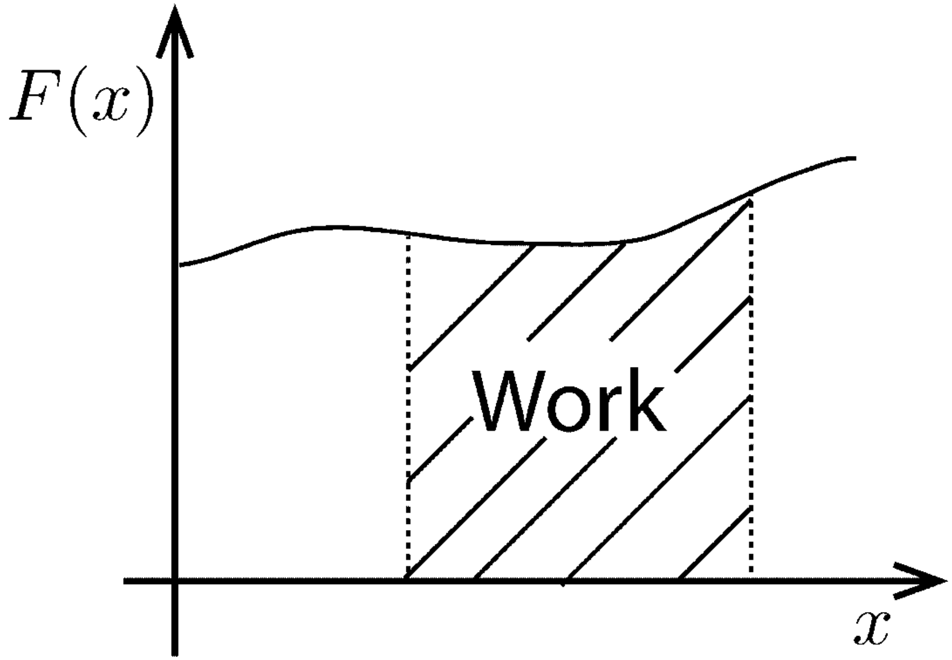
Work and Energy

Definition of Work

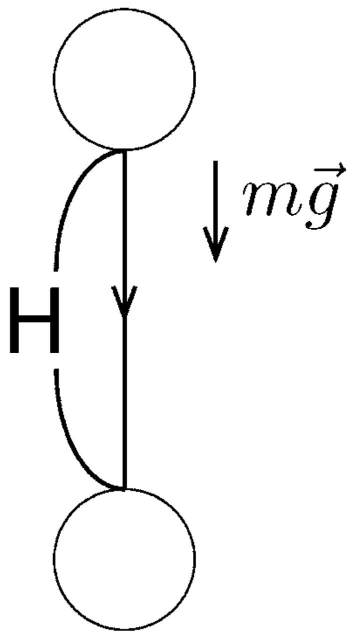
$$\text{Work} = \int F\,dx = \lim_{\Delta x_i \to 0} \sum_i F(x_i)\Delta x_i$$



$$\text{Work} > 0 \ \rightarrow \ \frac{1}{2}mv^2 \text{ increases}$$

$$\text{Work} < 0 \ \rightarrow \ \frac{1}{2}mv^2 \text{ decreases}$$

Work on a Falling body



$$a = g$$

$$v = v_0 + gt$$

$$v^2 = v_0^2 + 2gH$$

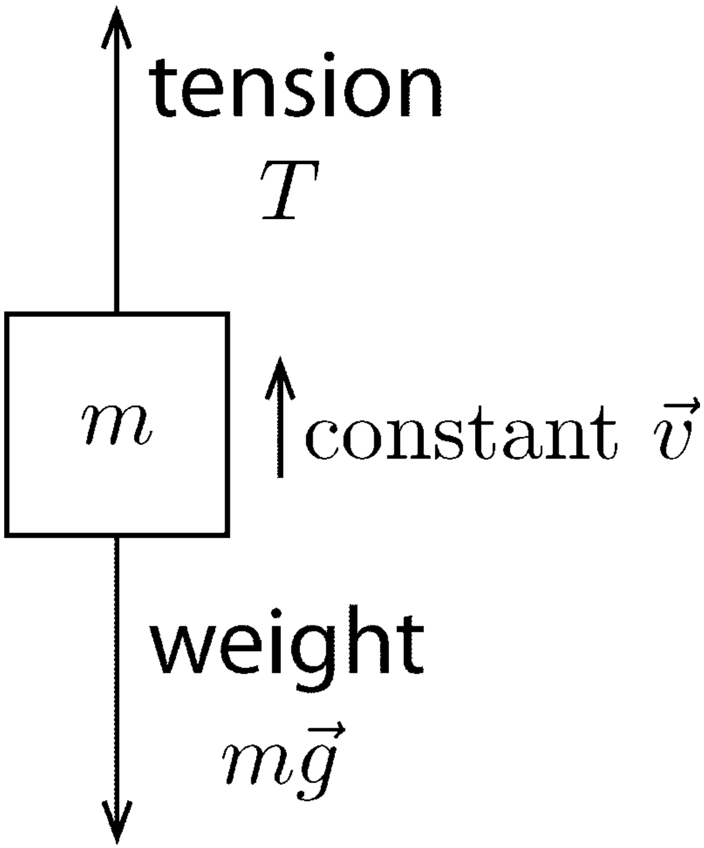
$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = mgH$$

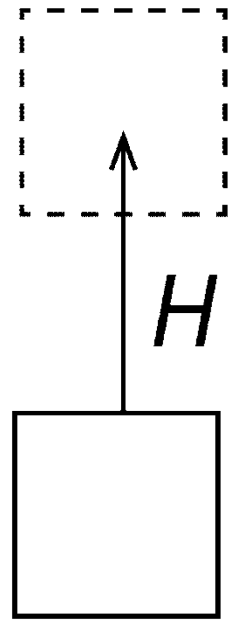
$\frac{1}{2}mv^2$ → **T(final)**

$\frac{1}{2}mv_0^2$ → **T(initial)**

mgH → **Work done by the gravity**

Lifting





$$W_T = mgH > 0$$

$$W_g = -mgH < 0$$

Work-Kinetic Energy Theorem

$$\Delta W = \Sigma F \Delta x \longleftarrow F = ma = m\frac{\Delta v}{\Delta t}$$

$$= \Sigma m \frac{\Delta v}{\Delta t}\Delta x \longleftarrow \Delta x = v\Delta t$$

$$= \Sigma m v \Delta v$$

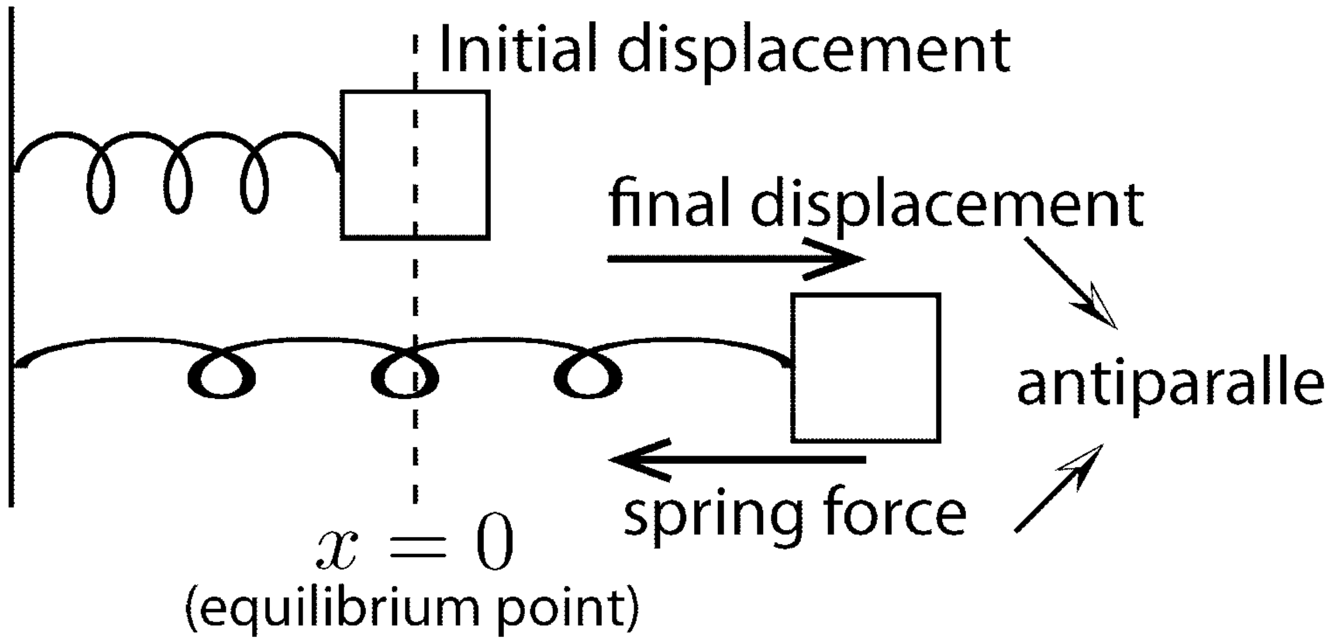
$$= m\int v\, dv$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

$$= \boldsymbol{T}(\mathbf{final}) - \boldsymbol{T}(\mathbf{initial})$$

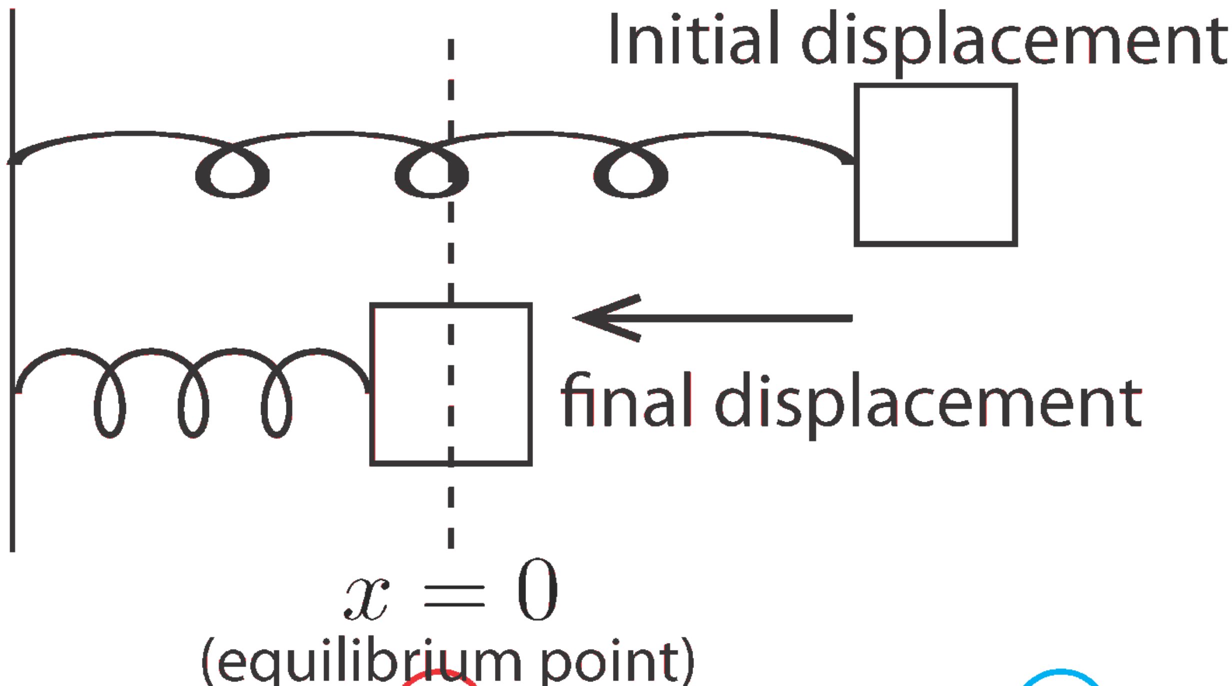
Work done on a massive object is the same as the increment of the kinetic energy

Work done bye the Spring force



$$W = \int F_s \, dx = \int_0^X (-kx) \, dx = -k \int_0^X x \, dx$$
$$= -\frac{1}{2} k X^2$$

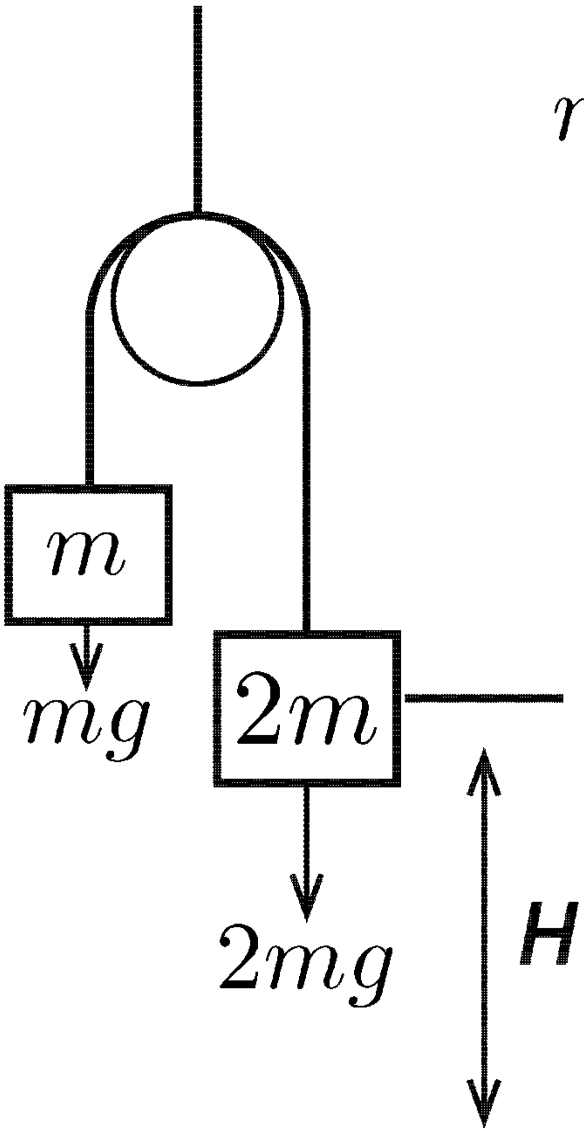
Work done bye the Spring force



$$W = \int F_s \, dx = \int_X^0 (-kx)\, dx = +k \int_0^X x \, dx$$

$$= \frac{1}{2} k X^2$$

Exercise



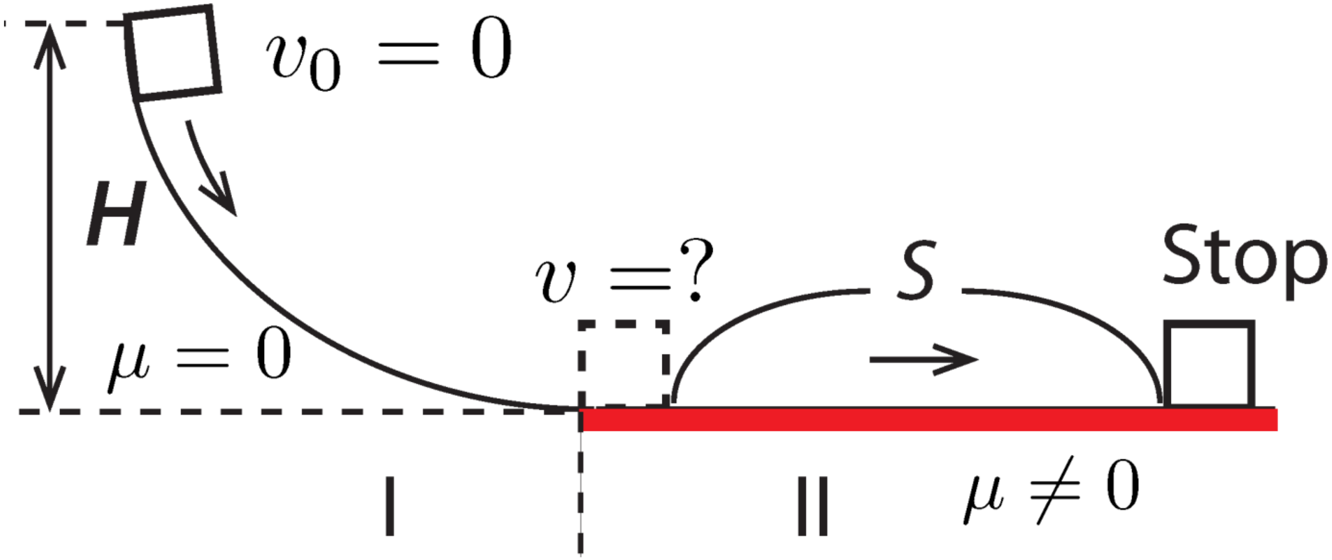
$$F_{net} = 2mg - mg = mg$$

$$\therefore a = \frac{mg}{3m} = \frac{1}{3}g$$

$$\Delta W = 2mgH - mgH = mgH$$

$$= \frac{1}{2}(3m)(v^2 - v_0^2)$$

Exercise



$$\Delta W_I = mgH = \frac{1}{2}mv^2 \rightarrow v = \sqrt{2gH}$$

$$\Delta W_{II} = -mgH = -\mu mgS$$

$$\rightarrow S = \frac{H}{\mu} = \frac{v^2}{2\mu g}$$