Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8^{th} and 9^{th} Ed.
- The rest is made by me.

$$\overbrace{x(t)}^{\text{Displacemnt at time } t} = \underbrace{x_m}_{\text{Amplitude}} \cos\overbrace{(\underbrace{\omega}_{\text{Angular frequency}} \underbrace{t}_{\text{Time}} + \underbrace{\phi}_{\text{Phase constant or phase angle}})}^{\text{Phase}}$$

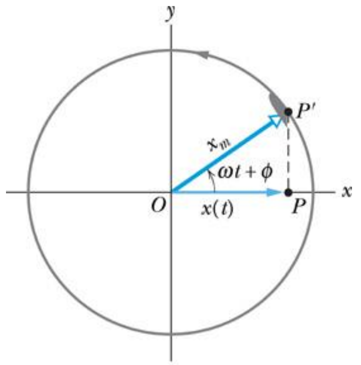
$$x_m \cos \omega t = x_m \cos \omega (t + T)$$

$$\omega (t + T) = \omega t + 2\pi$$

$$\omega T = 2\pi$$

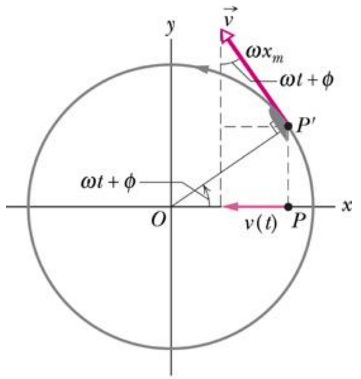
$$\boxed{\omega = \frac{2\pi}{T} = 2\pi f}$$

SHO and uniform CM



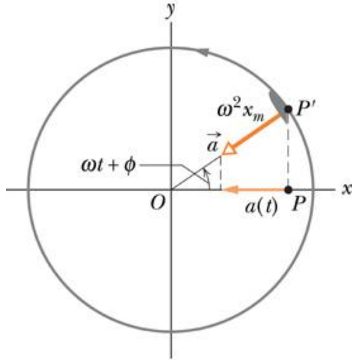
(a)

$$x(t) = x_m \cos(\omega t + \phi)$$



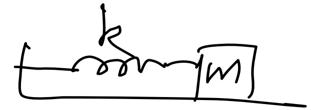
(b)

$$v(t) = -\omega x_m \sin(\omega t + \phi)$$

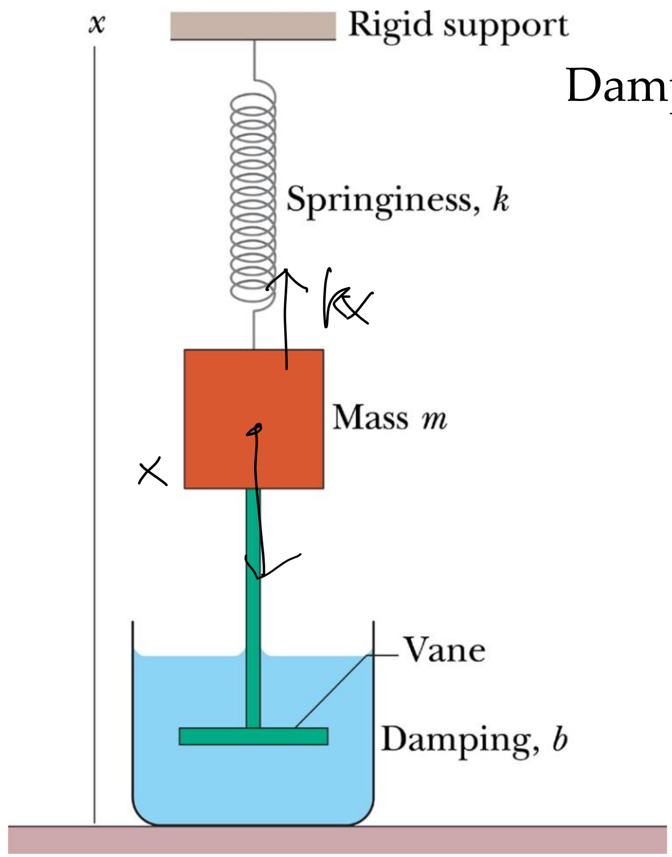


(c)

$$a(t) = -\omega^2 x_m \cos(\omega t + \phi)$$



Damped simple harmonic motion



Damping force가 $F_d = -bv$ 라고 가정하면

$$-bv - kx = ma$$

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = 0$$

$$-ma^2 + iab + k = 0$$

$$x = Ae^{iat}$$

$$ma^2 - iba - k = 0$$

$$a = \frac{1}{2m}\left(ib \pm \sqrt{4mk - b^2}\right)$$

$$= i\frac{b}{2m} \pm \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$kx - mg = ma$

$x = \frac{mg}{k}$

$k(x - mg) = ma$

$x' = x - mg$

$kx' = ma$

간단한 미분 방정식 풀기

$$\frac{dx^2}{dt^2} + \omega^2 x = 0$$

$$m\frac{d^2x}{dt^2} + m\omega^2 x = 0$$

$$x(t) = x_m \cos\omega t,\ \ x_m \sin\omega t$$

$$x(t) = x_m \cos(\omega t + \phi)$$

$$x = Ae^{iat} \rightarrow a^2 = \omega^2 \rightarrow a = \pm\omega$$

$$\cancel{e^{i\omega t}}$$

$$\cos\omega t = \frac{1}{2}(e^{i\omega t} + e^{-i\omega t})$$

$$\sin\omega t = \frac{1}{2i}(e^{i\omega t} - e^{-i\omega t})$$

$$x(t) = Ae^{i\omega t} + Be^{-i\omega t}$$

$$*\ e^{i\omega t} = \cos\omega t + i\sin\omega t$$

$$x(t) = (A+B)\cos\omega t + i(A-B)\sin\omega t$$

Small damping

$$F_d = -bv = -b\frac{dx}{dt}$$

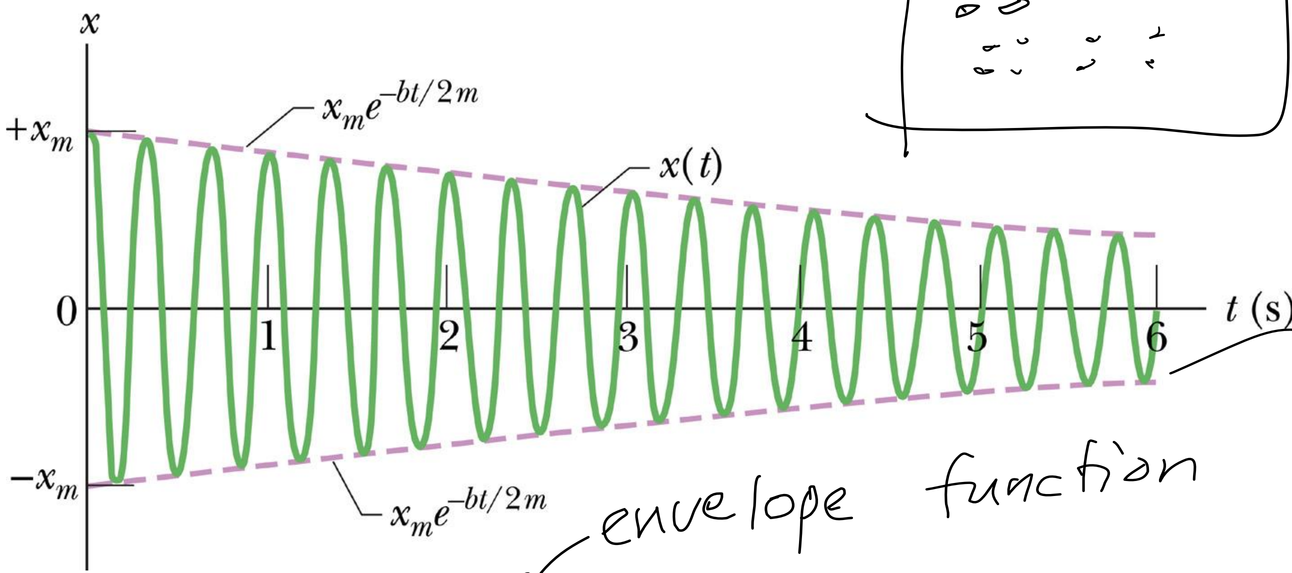
$$m\frac{d^2x}{dt^2} = -b\frac{dx}{dt} - kx \longrightarrow \frac{d^2x}{dt^2} + \frac{b}{m}\frac{dx}{dt} + \frac{k}{m}x = 0$$

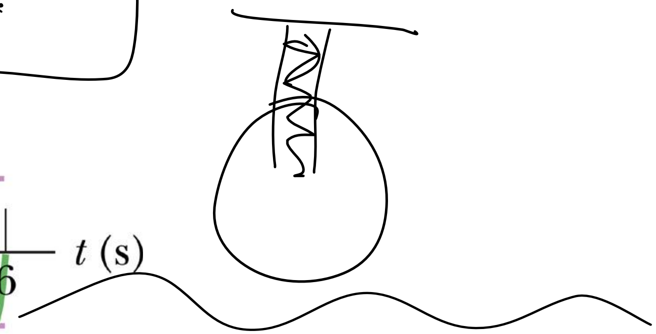
$$x = Ae^{at} \rightarrow a^2 + \frac{b}{m}a + \frac{k}{m}a = 0$$

ω'

$$a = -\frac{b}{2m} \pm \sqrt{\left(\frac{b}{2m}\right)^2 - \frac{k}{m}} = -\frac{b}{2m} \pm i\sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$$

$$\boxed{x(t) = Ae^{-bt/2m}\cos(\omega' t + \phi)}$$





envelope function

$$x(t) = x_m e^{-bt/2m} \cos(\omega' t + \phi)$$

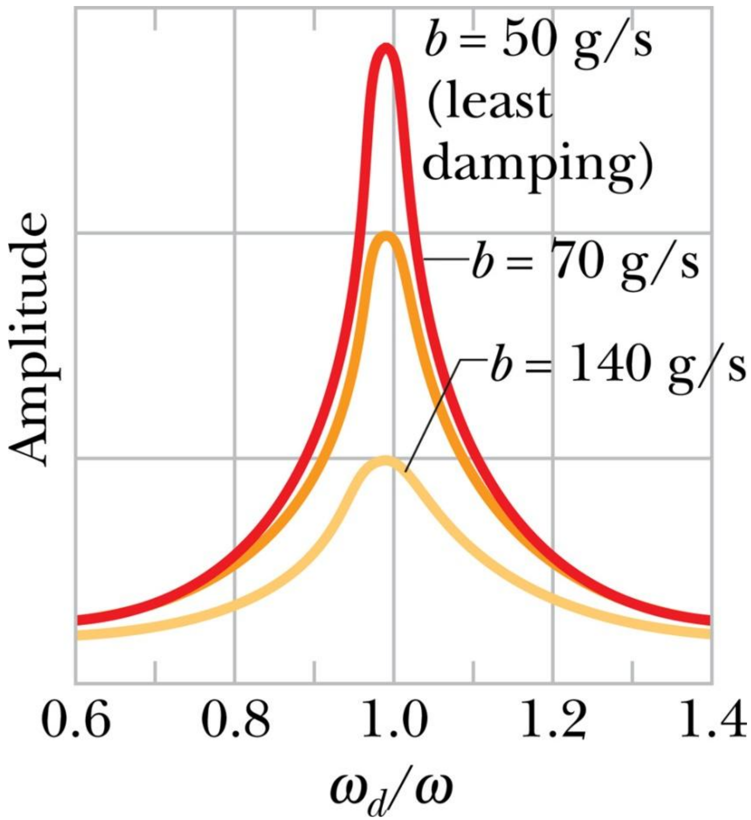
$$\omega' = \sqrt{\frac{k}{m} - \frac{b^2}{4m^2}}$$

$$k/m - b^2/4m^2 > 0 \quad \text{underdamping}$$

$$k/m - b^2/4m^2 = 0 \quad \text{critical damping}$$

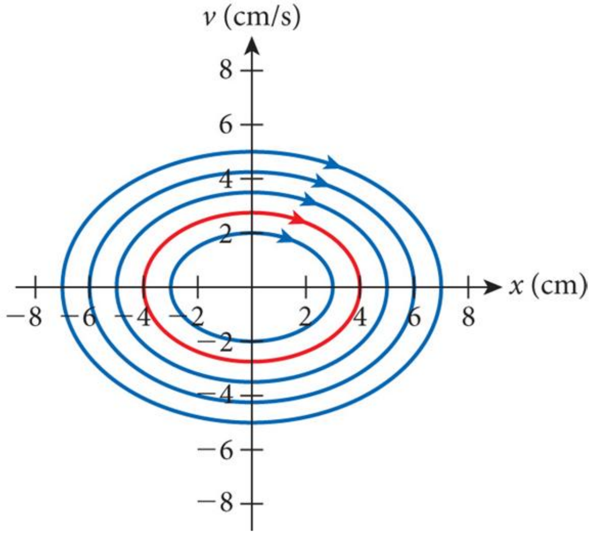
$$k/m - b^2/4m^2 < 0 \quad \text{overdamping}$$

Forced oscillations and resonance



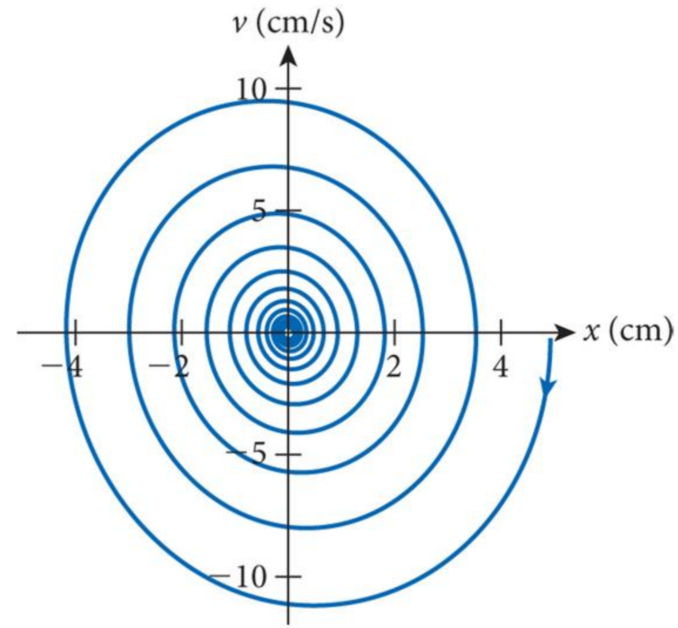
$$x(t) = x_m \cos(\omega_d t + \phi)$$

Phase space



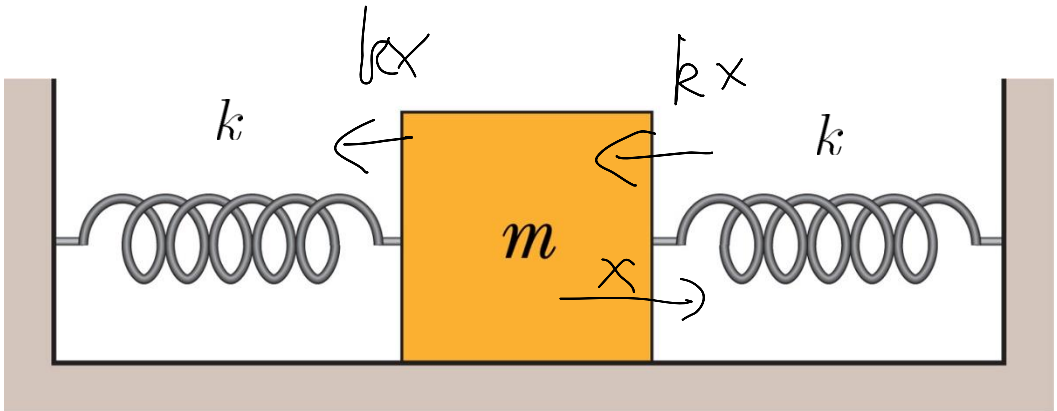
Simple harmonic oscillator

$E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$



Damped oscillator

Problem 1



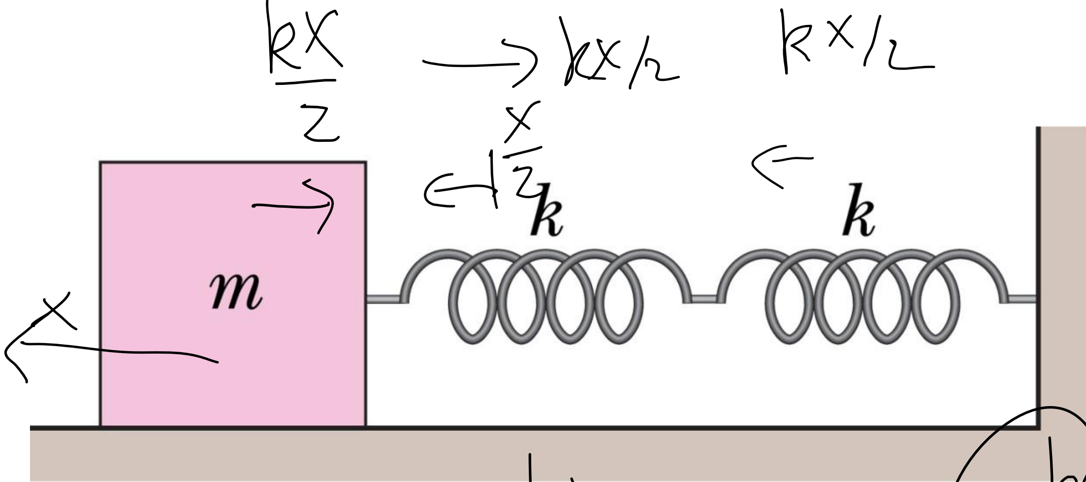
$\omega = ?$

ω^2

$$ma = -2kx \qquad a = -\frac{2k}{m}x$$

$$\omega = \sqrt{\frac{2k}{m}}$$

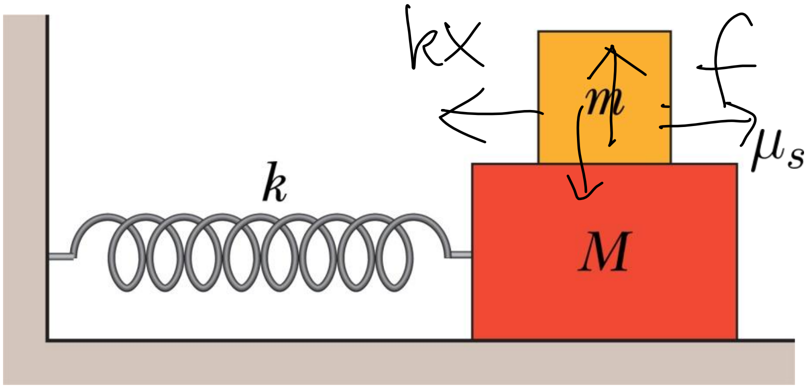
Problem 2



$$ma = -\frac{kx}{2} \qquad a = -\frac{k}{2m}x$$

ω^2

Problem 3



$$kx = (M+m)\,a$$

$$\omega^2 = \frac{k}{M+m}$$

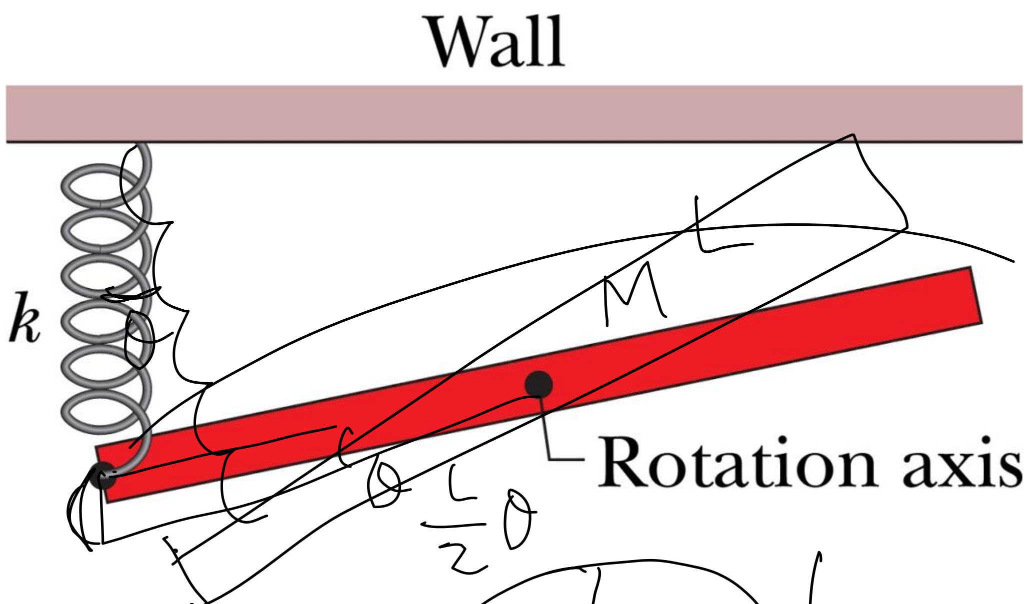
$$kx_{max} = \mu_s m g$$

$$x_{max} = \frac{\mu_s m g}{k}$$

$F = -kx$

$a = -\omega^2 x$

Problem 4



$$\tau = -\kappa\theta = I\alpha$$

$$\alpha = -\underbrace{\frac{\kappa}{I}}_{\omega^2}\theta$$

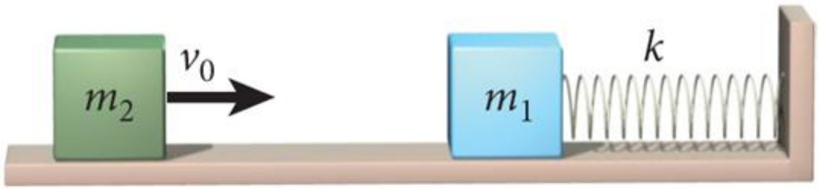
$$\omega = \sqrt{\frac{3k}{M}}$$

$$\tau = -k\frac{L}{2}\theta\,\frac{L}{2} = -k\frac{L^2}{4}\theta = I\alpha = \frac{ML^2}{12}\alpha$$

$I =$

$$\alpha = -k\frac{\cancel{L^2}}{\cancel{4}}\,\frac{\cancel{12}\,3}{M\cancel{L^2}}\,\theta = -\underbrace{\frac{3k}{M}}_{\omega^2}\theta$$

Problem 5



Max. compression and the time to get there

$$m_2 v_0 = (m_1+m_2)\,V$$

$$V = \frac{m_2}{m_1+m_2} v_0$$

$$\cancel{\tfrac{1}{2}}(m_1+m_2)V^2 = \cancel{\tfrac{1}{2}}\,k x_{max}^2$$

$$x_{max}^2 = \frac{1}{k}\,(\cancel{m_1+}m_2)\,\frac{m_2^2 v_0^2}{(m_1+m_2)^{\cancel{2}}}$$

$$= \frac{m_2^2 v_0^2}{k(m_1+m_2)}$$

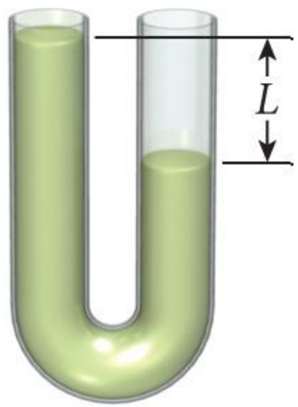
$$x_{max} = \frac{m_2 v_0}{\sqrt{k(m_1+m_2)}}$$

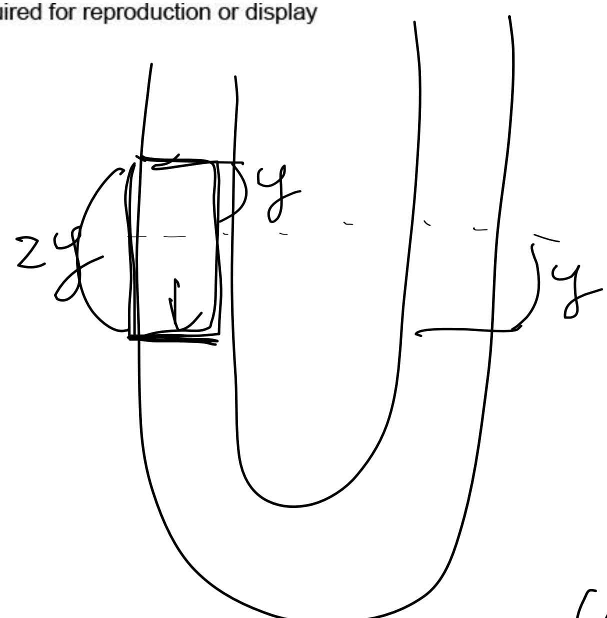
$$-kx = (m_1+m_2)\,a \qquad \omega = \sqrt{\frac{k}{m_1+m_2}}$$

$$T = 2\pi\sqrt{\frac{m_1+m_2}{k}}$$

$$t = \frac{T}{4} = \frac{\pi}{2}\sqrt{\frac{m_1+m_2}{k}}$$

Problem 6





$$\rho A g \cdot 2y = M a$$

$$a = \frac{2\rho g y}{M} A = \omega^2$$

$$\omega = \sqrt{\frac{2\rho g A}{M}}$$

Ch. 15 Waves

Waves

- 물체(매질)은 제자리에서 진동을 하며 에너지를 전달하는 현상

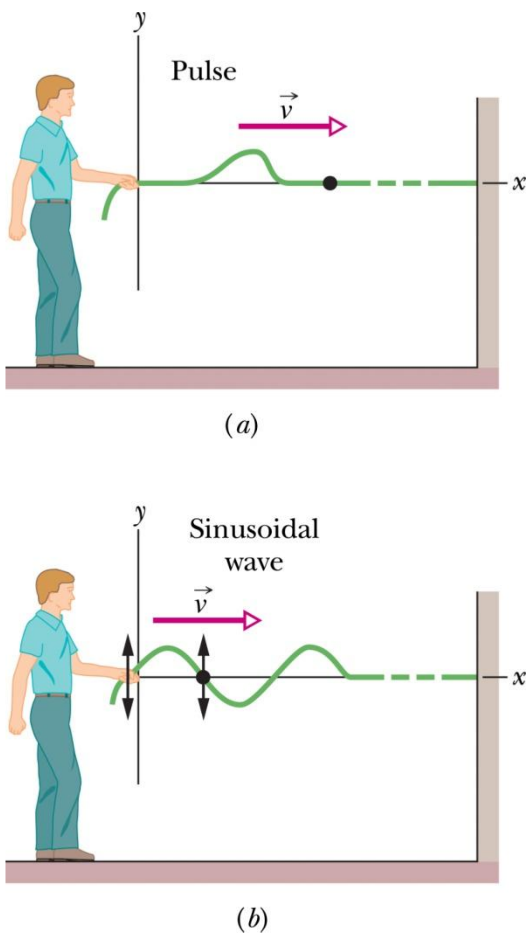
- 파동의 종류

(1) 역학적 파동: 수면파, 음파, 줄의 파동

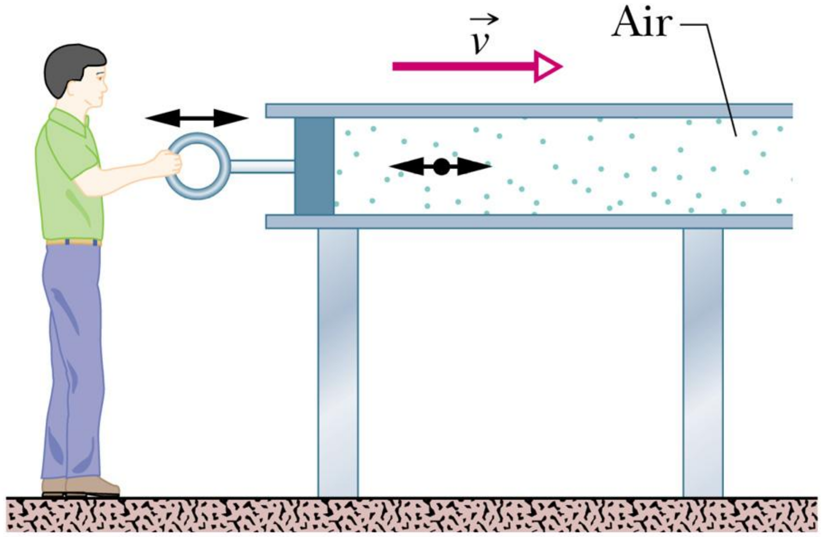
(2) 전자기파: 전기장과 자기장이 진동

(3) 물질파: 입자의 양자역학적 현상

Transverse wave



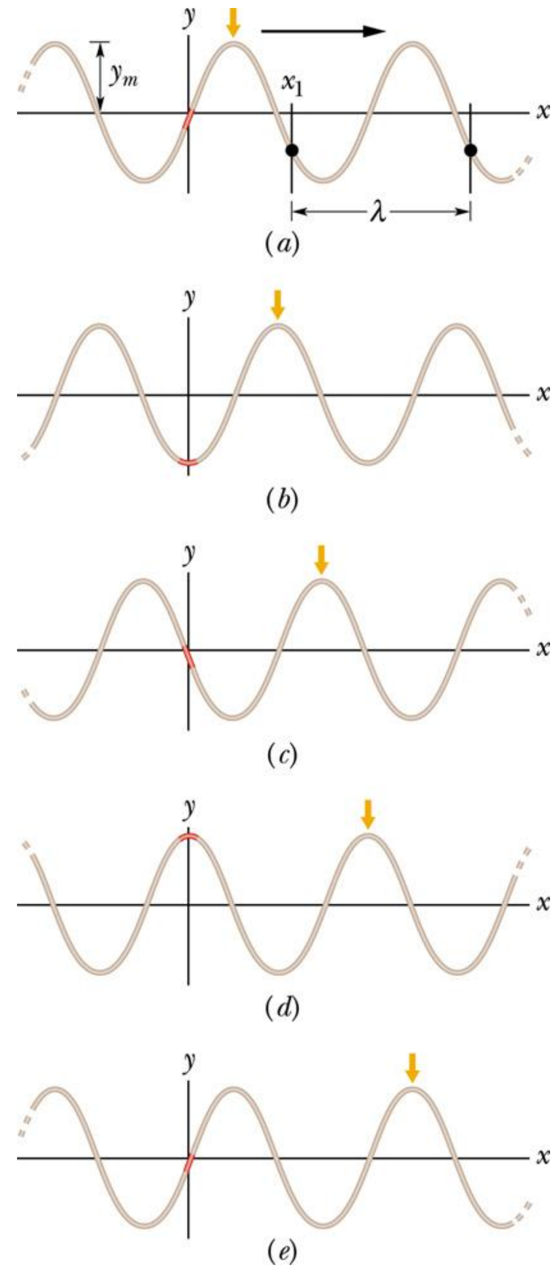
Longitudinal wave



Wavelength and frequency

$$y(x,t) = y_m \sin(kx - \omega t)$$

displacement amplitude angular wave number phase ang. freq.



Angular wave number $k = \dfrac{2\pi}{\lambda}$

Angular frequency $\omega = \dfrac{2\pi}{T}$

$$k\lambda = 2\pi = \omega T$$

frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$