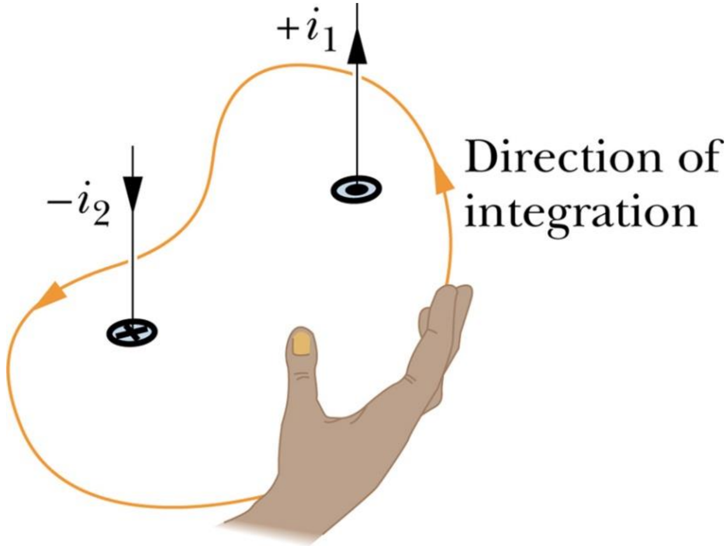

$$K = \frac{\mu_0 L i^2}{\pi} \ln \frac{d - r}{r}$$

# Ampere's law




$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

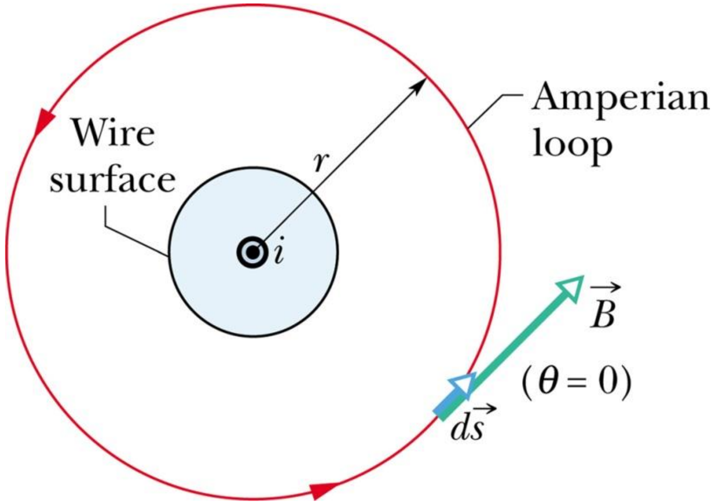
$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta ds = \mu_0 i_{\text{enc}}$$




$$i_{\text{enc}} = i_1 - i_2$$

$$\oint B \cos\theta ds = \mu_0 (i_1 - i_2)$$

# Magnetic field due to a long wire

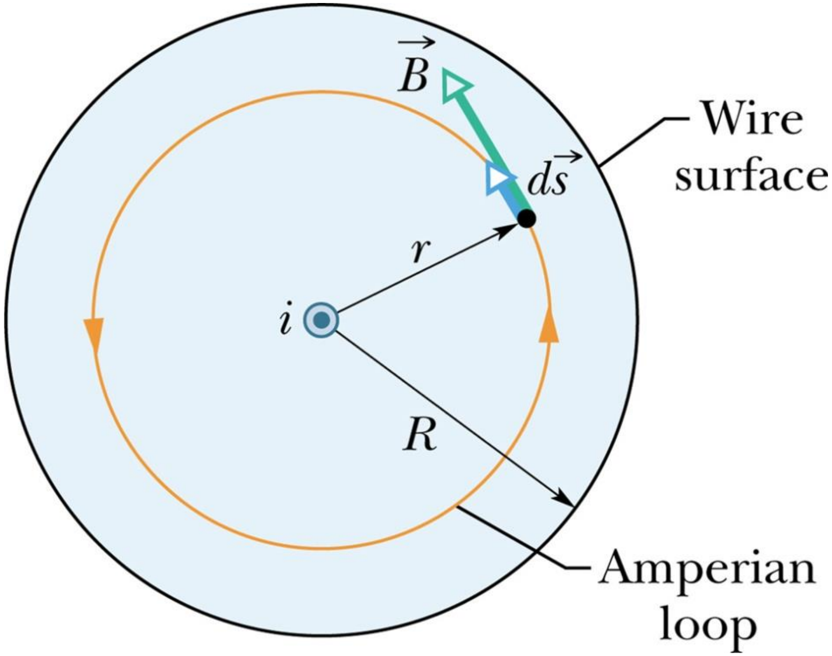


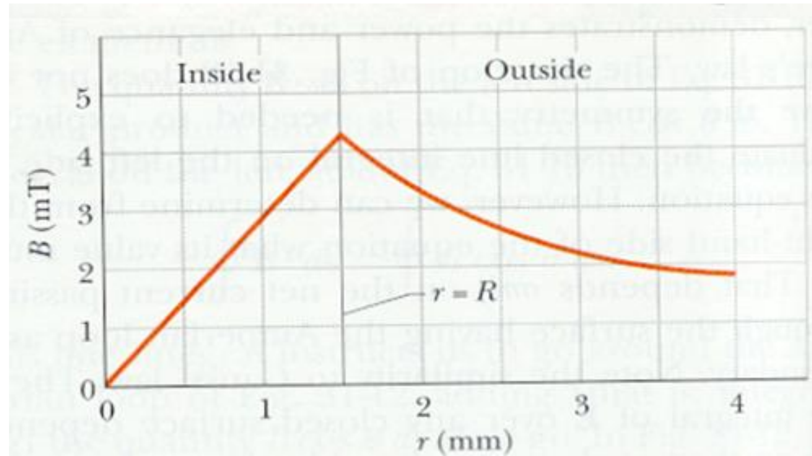

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta ds = B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i$$

$$B = \frac{\mu_0 i}{2\pi r}$$

# Magnetic field inside a long wire



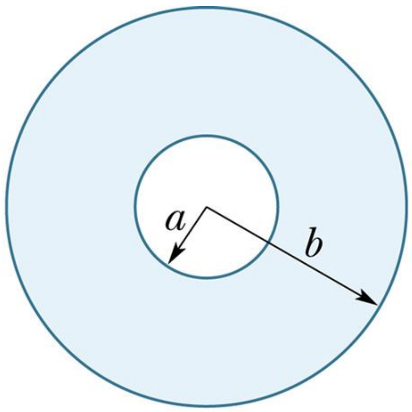

$$\oint \vec{B} \cdot d\vec{s} = B \oint ds = B(2\pi r)$$

$$B(2\pi r) = \mu_0 i \frac{r^2}{R^2}$$

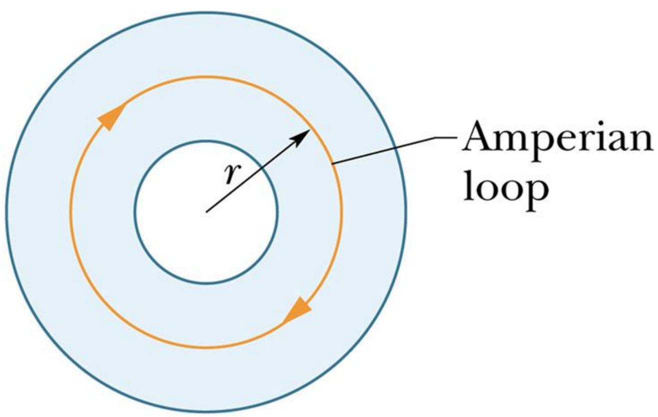
$$i_{\text{enc}} = i \frac{\pi r^2}{\pi R^2}$$

$$B = \left( \frac{\mu_0 i}{2\pi R^2} \right) r$$

# Example

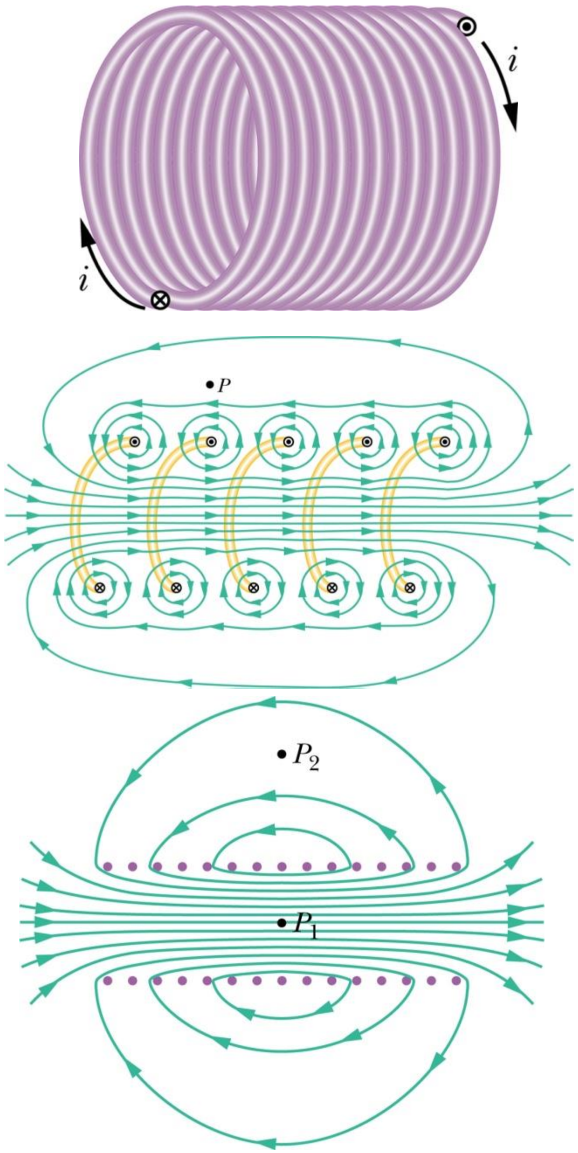


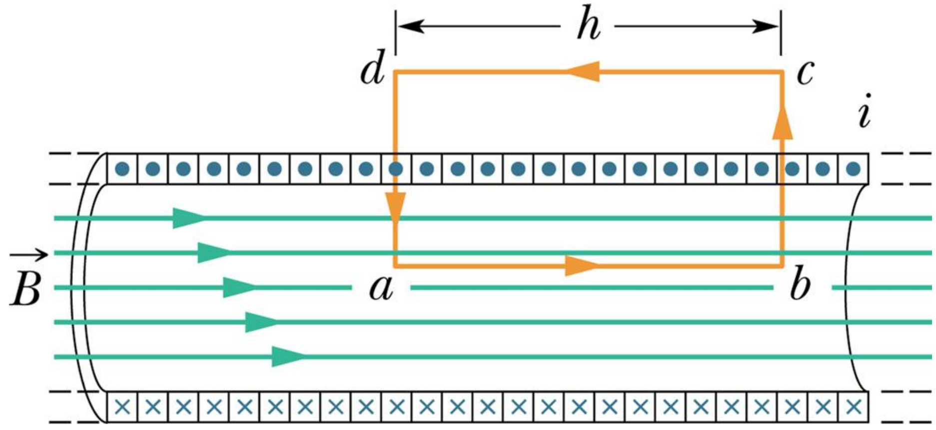

(a)




(b)

# Solenoid



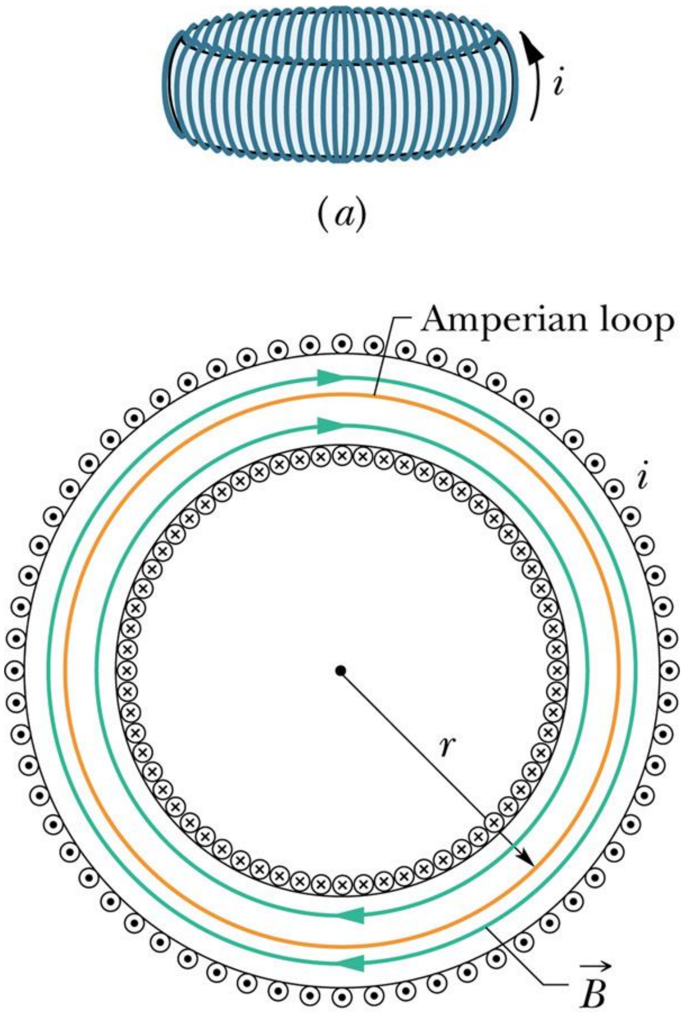

$$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{\text{enc}}$$

$$Bh = \mu_o inh$$

$$\boxed{B = \mu_0 in}$$

Uniform magnetic field

# Toroid



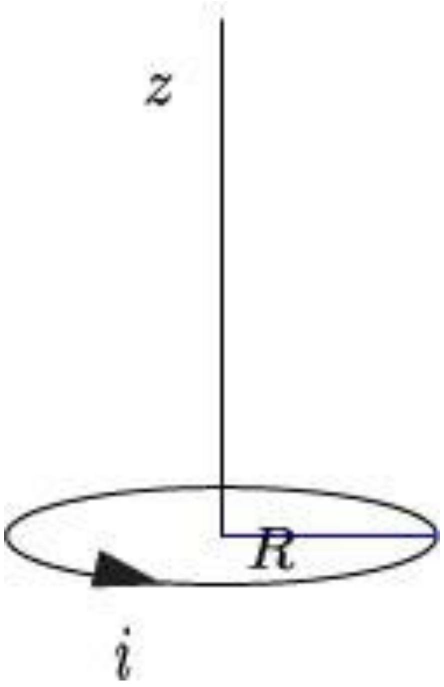

(a)

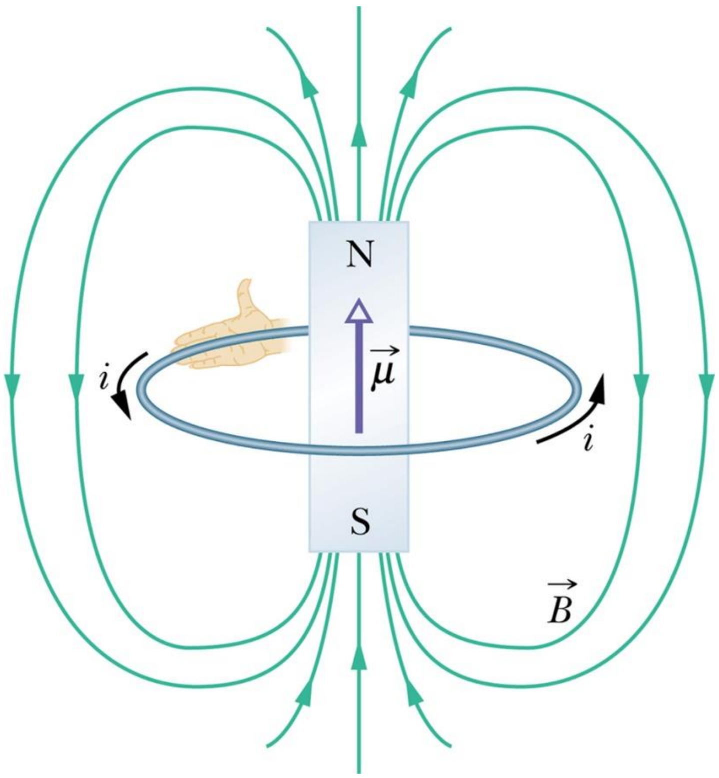
(b)

$$B(2\pi r) = \mu_0 i N$$

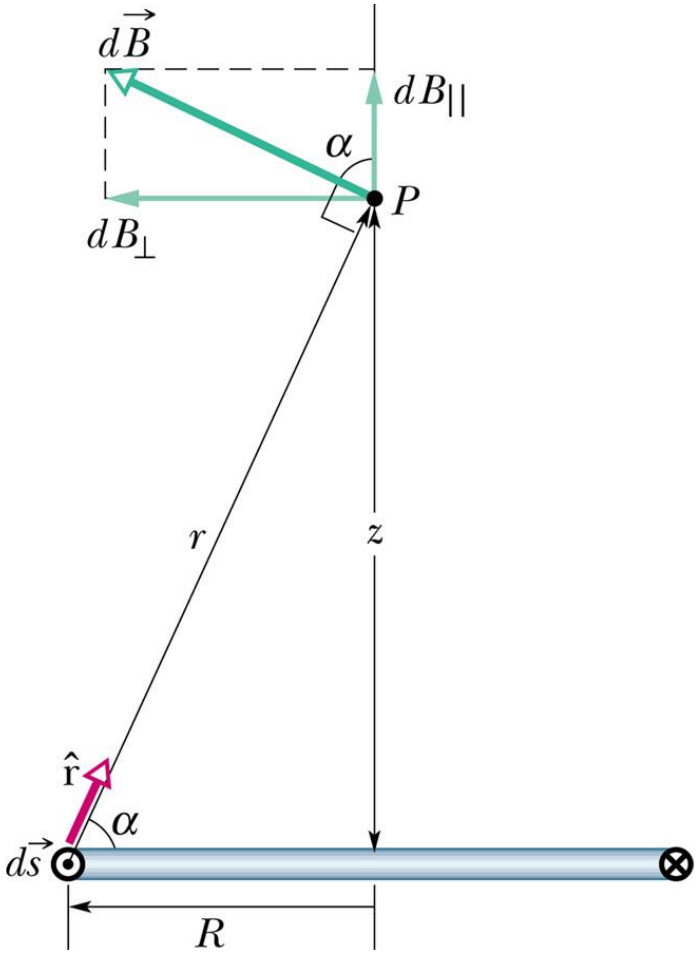
$$B = \frac{\mu_0 i N}{2\pi} \frac{1}{r}$$

# Magnetic field due to a loop



# Magnetic dipole moment




$$dB = \frac{\mu_0}{4\pi}\frac{i\,ds\sin 90^\circ}{r^2}$$

$$dB_{||} = dB\cos\alpha = \frac{\mu_0 i\cos\alpha\,ds}{4\pi r^2}$$

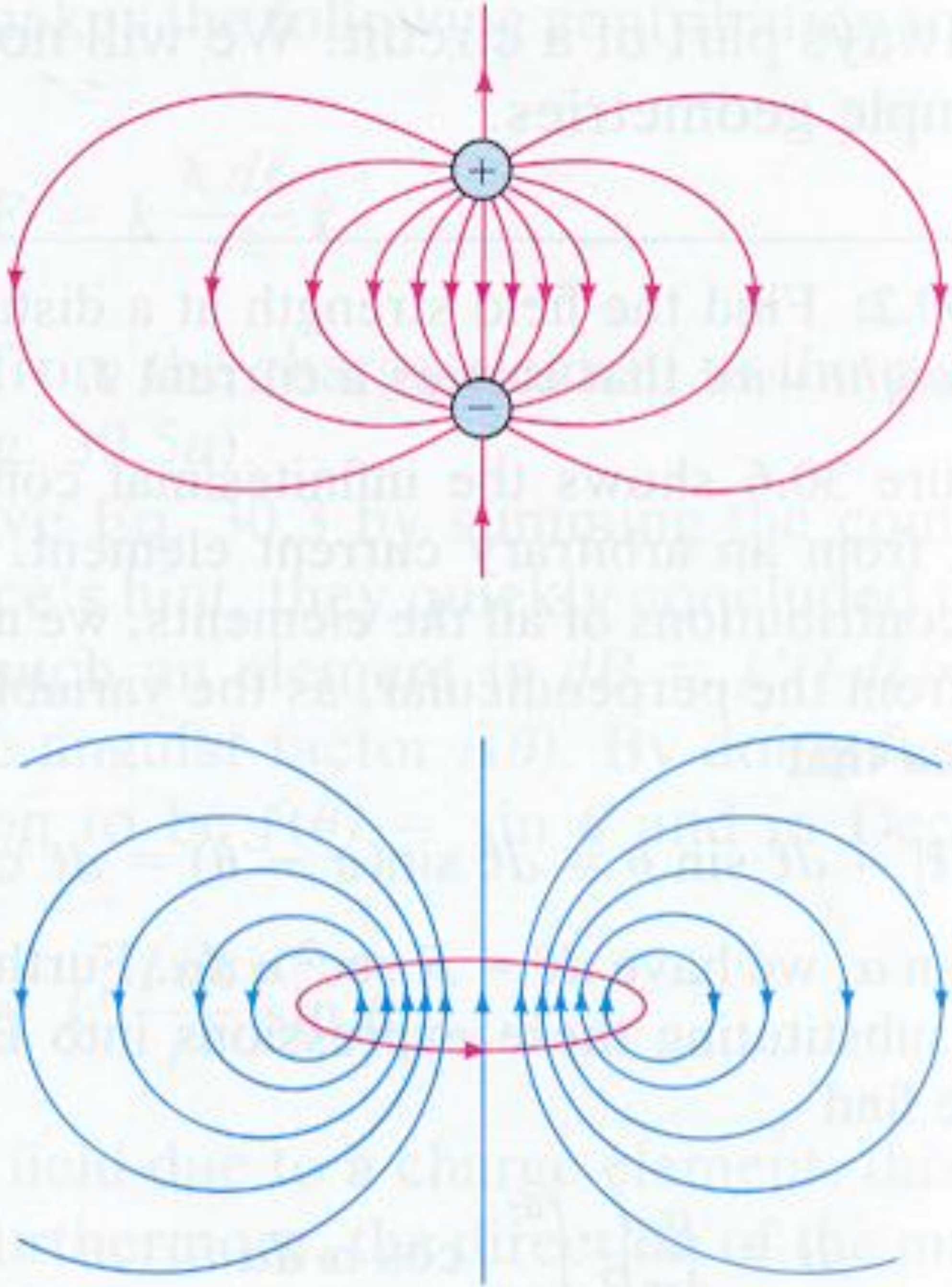
$$dB_{||} = \frac{\mu_0 i R}{4\pi(R^2+z^2)^{3/2}}ds$$

$$B = \int dB_{||} = \frac{\mu_0 i R^2}{2(R^2+z^2)^{3/2}}$$

$$z \gg R$$

$$B(z) \approx \frac{\mu_0 i R^2}{2z^3} \rightarrow \frac{\mu_0}{2\pi}\frac{NiA}{z^3}$$

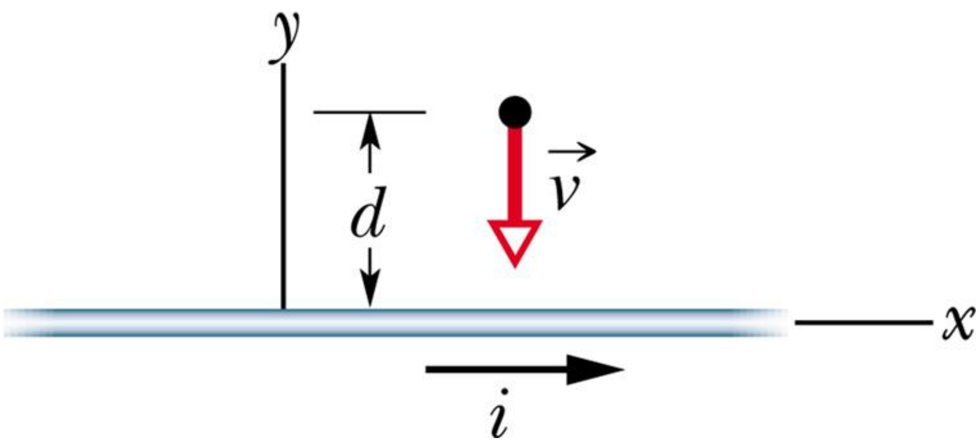
$$\mathbf{B}(z) = \frac{\mu_0}{2\pi}\frac{\vec{\mu}}{z^3}$$



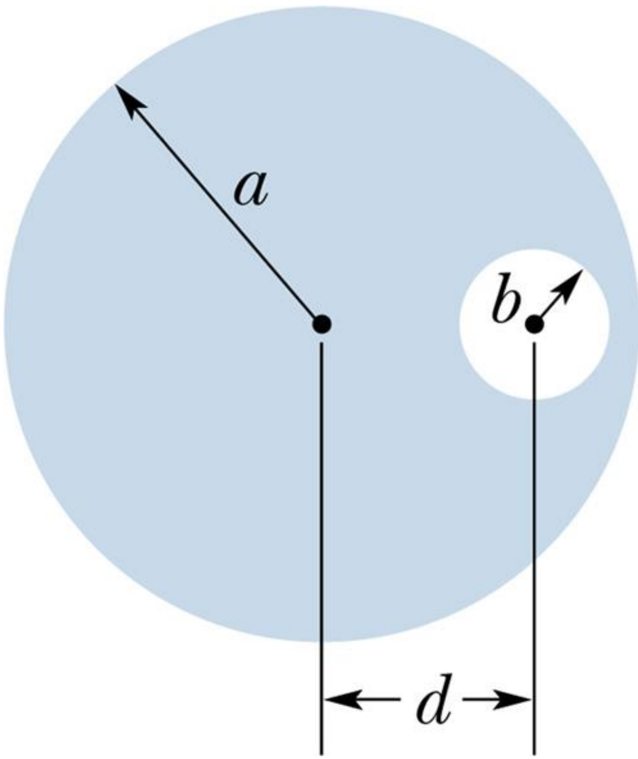
(a)

(b)

# Problem



# Problem



# Problem