Quantum Mechanics II

Assignment 6

Due: November 21 (Thursday), 2013

1. Neutrons are scattered by protons at such energy that only S and P waves need be considered. Assume that the scattering potential is spherically symmetric.

   (a) Show that the differential cross section can be written in the form

   $$\frac{d\sigma}{d\Omega} = a + b\cos\theta + c\cos^2\theta. \tag{1}$$

   (b) What are the values of a, b, and c in terms of the phase shifts?

   (c) What is the value of the total cross section in terms of a, b and c? What is the value of the forward scattering amplitude?

2. Consider a hard-sphere potential

   $$V(r) = \begin{cases} \infty, & r \le a \\ 0, & r > a. \end{cases} \tag{2}$$

   (a) What is the boundary condition of the wave function for the scattering with this potential?

   (b) As we did in class, find an expression for the phase shift δ_l in terms of the spherical Bessel functions j_l and the spherical Neumann functions n_l.

   (c) Consider the limit where $ka \ll 1$. What is the leading behavior of the partial-wave total scattering cross section σ_l?

   (d) For $l = 0$, show that the total scattering cross section is given by $4\pi a^2$, which is 4 times larger than the area of the potential.

3. Consider a low-energy scattering from a spherical shell given by

   $$V(r) = A\delta(r - a), \tag{3}$$

   where A and a are constants.

   Calculate the scattering amplitude $f(\theta)$, the differential scattering cross section, and the total cross section. Assume $ka \ll 1$ such that only the $l = 0$ term contributes.