GEST 011, Newton's Clock & Heisenberg's Dice, Fall 2013

The Quantum Theory of Measurement

Mahn-Soo Choi (Korea University)

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Fundamental Postulates

(The Copenhagen Interpretation)

1. The state of a particle in motion is described by the wave function $\psi(x, t)$.
2. The dynamics is governed by Schrödinger's equation of motion
$$i\hbar\frac{\partial}{\partial t}\psi(x, t) = H\psi(x, t)$$
3. Under a given condition, every physical quantity has a unique wave function* $\psi_m(x)$ for each observed value† m of it.
4. After a measurement, the wave function "collapses" into one of the eigenfunctions of the measured quantity.

*Called as eigenfunction

†Called as eigenvalue

Quantum Theory of Measurement

Measurement Hypothesis

After the measurement, the wave function "collapses" into one of the eigenstate of the quantity.

Measuring a Quantum Penny

$$\psi(t) = \psi_H(t)$$

$$|\psi_H(t)|^2 = 1$$

Measuring a Quantum Penny

$$\psi(t) = \psi_T(t)$$

$$|\psi_T(t)|^2 = 1$$

Measuring a Quantum Penny

$$\psi(t) = \frac{1}{\sqrt{3}}\psi_H(t) + \frac{\sqrt{2}}{\sqrt{3}}\psi_T(t)$$

Measuring a Quantum Penny

$$\psi(t) = \frac{1}{\sqrt{3}}\psi_H(t) + \frac{\sqrt{2}}{\sqrt{3}}\psi_T(t)$$

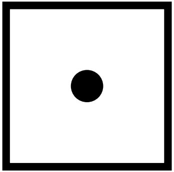
$\psi(t) \Longrightarrow$

$= \psi_H\,, \quad P_H = \frac{1}{3}$

$= \psi_T\,, \quad P_T = \frac{2}{3}$

Images from `http://stores.auction.co.kr/`

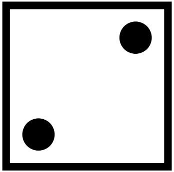
Measuring a Quantum Dice



$$\psi(t) = \psi_1(t)$$

$$|\psi_1(t)|^2 = 1$$

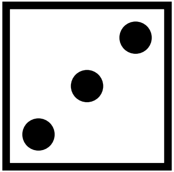
Measuring a Quantum Dice



$$\psi(t) = \psi_2(t)$$

$$|\psi_2(t)|^2 = 1$$

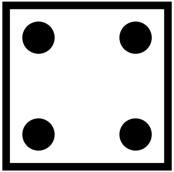
Measuring a Quantum Dice



$$\psi(t) = \psi_3(t)$$

$$|\psi_3(t)|^2 = 1$$

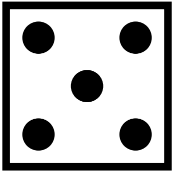
Measuring a Quantum Dice



$$\psi(t) = \psi_4(t)$$

$$|\psi_4(t)|^2 = 1$$

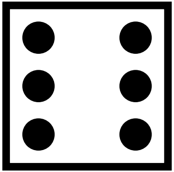
Measuring a Quantum Dice



$$\psi(t) = \psi_5(t)$$

$$|\psi_5(t)|^2 = 1$$

Measuring a Quantum Dice



$$\psi(t) = \psi_6(t)$$

$$|\psi_6(t)|^2 = 1$$

Measuring a Quantum Dice

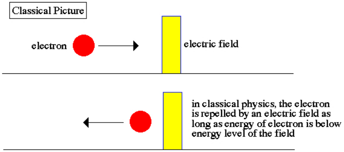
http://www.gmdice.com/

$$\psi(t) = \frac{\psi_4(t) + \sqrt{2}\psi_5(t) - \psi_6(t)}{2}$$

$$\psi(t) \Longrightarrow \begin{cases} ⚀ = \psi_1 \,, & P_1 = 0 \\ ⚁ = \psi_2 \,, & P_2 = 0 \\ ⚂ = \psi_3 \,, & P_3 = 0 \\ ⚃ = \psi_4 \,, & P_4 = 1/4 \\ ⚄ = \psi_5 \,, & P_5 = 2/4 \\ ⚅ = \psi_6 \,, & P_6 = 1/4 \end{cases}$$

Quantum Tunneling

Quantum Tunneling



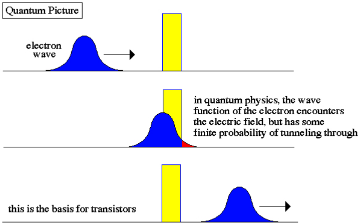


Image courtesy of `http://abyss.uoregon.edu/~js/`

Quantum Tunneling

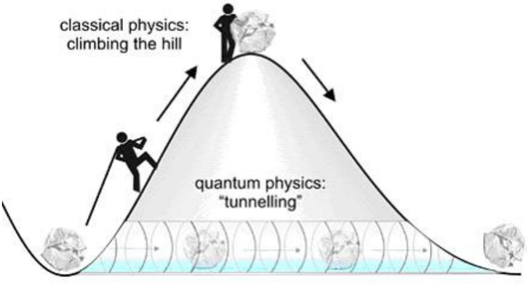
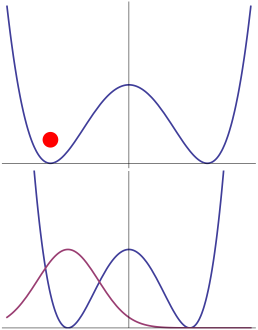


Image courtesy of `http://www.intomobile.com/`

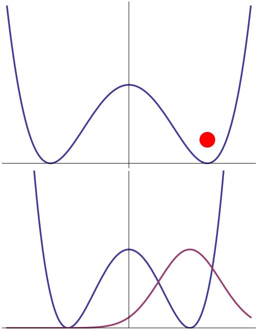
Measuring a Quantum Penny



$$\psi(t) = \psi_L(x,t)$$

$$\int dx\, |\psi_L(x,t)|^2 = 1$$

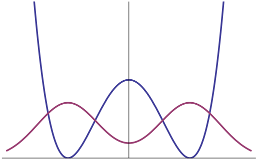
Measuring a Quantum Penny



$$\psi(t) = \psi_R(x,t)$$

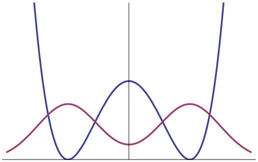
$$\int dx\, |\psi_R(x,t)|^2 = 1$$

Measuring a Quantum Penny



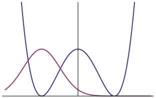
$$\psi(t) = \frac{1}{\sqrt{3}}\psi_L(t) + \frac{\sqrt{2}}{\sqrt{3}}\psi_R(t)$$

Measuring a Quantum Penny

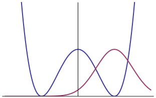


$$\psi(t) = \frac{1}{\sqrt{3}}\psi_L(t) + \frac{\sqrt{2}}{\sqrt{3}}\psi_R(t)$$

$$\psi(t) \Longrightarrow$$

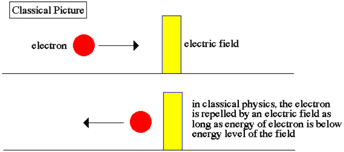


$$= \psi_L(x,t)\,, \quad P_L = \tfrac{1}{3}$$



$$= \psi_R(x,t)\,, \quad P_R = \tfrac{2}{3}$$

Quantum Tunneling



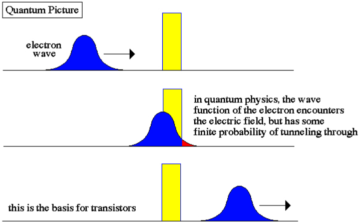
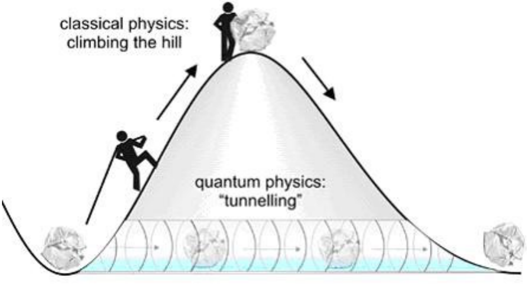


Image courtesy of `http://abyss.uoregon.edu/~js/`

Quantum Tunneling



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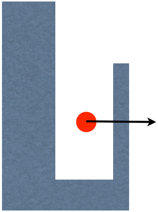


"An unstable particle, if observed continuously, will never decay."

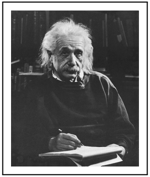
"A watched pot never boils."

http://www.vikingasia.org/

Measurement & Complementarity Principle

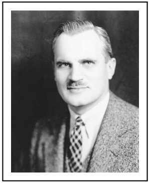
Wave-Particle Duality

(A. Einstein, 1902; A. H. Compton, 1923; L. de Broglie, 1924)



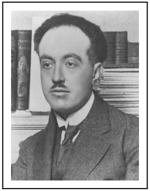
Let light (wave) have discrete energies!

$$(\text{energy}) = h \times (\text{frequency})$$



Let light (wave) have momentum!

$$(\text{momentum}) = \frac{h}{(\text{wavelength})}$$



Let particles behave like a wave with:

$$(\text{frequency}) = h \times (\text{energy}) \,,$$

$$(\text{wavelength}) = \frac{h}{(\text{momentum})}$$

Imeages courtesy of Wikipedia

Baron Ashura vs Jekyll & Hyde

(bad and good analogies of wave-particle duality)

Images from http://blog.naver.com/, http://www.weblo.com/, & http://en.wikipedia.org/

Baron Ashura vs Jekyll & Hyde

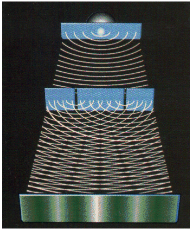
(bad and good analogies of wave-particle duality)

Complementarity principle (Niels Bohr)

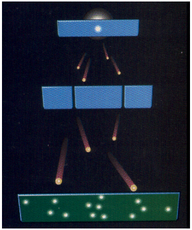
Once wave-like, not like paricles!
Once particle-like, not like waves!

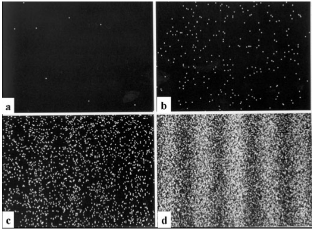
Images from http://blog.naver.com/, http://www.weblo.com/, & http://en.wikipedia.org/

Double-Slit Interference (Revisited)



vs

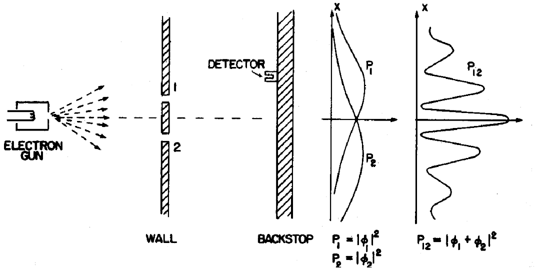




$$\Psi = \frac{1}{\sqrt{2}}\left(\Psi_L + \Psi_R\right)$$

The Complementarity Principle

(either, but not both)



Either (not both) particle-like or wave-like

Image courtesy of `http://quantummechanics.ucsd.edu/`

The 5th Solvay International Conference (1927)

(on photons and electrons)

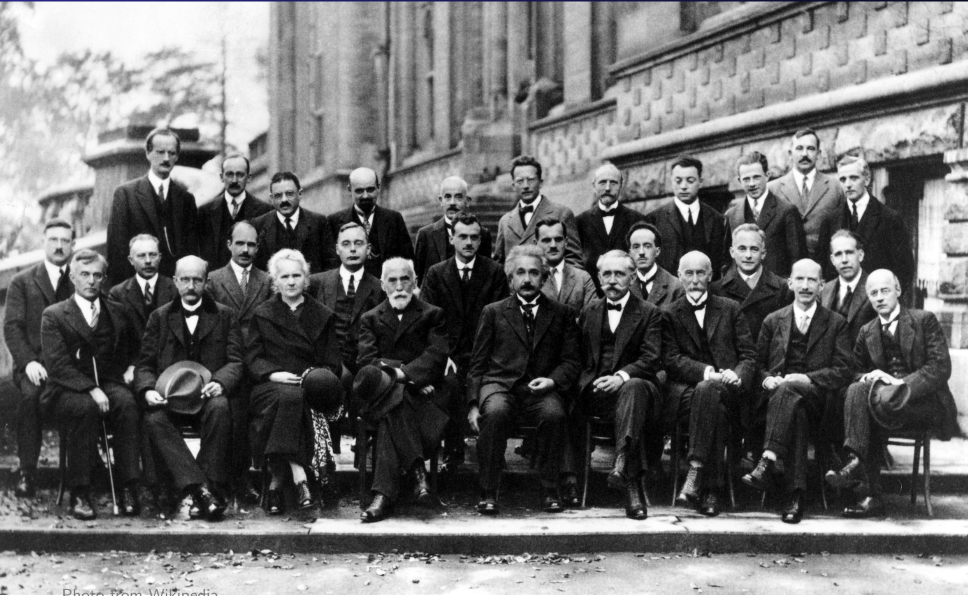


Photo from Wikipedia

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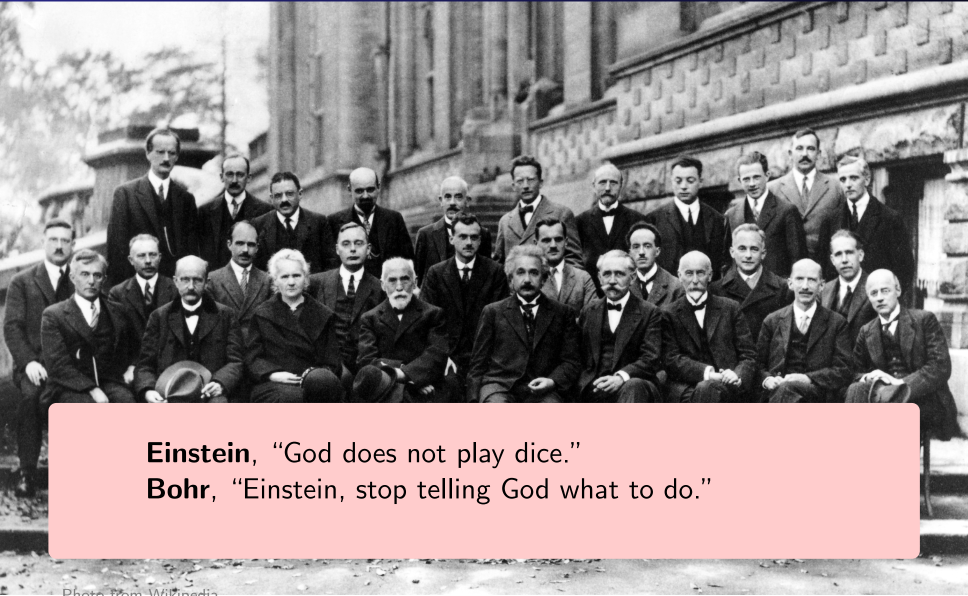
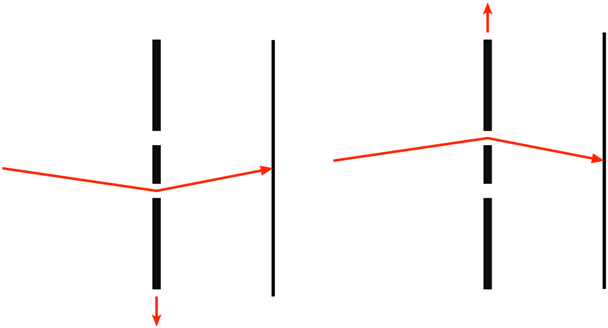


Einstein, "God does not play dice."
Bohr, "Einstein, stop telling God what to do."

Photo from Wikipedia

The 5th Solvay International Conference (1927)

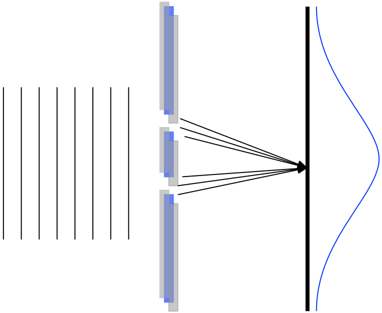
(A Challenge to the Complementarity Principle)



Einstein, "Both which-path detection and interference!"

The 5th Solvay International Conference (1927)

(A Challenge to the Complementarity Principle)



Einstein, "Both which-path detection and interference!"

Bohr, "No interference because of the uncertainty."

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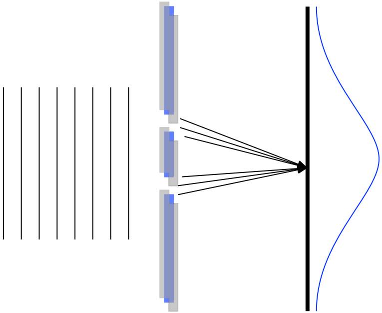
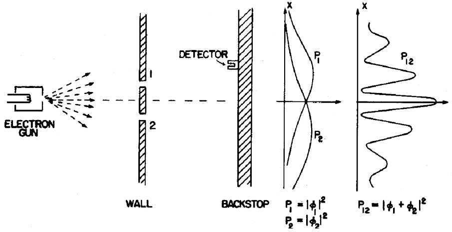


Einstein, "Both which-path detection and interference!"

Bohr, "No interference because of the uncertainty."

Enough?

The Complementarity Principle?



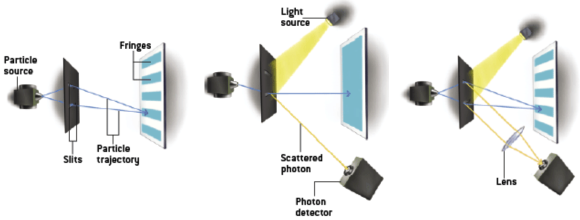
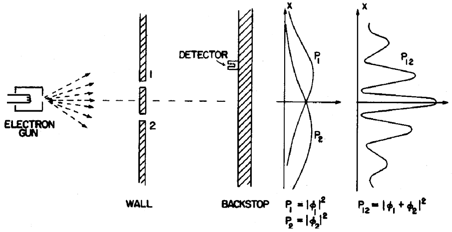
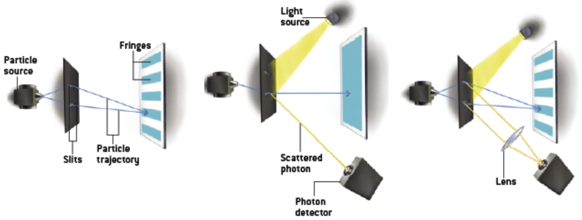


Image courtesy of `http://quantummechanics.ucsd.edu/` (top) and Hillmer & Kwiat, Scientific American (2007) (bottom).

Quantum Eraser

(Hillmer & Kwiat, Scientific American, 2007)





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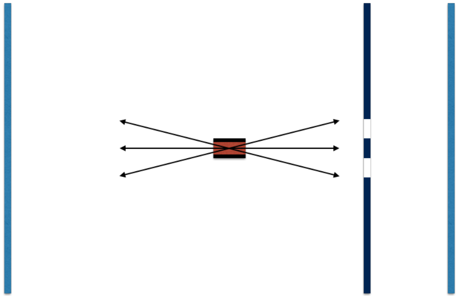
Quantum Eraser: Another Example

(Dopfer 1998)



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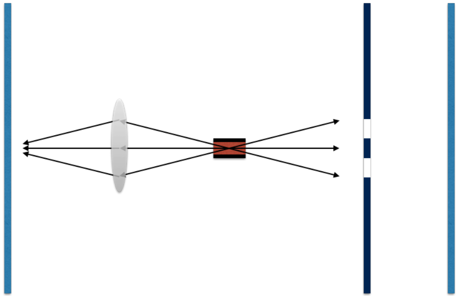

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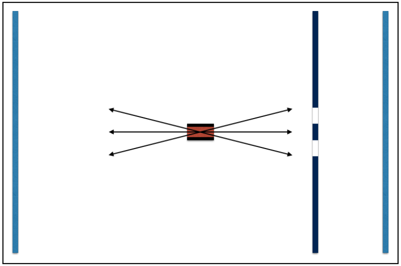

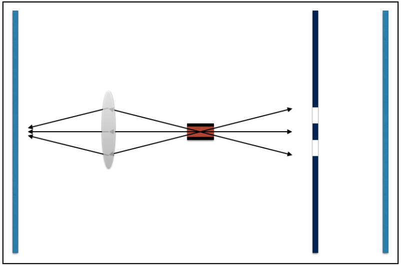

Complementarity Principle

Wave function callapses as much as the information can be acquired.

References

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