Chapter 1

Newton's Law of Motion

1.1 Introduction

1.1.1 Postulate of Classical Mechanics

1. there are three spatial directions which are orthogonal among one another.
2. the space is homogeneous so that it is invariant under translation.
3. the space is isotropic so that it is invariant under rotation.
4. time flows homogeneously
5. physics law are invariant whenever and wherever we are.

$*$ meaning of homogeneous is similar to uniform, and that of isotropic is smilar to indepent of direction.

1.1.2 Position Vector

1. The position of particle can be expressed as a function of time

$$\boldsymbol{x}(t) = \sum_{i=1}^{3} x_i(t)\hat{\boldsymbol{e}}_i \tag{1.1}$$

,where $\boldsymbol{e}_i$ is unit vector of x_i-axis in three-dimensional Euclidean space.

1.1.3 Velocity

1. The velocity vector $\boldsymbol{v}(t)$ is the first-order time derivative of the position.

$$\boldsymbol{v}(t) \equiv \frac{d\boldsymbol{x}(t)}{dt}. \tag{1.2}$$

1.1.4 Acceleration

1. The acceleration vector $\boldsymbol{a}(t)$ is the second-order time derivative of the position.

$$\boldsymbol{a}(t) \equiv \frac{d^2\boldsymbol{x}(t)}{dt^2} = \frac{d\boldsymbol{v}(t)}{dt}. \tag{1.3}$$

1.1.5 Momentum

1. The momentum vector $\boldsymbol{p}$ is the product of mass and velocity.

$$\boldsymbol{p} = m\boldsymbol{v}. \tag{1.4}$$

1.1.6 Force

1. The force $\boldsymbol{F}$ is the first-order time derivative of the momentum

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt} = \frac{d(m\boldsymbol{v})}{dt} = m\frac{d\boldsymbol{v}}{dt} + \boldsymbol{v}\frac{dm}{dt} = m\frac{d\boldsymbol{v}}{dt} = m\boldsymbol{a} \tag{1.5}$$

,where we assume that $\frac{dm}{dt} = 0$.

1.2 Newton's Laws of Motion

1. The velocity of an object is invariant if the object is free. Hence, if the velocity is constant, the net external force is zero.

2. The net external force is product of mass and acceleration.

$$\boldsymbol{F} = m\boldsymbol{a}. \tag{1.6}$$

3. If two objects 1 and 2 exert force on each other, the force on object is identical to that on object 2 except that the direction is opposite.

$$\boldsymbol{F}_{2\to1} = \boldsymbol{F}_{1\to2}. \tag{1.7}$$

1.3 Frame of Reference[1]

1.3.1 Reference Frame

1. A reference frame is a frame where the positon of a particle with respect to time is measured.

1.3.2 Inertial Frame

1. An inertial frame is a frame where Newton's laws are valid.

1.3.3 Galilean Relativity

1. A Galilean transformation is the tranformation of the space-time coordinates between two inertial reference frame. Between two frames, space and time are homogeneous, mass is indetical.

2. Under Galilean tranformation, position, velocity, kinectic engergy are covariant. Otherwise, force and Newton's third law are invariant under Galilian transformation.

3. Let us consider two frames S and S'. The space-time coor dinates of a particle moving around the $\boldsymbol{x}(t)$ and $\boldsymbol{x}'(t)$. If the relative velocity of two frames is $\boldsymbol{V}$, then,

$$t = t', \tag{1.8a}$$
$$\boldsymbol{x}'(t) = \boldsymbol{x}(t) + \boldsymbol{V}t, \tag{1.8b}$$
$$\dot{\boldsymbol{x}}'(t) = \dot{\boldsymbol{x}}(t) + \boldsymbol{V}, \tag{1.8c}$$
$$\ddot{\boldsymbol{x}}'(t) = \ddot{\boldsymbol{x}}(t), \tag{1.8d}$$
$$\boldsymbol{F}' = \boldsymbol{F}, \tag{1.8e}$$
$$m' = m, \tag{1.8f}$$
$$T' \neq T. \tag{1.8g}$$

[1] Seeing full details, recommend to see lecture video.

1.4 Rotational version of Newton's Laws

1.4.1 Angular Momentum

1. The angular momentum of particle is defined by

$$\boldsymbol{L} = \boldsymbol{r} \times \boldsymbol{p} \tag{1.9}$$

,where $\boldsymbol{r}$ is the position of particle.

1.4.2 Torque

1. The torque of particle is defined by

$$\boldsymbol{\tau} = \boldsymbol{r} \times \boldsymbol{F}. \tag{1.10}$$

2. Other definition of the torque is the first-order time derivative of the angular momentum.

$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{\tau}. \tag{1.11}$$

1.4.3 Central Force

1. The central force is a force on a particle is collinear to the postion vector.

$$\boldsymbol{F} = F\hat{\boldsymbol{r}}. \tag{1.12}$$

2. If the net external force on a particle is central force,

$$\frac{d\boldsymbol{L}}{dt} = \boldsymbol{r} \times \boldsymbol{F} = \boldsymbol{r} \times \hat{\boldsymbol{r}}F = 0. \tag{1.13}$$

This means that the angular momentum is conserved.