Quantum Mechanics 1

Assignment 1

Due: March 19 (Tuesday), 2013

1. Some students may have wondered if the choice of arbitrary boudary conditions did not really matter in computing the number of states for blackbody radiation. Consider a cubic blackbody with sides L, and apply the fixed-end boundary conditions. That is, the electric fields are zero at the surface. By following the methods used in class, show that the number of states between ν and $\nu + d\nu$ is the same as the case with the periodic boudary conditions.

   Hint: The electromagnetic wave can be written in the form $A \sin k_1 x \sin k_2 y \sin k_3 z$. And since $\sin(-k_1 x) = -\sin k_1 x$ is not an independent function, only the positive numbers of k_1 should be included.

2. In class, we derived the energy density $u(\nu, T)$ of a blackbody à la Planck. Let us consider a two-dimensional blackbody.

   (a) Obtain the energy density $u_2(\nu, T)$ of the two-dimensional blackbody. Assume that there are two possible polarizations for the electromagnetic waves even in two dimensions.

   (b) The total energy $U_2(T)$ can be written as AT^b. Obtain A and b.

3. *Taste of statistical mechanics*

   In the derivation of the Planck's formula, we have computed the average energy $\langle E_n \rangle = \langle n \rangle h\nu$ for a given frequency ν at temperature T.

   (a) Compute the energy fluctuation defined by

$$\Delta E_n = \sqrt{\langle E_n^2 \rangle - \langle E_n \rangle^2}. \quad (1)$$

   (b) Write ΔE_n in the low-frequency limit, and in the high-frequency limit.

4. We briefly reviewed the Bohr's model of a hydrogen atom. From spectroscopy, the wavelength λ of the absorbed line spectrum from the state n_1 to the state n_2 is given by

$$\frac{1}{\lambda} = \text{Ry.}\Big(\frac{1}{n_1^2} - \frac{1}{n_2^2}\Big), \tag{2}$$

where Ry. is called the Rydberg constant. By deriving it carefully again, obtain the Rydberg constant and express it in SI units.