Quantum Mechanics II

Assignment 1

Due: September 24 (Tuesday), 2013

1. As discussed in class, the first-order correction to the unperturbed eigenstate can be written as

$$|\psi_n\rangle = N(\lambda)\Big(|\phi_n\rangle + \lambda \sum_{k\neq n} C_{nk}^{(1)}|\phi_k\rangle + \lambda^2 \sum_{k\neq n} C_{nk}^{(2)}|\phi_k\rangle + \cdots\Big), \tag{1}$$

where λ is a small parameter in $H = H_0 + \lambda H_1$. Obtain $C_{nk}^{(2)}$ as suggested in class.

2. *Gaussian perturbation*

Suppose the total Hamiltonian is given by

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2 + V e^{-\alpha x^2}, \tag{2}$$

where V and α are positive constants, and the last term can be regarded as a perturbation. Find the first-order correction to the ground state energy.

3. *anharmonic oscillator*

Consider an anharmonic oscillator described by the Hamiltonian

$$H = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2\Big(1 + \frac{x^2}{b^2}\Big), \tag{3}$$

where the last term is regarded as a perturbation. Obtain the energy to first order for the unperturbed eigenstate $|n\rangle$ with the energy eigenvalue $E_n^{(0)} = (n + 1/2)\hbar\omega$.

4. Consider an isotropic harmonic oscillator with the Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2 \mathbf{r}^2. \tag{4}$$

Suppose that the particle has the electric charge q, and is placed in a uniform electric field $E\hat{x}$. Then the perturbation is given by $\lambda H_1 = -qEx$.

(a) Compute the energy shift of the ground state to nontrivial order in perturbation.

(b) Compute the energy shift of the first excited states to first order in perturbation.

5. Suppose that the unperturbed Hamiltonian is given by a 3×3 matrix as

$$H_0 = A \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{5}$$

where A is a positive number. In this basis the perturbation is given by

$$H_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & \delta \\ 0 & \delta & 0 \end{pmatrix}, \tag{6}$$

where δ is small. Compute the energy eigenvalues of the unperturbed states to lowest nontrivial order and obtain the corresponding eigenstates in terms of the eigenstates of the unperturbed Hamiltonian.