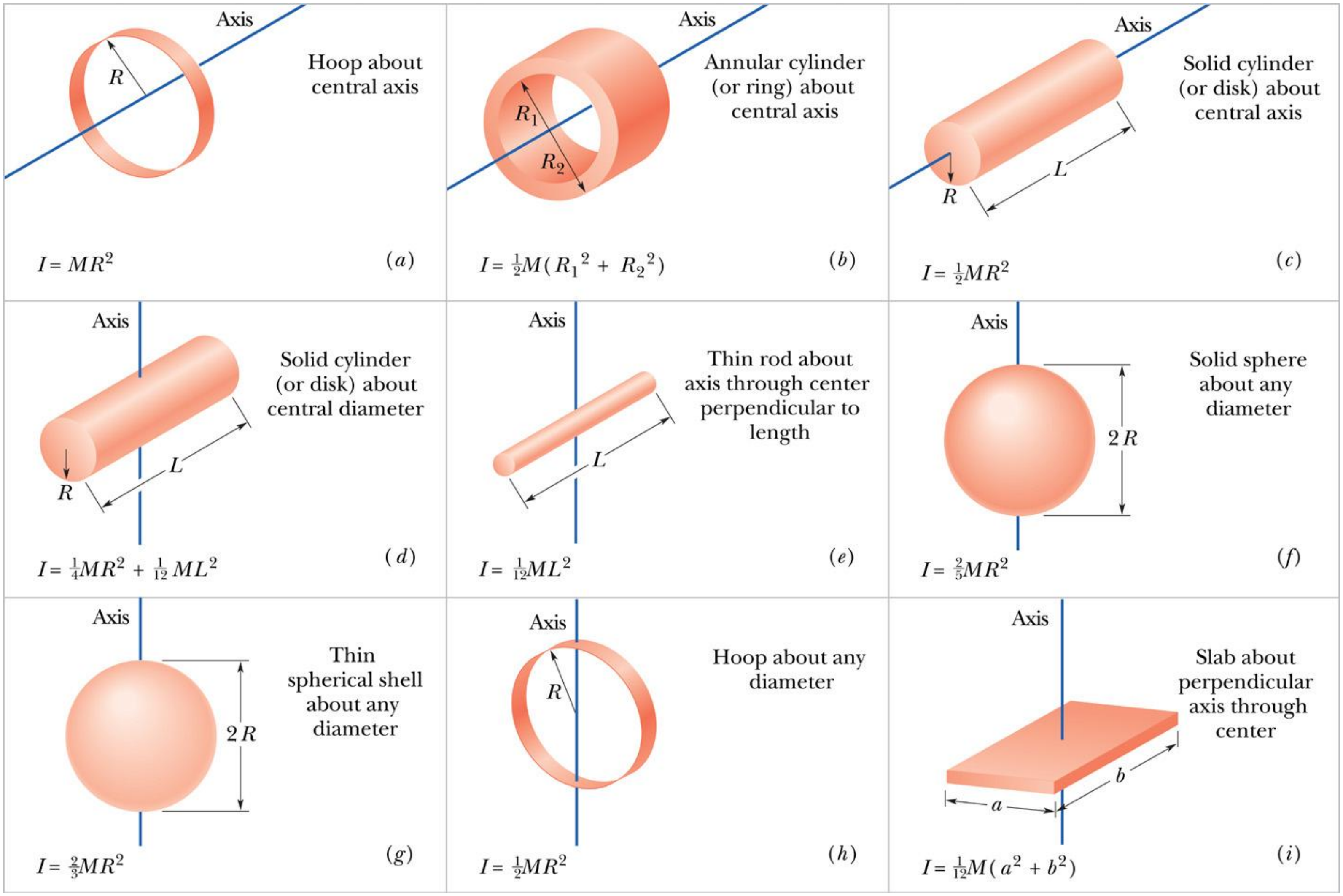
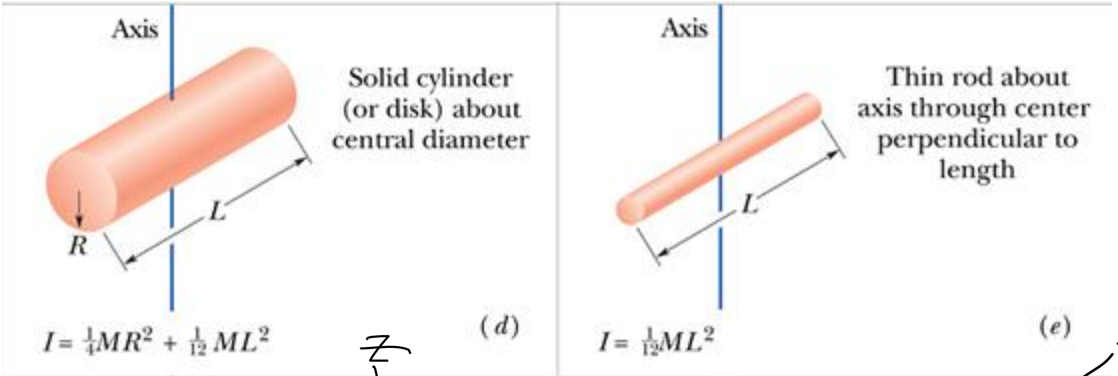
Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

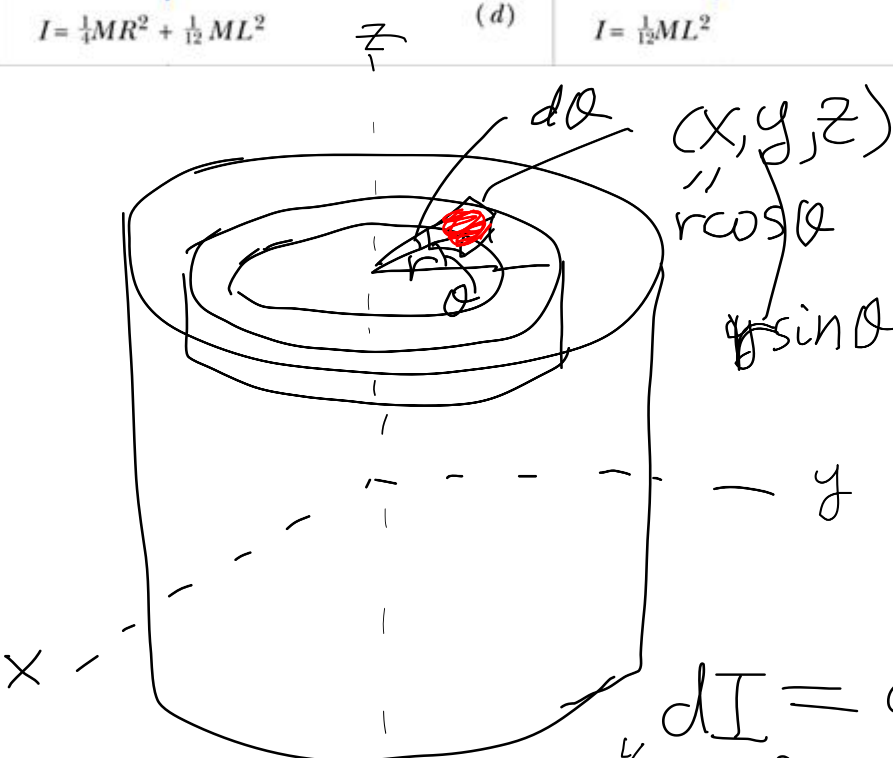
MOI 계산하기

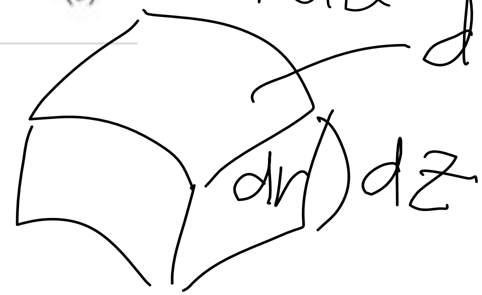


Axis
Hoop about central axis
R
$I = MR^2$
(a)
Axis
Annular cylinder (or ring) about central axis
R_1
R_2
$I = \frac{1}{2}M(R_1{}^2 + R_2{}^2)$
(b)
Axis
Solid cylinder (or disk) about central axis
L
R
$I = \frac{1}{2}MR^2$
(c)
Axis
Solid cylinder (or disk) about central diameter
L
R
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)
Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)
Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)
Axis
Hoop about any diameter
R
$I = \frac{1}{2}MR^2$
(h)
Axis
Slab about perpendicular axis through center
b
a
$I = \frac{1}{12}M(a^2 + b^2)$
(i)



$$\rho = \frac{M}{\pi R^2 L}$$





$$dV = r\,dr\,d\theta\,dz$$

$$dm = \rho\,dV$$

$$dI = dm\,(r^2\sin^2\theta + z^2)$$

$$I = 2\rho\int_0^{L/2} dz \int_0^{R} dr \int_0^{2\pi} d\theta\,(r^2\sin^2\theta + z^2)\,r\,dr\,d\theta\,dz$$

$$= 2\rho\pi\int_0^{L/2} dz\int_0^{R} dr\,(r^3 + 2z^2 r)\,dr\,dz$$

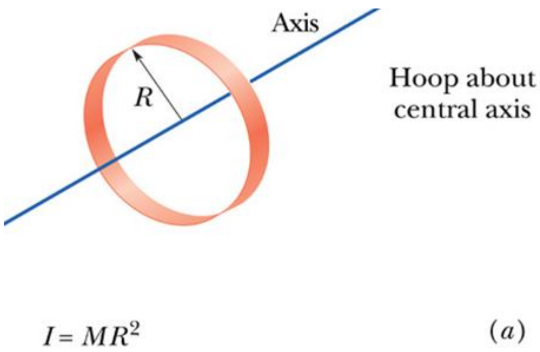
$$2\rho\pi\int_0^{L/2} dz\left(\frac{R^4}{4} + z^2R^2\right)$$

$$= 2\rho\pi\left(\frac{R^4 L}{8} + R^2\frac{L^3}{24}\right)$$

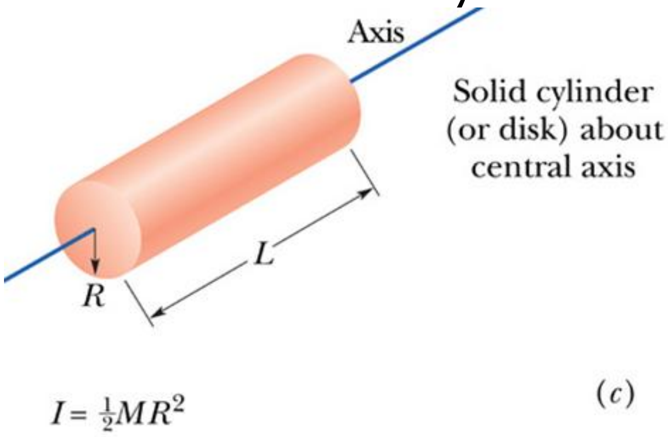
$$= \rho\pi R^2 L\left[\frac{R^2}{4} + \frac{L^2}{12}\right]$$

$$= \frac{MR^2}{4} + \frac{ML^2}{12}$$

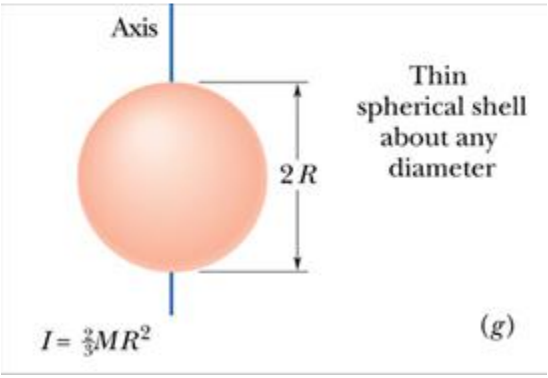
Summary



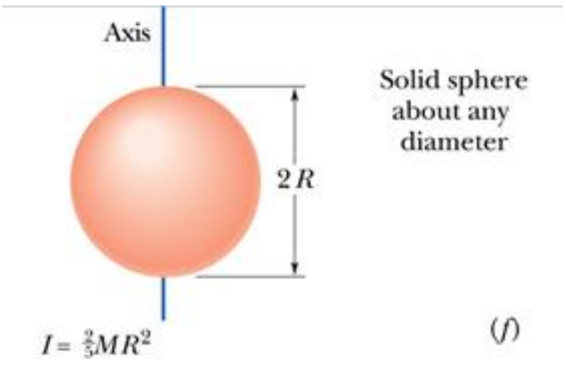
Axis
R
Hoop about central axis
$I = MR^2$
(a)



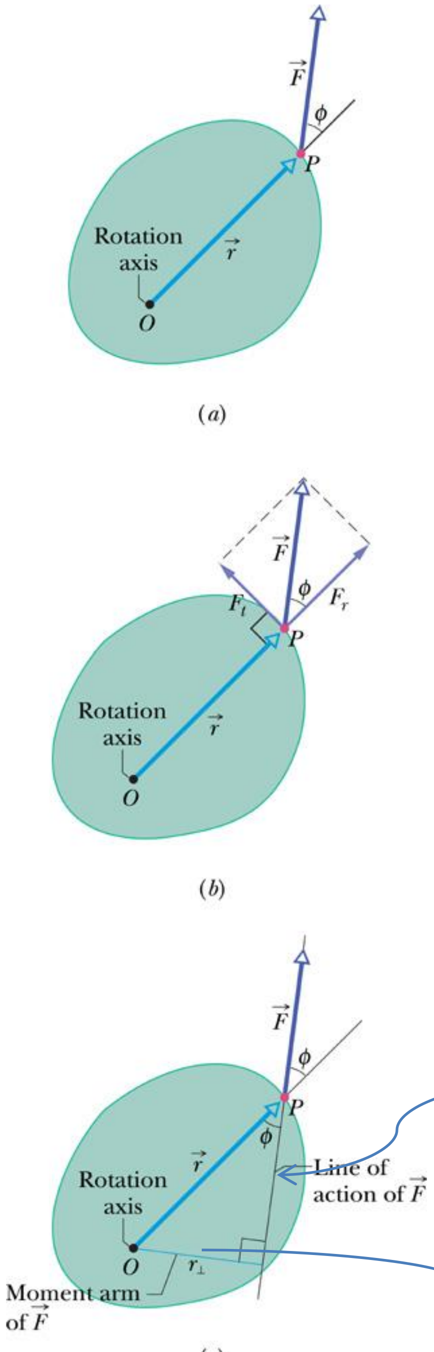
Axis
Solid cylinder (or disk) about central axis
R
L
$I = \frac{1}{2}MR^2$
(c)



Axis
Thin spherical shell about any diameter
2R
$I = \frac{2}{3}MR^2$
(g)



Axis
Solid sphere about any diameter
2R
$I = \frac{2}{5}MR^2$
(f)



torque

$$\vec{\tau} = \mathbf{r} \times \mathbf{F}$$

$$\tau = (r)(F \sin\phi) = rF_t$$

$$\tau = (r\sin\phi)(F) = r_\perp F$$

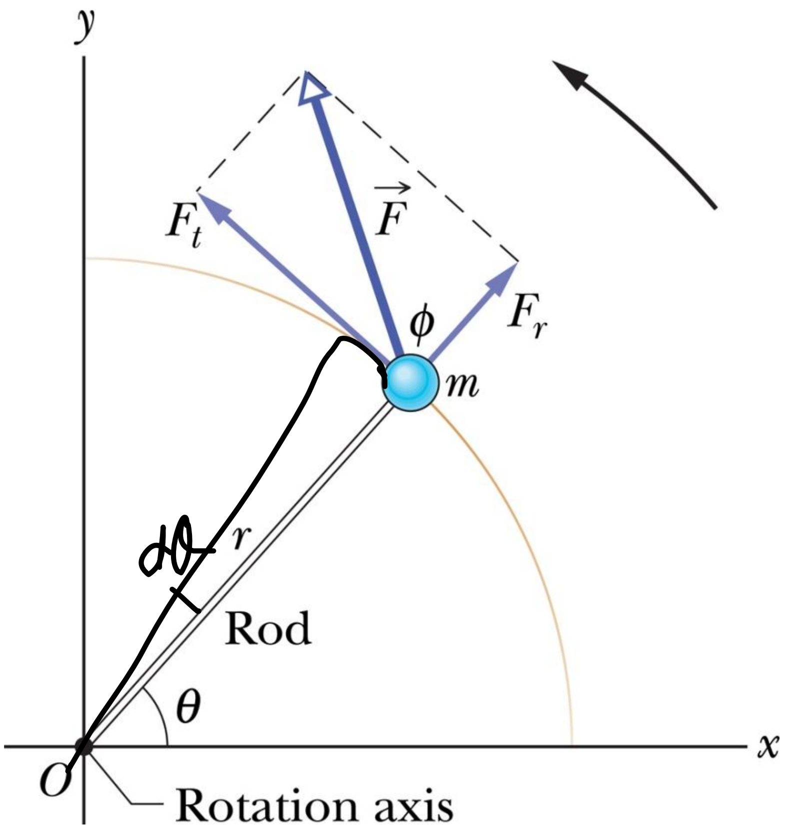
SI 단위: $\mathrm{N \cdot m}$

*일과 차원이 같으나 torque의 단위를 $\mathbf{J}$ 로 쓰지는 않는다.

힘의 작용선

힘의 모멘트팔

회전에 대한 Newton의 제2법칙

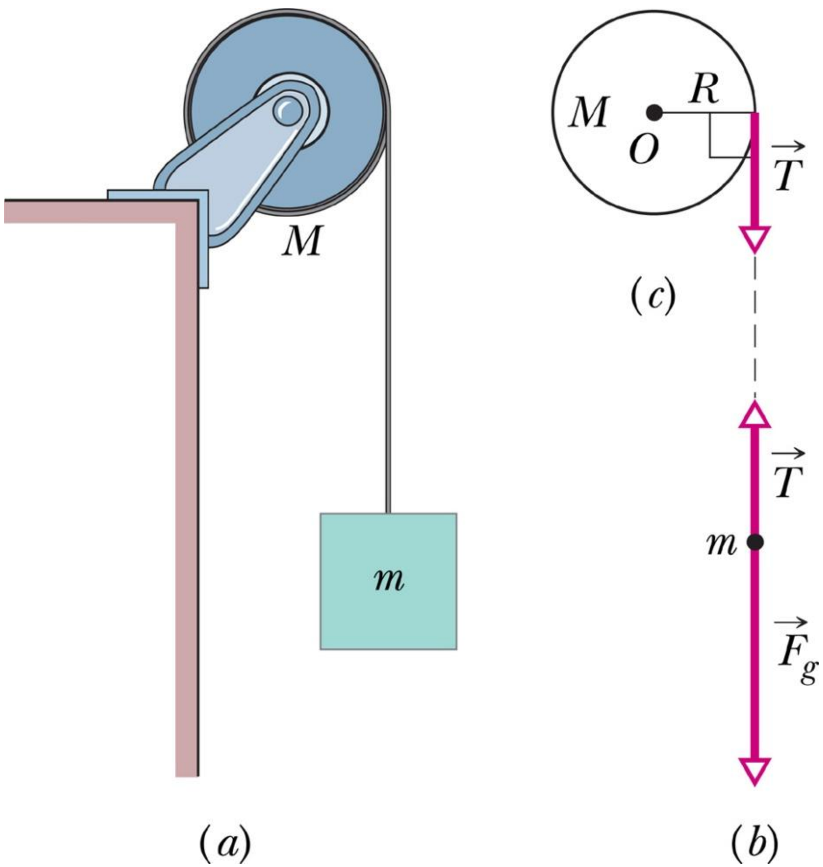


$$\tau = rF\sin\phi = rF_t = r(ma_t)$$
$$= rm(r\alpha) = (mr^2)\alpha$$

$$\tau = I\alpha$$

Sample prob.

$T = \frac{Ma}{2}$



$-T + mg = ma \rightarrow \left(\frac{M}{2} + m\right)a = mg$

$$\tau = RT = I\alpha = \frac{1}{2}MR^2\frac{a}{R}$$

$$a = g\frac{2m}{M + 2m} = 4.8\ (\mathrm{m/s^2})$$

$$T = \frac{1}{2}Ma = 6.0\ \mathrm{N}$$

$$\alpha = \frac{a}{R} = 24\ \mathrm{rad/s^2}$$

$M = 2.5\ \mathrm{kg},\ R = 20\ \mathrm{cm},\ m = 1.2\ \mathrm{kg}$

$a = \frac{m}{\frac{M}{2} + m}g = \frac{2m}{M + 2m}g$

Work & rotational kin. energy

일과 운동에너지 정리

$$\begin{aligned}\Delta K &= K_f - K_i = \frac{1}{2}mr^2\omega_f^2 - \frac{1}{2}mr^2\omega_i^2 \\ &= \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_f^2 = W\end{aligned}$$

$$dW = F_t r d\theta = \tau d\theta \qquad W = \int_{\theta_i}^{\theta_f} \tau d\theta$$

$$\begin{aligned}W &= \int \tau d\theta = \int I\alpha d\theta = \int I\frac{d\omega}{dt}d\theta = \int I\omega d\omega \\ &= \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 = \Delta K\end{aligned}$$

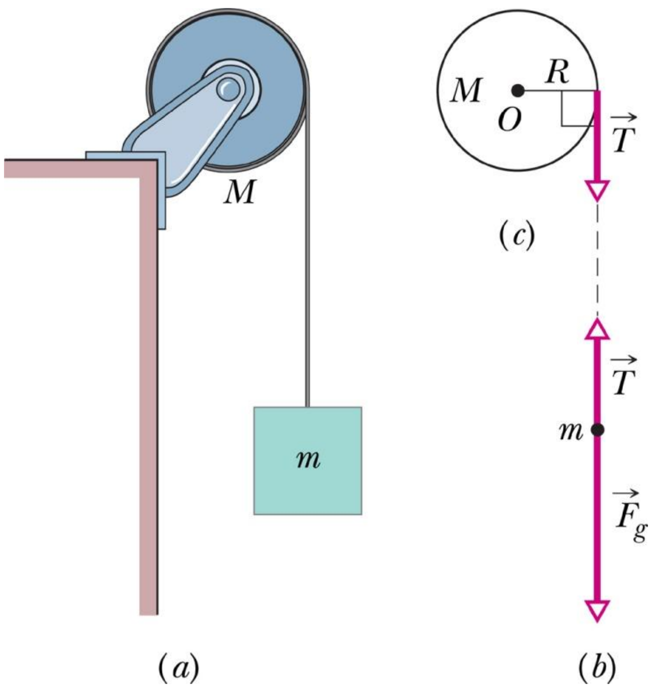
power

$$P = \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$$

$v^2 = 2as$

$s = \frac{v^2}{2a}$

Sample probs.



(1) 토막, 원판의 가속도 및 줄의 장력

$$RT = \frac{1}{2}MR^2\alpha$$

$$\alpha = \frac{a}{R}$$

$$mg - T = ma$$

$$T = \frac{1}{2}Ma$$

$$a = g\frac{2m}{M+2m}$$

$$T = \frac{mM}{M+2m}g$$

$M = 2.5$ kg, $R = 20$ cm, $m = 1.2$ kg

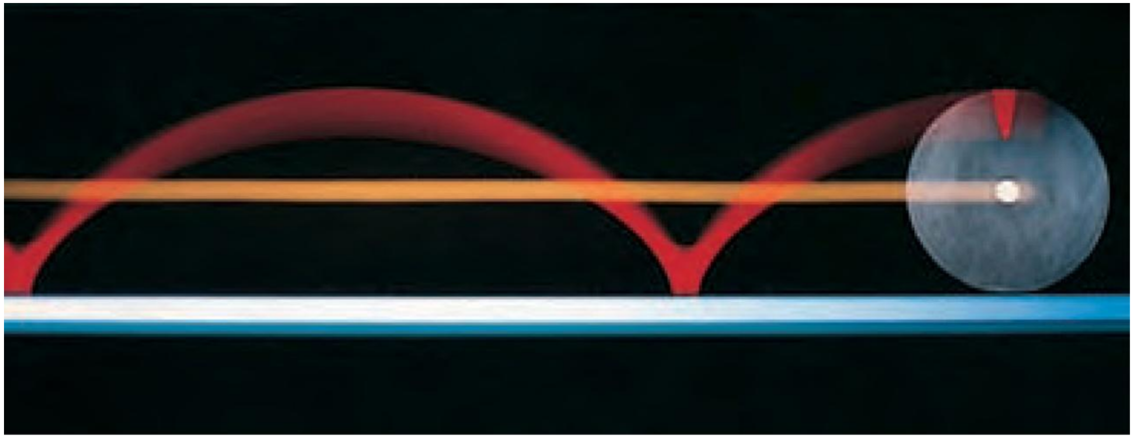
(2) 시간 t만큼 지난 후 역학적에너지

$$\omega = \alpha t \quad K_M = \frac{1}{2}(\frac{1}{2}MR^2)(\alpha t)^2 = \frac{1}{4}M(R\alpha t)^2$$

$$K_m = \frac{1}{2}m(at)^2 \quad U = -\frac{mg}{2a}(at)^2$$

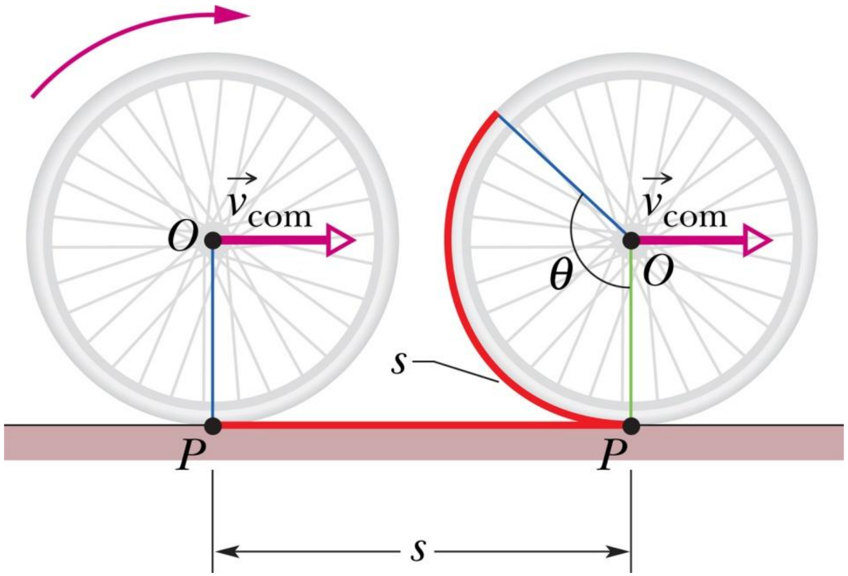
$U = mg$

Invention of a wheel



Translation of com
Rotation w.r.t. com

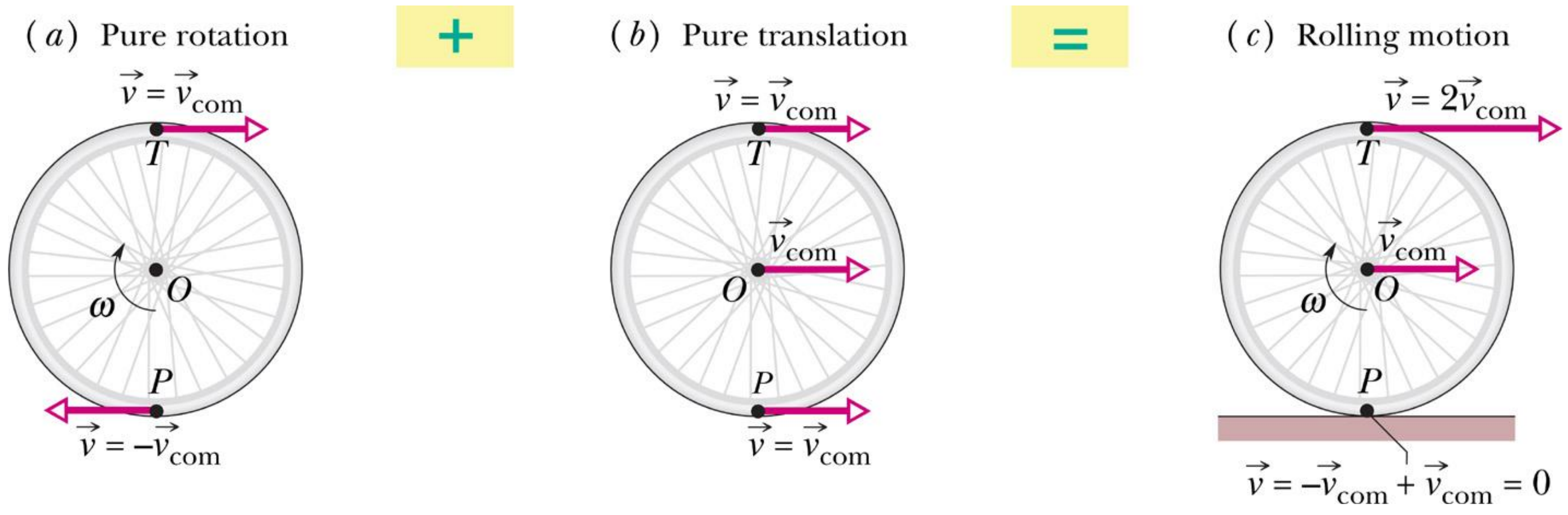
Contact point at rest
w.r.t. the ground
- Static friction



$$s = R\theta$$

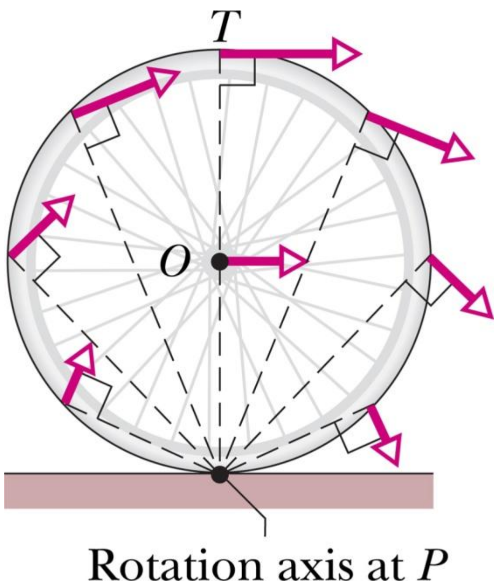
$$v_{com} = \omega R$$

Rolling



(a) Pure rotation
+
(b) Pure translation
=
(c) Rolling motion
$\vec{v} = \vec{v}_{com}$
T
O
ω
P
$\vec{v} = -\vec{v}_{com}$
$\vec{v} = \vec{v}_{com}$
T
$\vec{v}_{com}$
O
P
$\vec{v} = \vec{v}_{com}$
$\vec{v} = 2\vec{v}_{com}$
T
$\vec{v}_{com}$
ω
O
P
$\vec{v} = -\vec{v}_{com} + \vec{v}_{com} = 0$

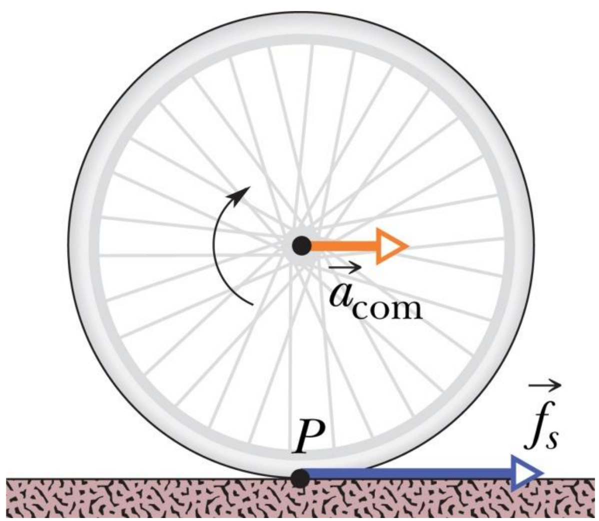
Rolling as pure rotation



$$v_{\text{top}} = \omega(2R) = 2R\omega = 2v_{\text{com}}$$

Kinetic energy

$$\begin{aligned} K &= \frac{1}{2}I_P\omega^2 = \frac{1}{2}(I_{\text{com}} + MR^2)\omega^2 \\ &= \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 \\ &= K_{\text{rot}} + K_{\text{trans}} \end{aligned}$$

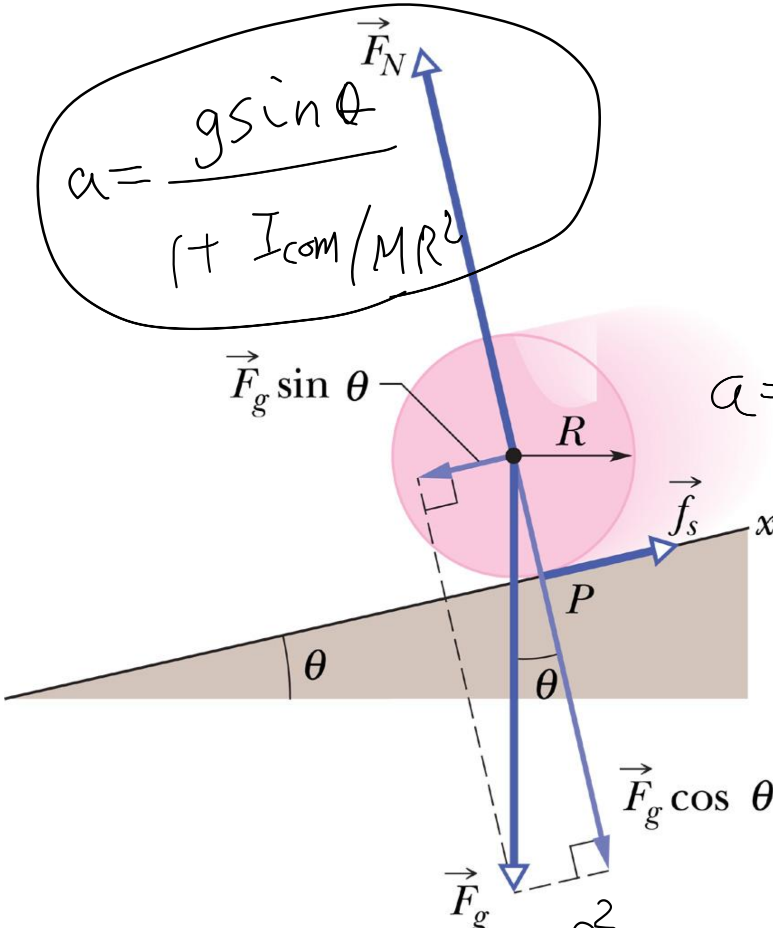


$a_{\text{com}} = \alpha R$ (no slipping)

$Mg\sin\theta = a\left(M + \frac{I_{com}}{MR^2}\right)$

Rolling down a ramp

$$a = \frac{g\sin\theta}{1 + I_{com}/MR^2}$$



force $\quad Mg\sin\theta - f_s = Ma$

torque $\quad Rf_s = I_{com}\alpha = \frac{I_{com}}{R}a$

$$a = \frac{R^2 f_s}{I_{com}}$$

$$a = R\alpha \qquad f_s = \frac{I_{com}}{R^2}a$$

$$f_s = I_{com}\frac{a}{R^2}$$

$$a = \frac{g\sin\theta}{1 + I_{com}/MR^2}$$

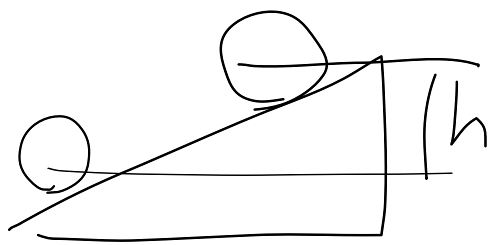
$$Mg\sin\theta = f_s + \frac{R^2}{I_{com}}Mf_s = f_s\left(1 + \frac{MR^2}{I_{com}}\right)$$

$$f_s = \frac{Mg\sin\theta}{1 + MR^2/I_{com}}$$

Sample prob.

질량 M인 꽉찬 공, 반지름 R, 회전각도 θ $I_{\text{com}} = \frac{2}{5}MR^2$

(a) 높이 h만큼 내려왔을 때 속력.



$$Mgh = \frac{1}{2}I\omega^2 + \frac{1}{2}Mv^2$$

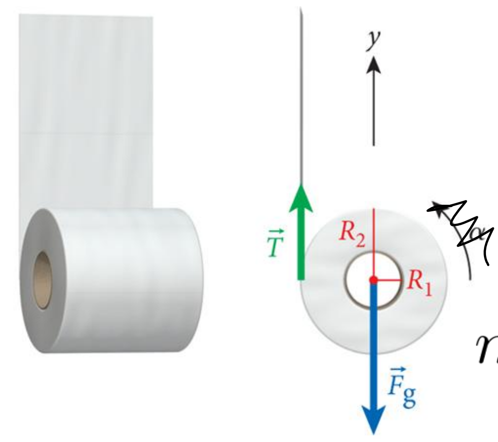
$$= \frac{1}{2}\frac{2}{5}M(R\omega)^2 + \frac{1}{2}Mv^2$$

(b) 구를 때 마찰력의 크기

$$= \frac{7}{10}Mv^2$$

$$v^2 = \frac{10gh}{7} \qquad v = \sqrt{\frac{10gh}{7}}$$

Ex. 10.3 toilet paper



(a) (b)

m, h

$$mg - T = ma$$

$$R_2\alpha = a$$

$$TR_2 = I\alpha \qquad T = I\frac{a}{R_2^2}$$

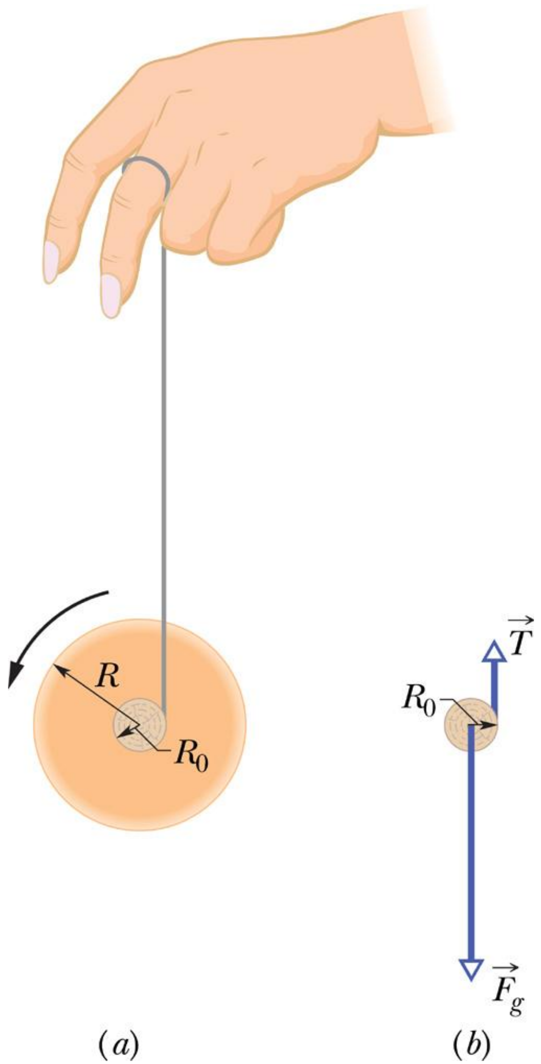
$$mg = a\left(m + \frac{I}{R_2^2}\right)$$

$$a = \frac{g}{1 + \frac{I}{mR_2^2}} = \frac{g}{1 + \frac{1}{mR_2^2}\frac{1}{2}m(R_1^2 + R_2^2)}$$

$$v^2 = 2ah$$

$$a_y = \frac{g}{3/2 + R_1^2/2R_2^2}$$

yoyo

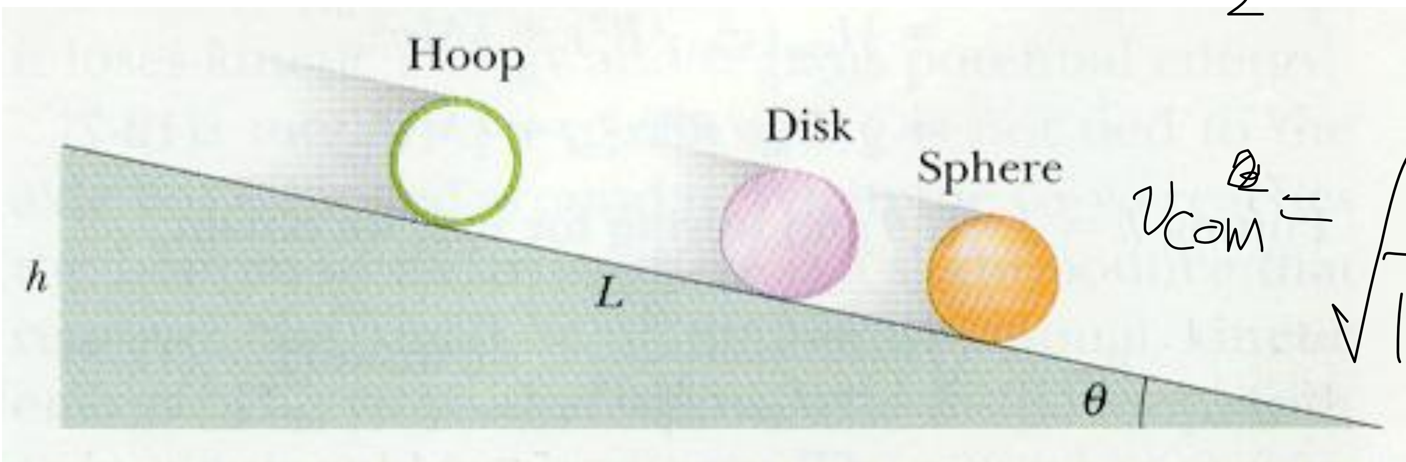


$$a_{\text{com}} = -\frac{g}{1 + I_{\text{com}}/MR_0^2}$$

Prob. 1

$$\frac{v_{com}^2}{2}\left(M + \frac{I_{com}}{MR^2}\right) = Mgh$$

$$v_{com} = \sqrt{\frac{2gh}{1+f}}$$



$$f = \frac{I_{com}}{MR^2}$$

$$\begin{aligned} Mgh &= \frac{1}{2} I_{com}\omega^2 + \frac{1}{2} M v_{com}^2 \\ &= \frac{1}{2} I_{com}(v_{com}/R)^2 + \frac{1}{2} M v_{com}^2 \end{aligned}$$

$$v_{com} = \sqrt{\frac{2gh}{1+f}}$$

$$\begin{aligned} \text{Hoop} &= 1 \\ \text{Disk} &= \frac{1}{2} \\ \text{Sphere} &= \frac{2}{5} \\ \text{Shell} &= \frac{2}{3} \end{aligned}$$