Chapter 3

Vector Integration

3.1 Basic Integrals

3.1.1 Integral

1. Consider a three-dimensional vector $\boldsymbol{A}$ which is function of a scalar s. The integral of the vector over the s is

$$\int \boldsymbol{A}(s)ds \equiv \hat{\boldsymbol{e}}_i \int A_i(S)ds. \tag{3.1}$$

3.1.2 Scalar line integral

1. Consider a three-dimensional vector $\boldsymbol{A}$ which is function of postion vector $\boldsymbol{r} = \hat{\boldsymbol{e}}_i x_i$. The scalar line integral of the vector over $\boldsymbol{r}$ from the poiont $\boldsymbol{a} = \hat{\boldsymbol{e}}_i a_i$ to $\boldsymbol{b} = \hat{\boldsymbol{e}}_i b_i$ is

$$\int \boldsymbol{A} \cdot d\boldsymbol{r} \equiv \int_{\boldsymbol{a}}^{\boldsymbol{b}} A_i(x_1, x_2, x_3)dx_i. \tag{3.2}$$

3.1.3 Closed integral

1. If the path C is closed, we use a special notation like

$$\oint_C \boldsymbol{A} \cdot d\boldsymbol{r}. \tag{3.3}$$

3.1.4 Surface integral

1. The surface integral is defined by

$$\int_S \boldsymbol{V} \cdot d\boldsymbol{A} \tag{3.4}$$

, where $d\boldsymbol{A} = \hat{\boldsymbol{n}}dA$, $\hat{\boldsymbol{n}}$ is normal vector of surface.

3.1.5 Volume integral

1. The volume integral of a vector $\boldsymbol{A}$ or a scalar $\boldsymbol{\phi}$ is defined by

$$\int_V \boldsymbol{A} d^3\boldsymbol{r}, \tag{3.5a}$$

$$\int_V \phi d^3\boldsymbol{r} \tag{3.5b}$$

, where $d^3\boldsymbol{r} = dx_1 dx_2 dx_3$.

3.2 Divergence Theorem

1. The divergence theorem is

$$\int_V \nabla \cdot \boldsymbol{V} d^3\boldsymbol{r} = \oint_S \boldsymbol{V} \cdot d\boldsymbol{A}. \tag{3.6}$$

, where a volume V is bounded by the closed surface S, $d\boldsymbol{A}$ is the surface elements of S.

3.3 Stokes' Theorem

1. The Stokes' theorem is

$$\int_S \nabla \times \boldsymbol{V} \cdot d\boldsymbol{A} = \oint_C \boldsymbol{V} \cdot d\boldsymbol{l} \tag{3.7}$$

, where a surface S is bounded by the closed path C.