Communication Signals

(Haykin Sec. 2.4 and Ziemer Sec.2.1-Sec. 2.2)

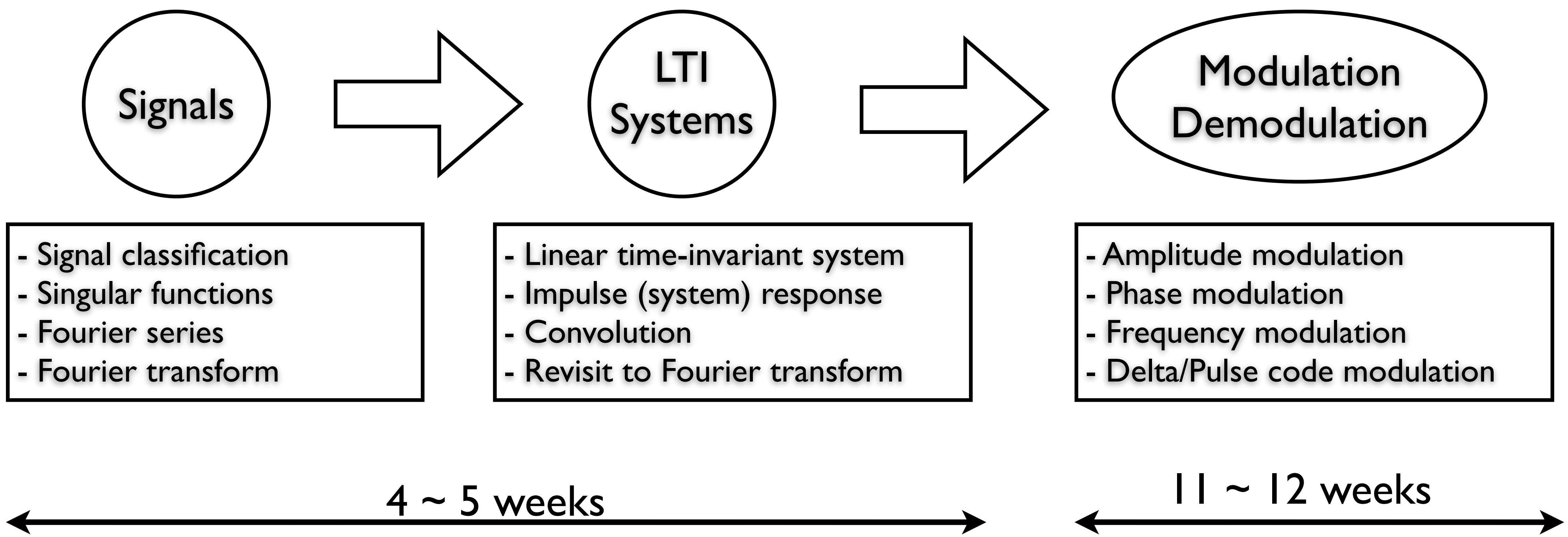
KECE321 Communication Systems I

Lecture #2, March 8, 2011

Prof. Young-Chai Ko

Summary of Today's Lecture

- Signal Classification
- Basic Continuous-Time Signals
- Singular functions



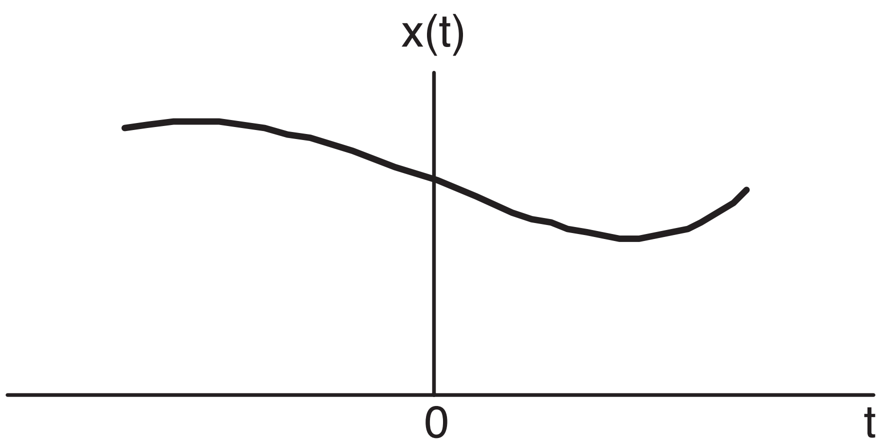
Signal Classification

- Continuous-Time and Discrete-Time signals
- Analog and Digital signals
- Real and Complex signals
- Deterministic and Random signals
- Even and Odd signals
- Periodic and Nonperiodic signals
- Energy and Power signals

Continuous-Time and Discrete-Time Signals

- Continuous-time signals
  - A signal $x(t)$ is continuous-time if t is a continuous variable.
- Discrete-time signals
  - If t is a discrete variable, that is, $x(t)$ is defined at discrete times, then $x(t)$ is a discrete-time signal.
  - Since a discrete time is defined at discrete times such as $t = nT$, a discrete-time signal is often identified as a sequence of numbers, denoted by $\{x_n\}$ or $x[n]$

Continuous-Time and Discrete-Time Signals



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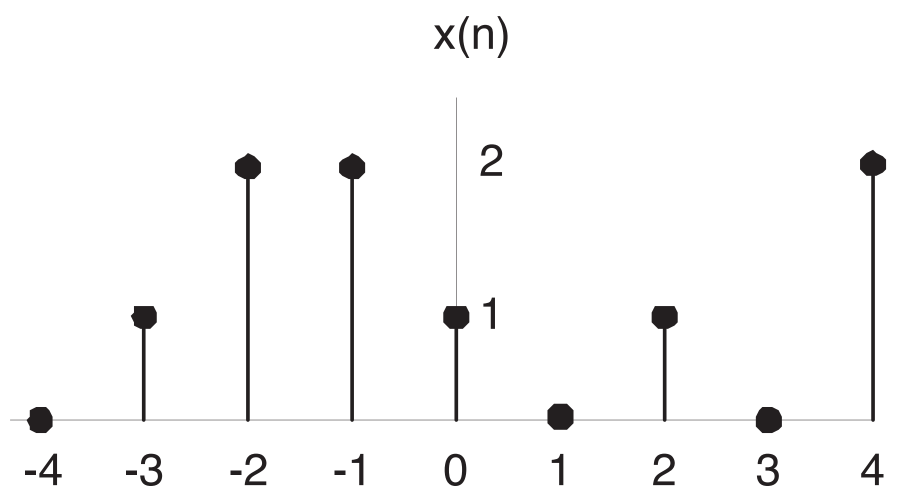

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Analog and Digital Signals

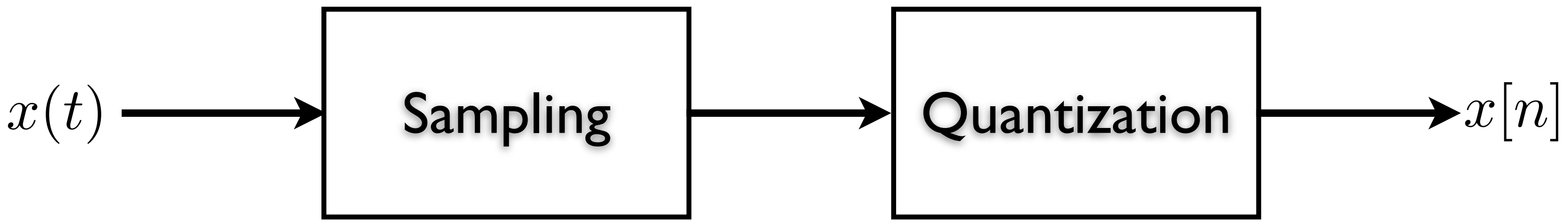
- **Analog signals**

$$-\infty < x(t) < \infty$$

- **Digital signals**

$$x[n] \in \{q_1, q_2, \cdots, q_n\}$$

- **Analog signals to Digital signals**



Real and Complex Signals

- Real signal
  - If $x(t)$ takes real number, it is a real signal
- Complex signal

$$x(t) = x_1(t) + jx_2(t)$$

  - Questions for fun
    - Is the complex signal real?
    - Does there really exist an imaginary part?

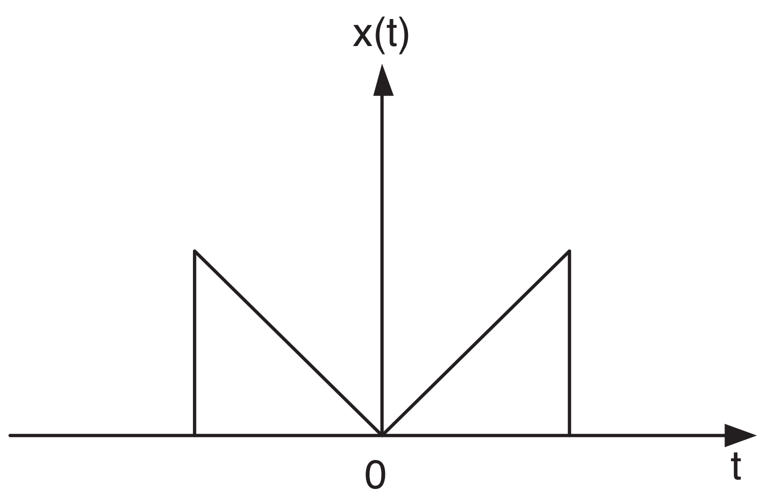
Even and Odd Signals

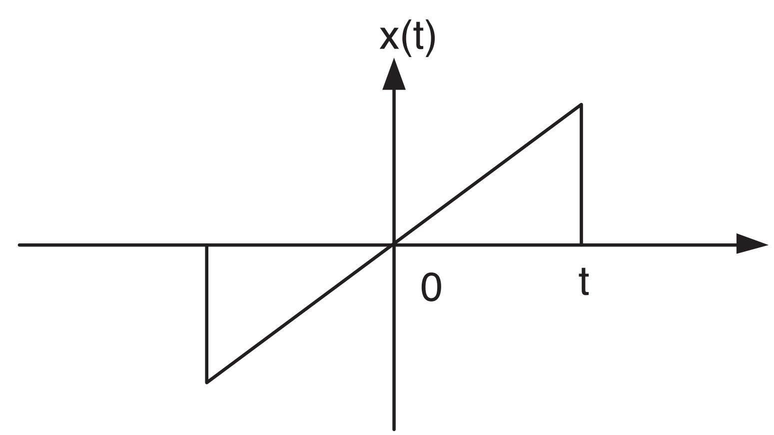
- **Even signal if**

$$x(-t) = x(t)$$

- **Odd signal if**

$$x(-t) = -x(t)$$





- Any signal $x(t)$ can be expressed as a sum of even and odd signals:

$$x(t) = x_e(t) + x_o(t)$$

- Even part and odd part of

$$x_e(t) = \frac{1}{2}\{x(t) + x(-t)\}$$
$$x_o(t) = \frac{1}{2}\{x(t) - x(-t)\}$$

Periodic and Nonperiodic Signals

- Periodic signal with period T if

$$x(t+T) = x(t) \quad \text{for all } t$$

- Fundamental period T_0
  - smallest positive value of T

$$T = mT_0 \quad \text{for any integer } m$$

Energy and Power Signals

- Energy of continuous time signal $x(t)$ is defined as

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- Normalized average power is defined as

$$P = \lim_{T \to \infty} \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- $x(t)$ is an energy signal if and only if

$$0 < E < \infty$$

- $x(t)$ is a power signal if and only if

$$0 < P < \infty$$

Phasor Signals and Spectra

- A useful periodic signal in system analysis is the complex signal

$$\tilde{x}(t) = Ae^{j(\omega_0 t+\theta)}, \qquad -\infty < t < \infty$$

  - A : amplitude
  - ω_0 : frequency in radian per second or $f_0 = \omega_0/2\pi$ hertz
  - θ_0 : phase in radians
  - We refer to $\tilde{x}(t)$ as a rotating phasor to distinguish from the phasor $e^{j\theta}$.

- We can show that $\tilde{x}(t) = \tilde{x}(t+T_0)$ with $T_0 = 2\pi/\omega_0 = 1/f_0$. Thus $\tilde{x}(t)$ is periodic signal with period $T_0 = 1/f_0$.

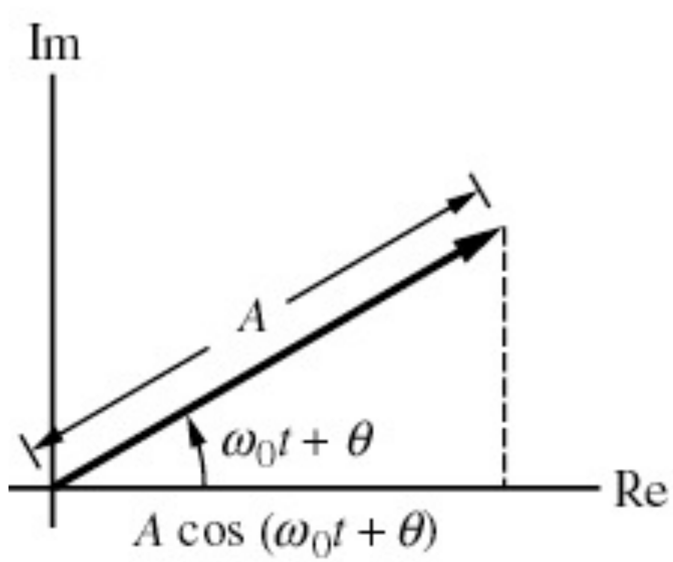
- A rotating phasor $Ae^{j(\omega_0 t+\theta)}$ can be related to a real, sinusoidal signal $A\cos(\omega_0 t+\theta)$ in two ways.

  - The first is by taking its real part,

$$\begin{aligned} x(t) &= A\cos(\omega_0 t+\theta) = \Re[\tilde{x}(t)] \\ &= \Re[Ae^{j(\omega_0 t+\theta)}] \end{aligned}$$

  - The second is by taking one-half of the sum of $\tilde{x}(t)$ and its complex conjugate,

$$\begin{aligned} A\cos(\omega_0 t+\theta) &= \frac{1}{2}\tilde{x}(t) + \frac{1}{2}\tilde{x}^*(t) \\ &= \frac{1}{2}Ae^{j(\omega_0 t+\theta)} + \frac{1}{2}Ae^{-j(\omega_0 t+\theta)} \end{aligned}$$



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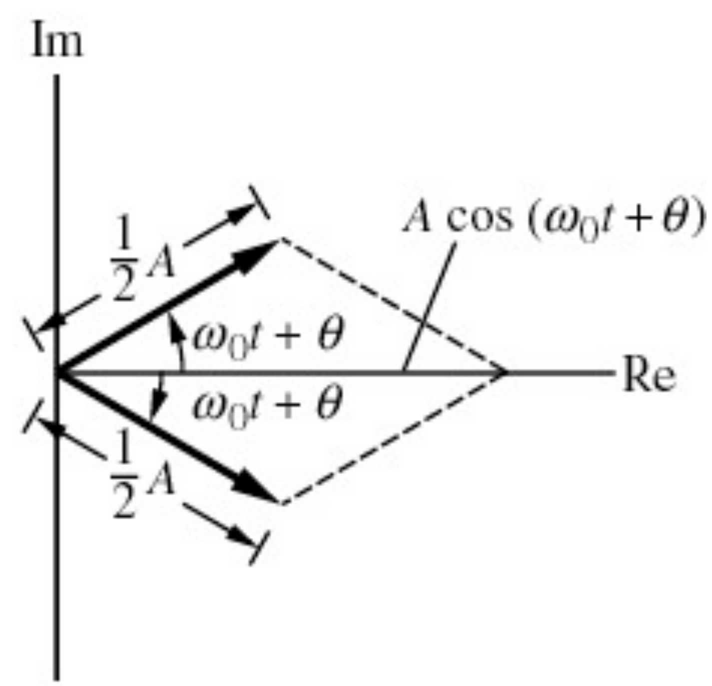

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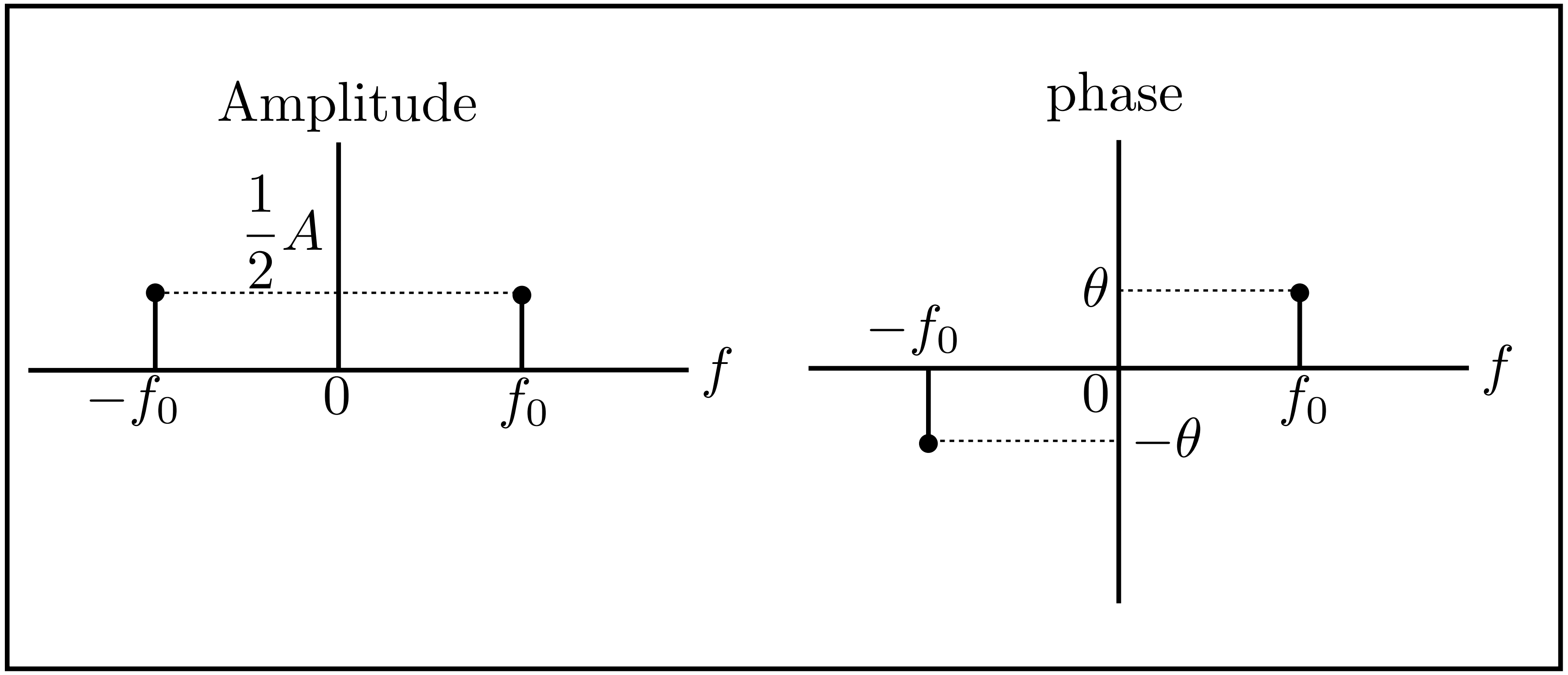
- Let $x(t) = A\cos(\omega_0 t + \theta)$. Then we showed that

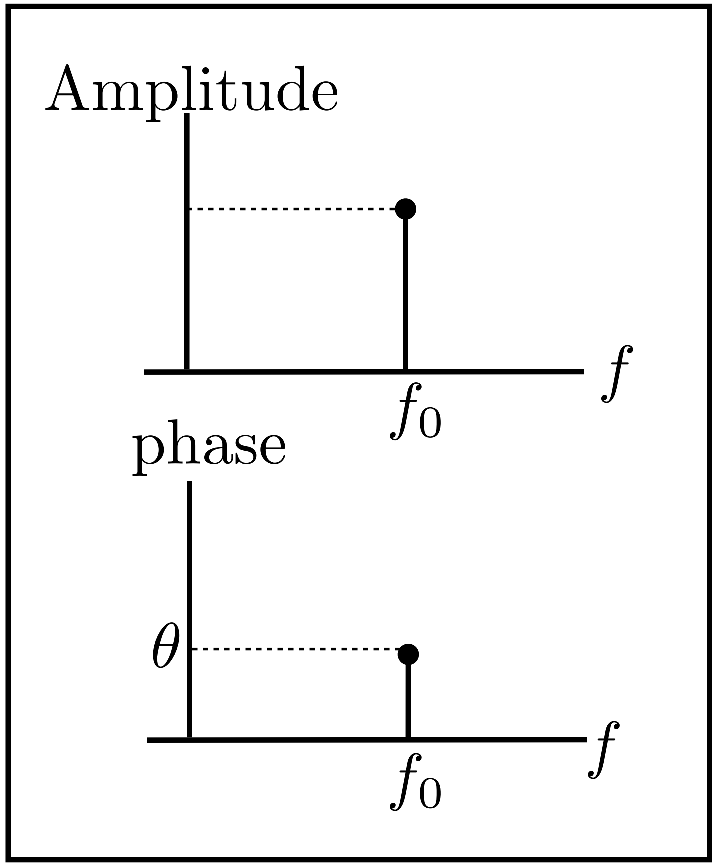
$$x(t) = \Re[\tilde{x}(t)] = \frac{1}{2}\tilde{x}(t) + \tilde{x}^*(t)$$

  - Two equivalent representation of $x(t)$ in the frequency domain may be obtained by noting that the rotating phasor signal is completely specified if the parameters, A and θ, are given for a particular f_0.

  - Thus plots of the magnitude and angle of $Ae^{j\theta}$ versus frequency gives sufficient information to characterize $x(t)$ completely.



Double-sided spectra



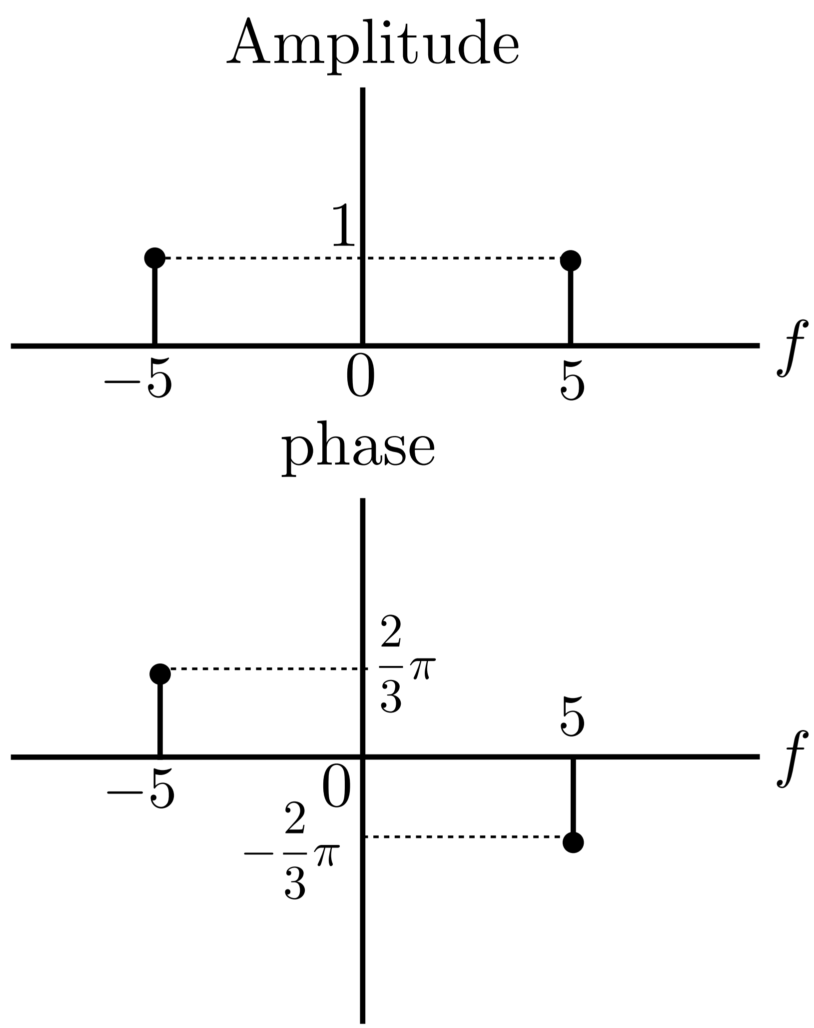
Single-sided spectra

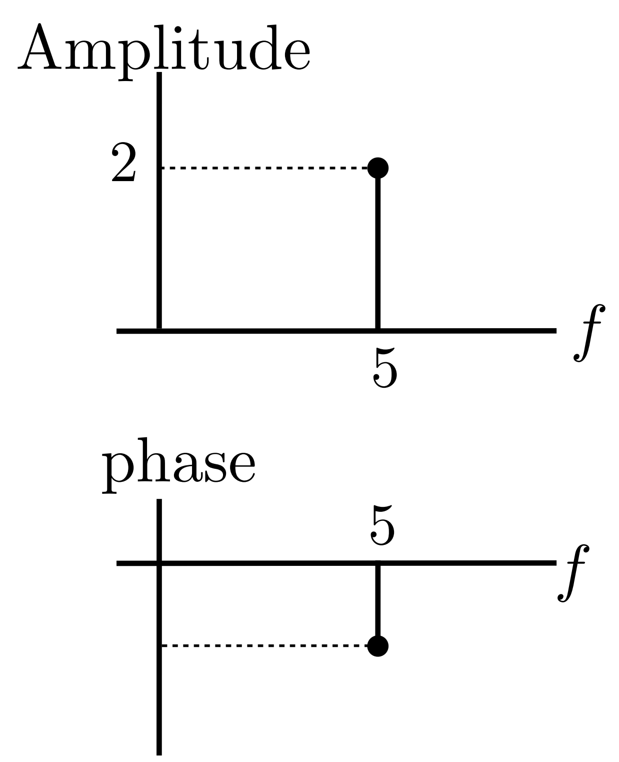
- *Example 1*, Sketch the single-sided spectra of

$$x(t) = 2\sin\left(10\pi t - \frac{1}{6}\pi\right).$$

- We note that $x(t)$ can be written as

$$\begin{aligned} x(t) &= 2\cos\left(10\pi t - \frac{1}{6}\pi - \frac{1}{2}\pi\right) = 2\cos\left(10\pi t - \frac{2}{3}\pi\right) \\ &= \Re\left[2e^{j(10\pi t - 2\pi/3)}\right] = e^{j(10\pi t - 2\pi/3)} + e^{-j(10\pi t - 2\pi/3)} \end{aligned}$$



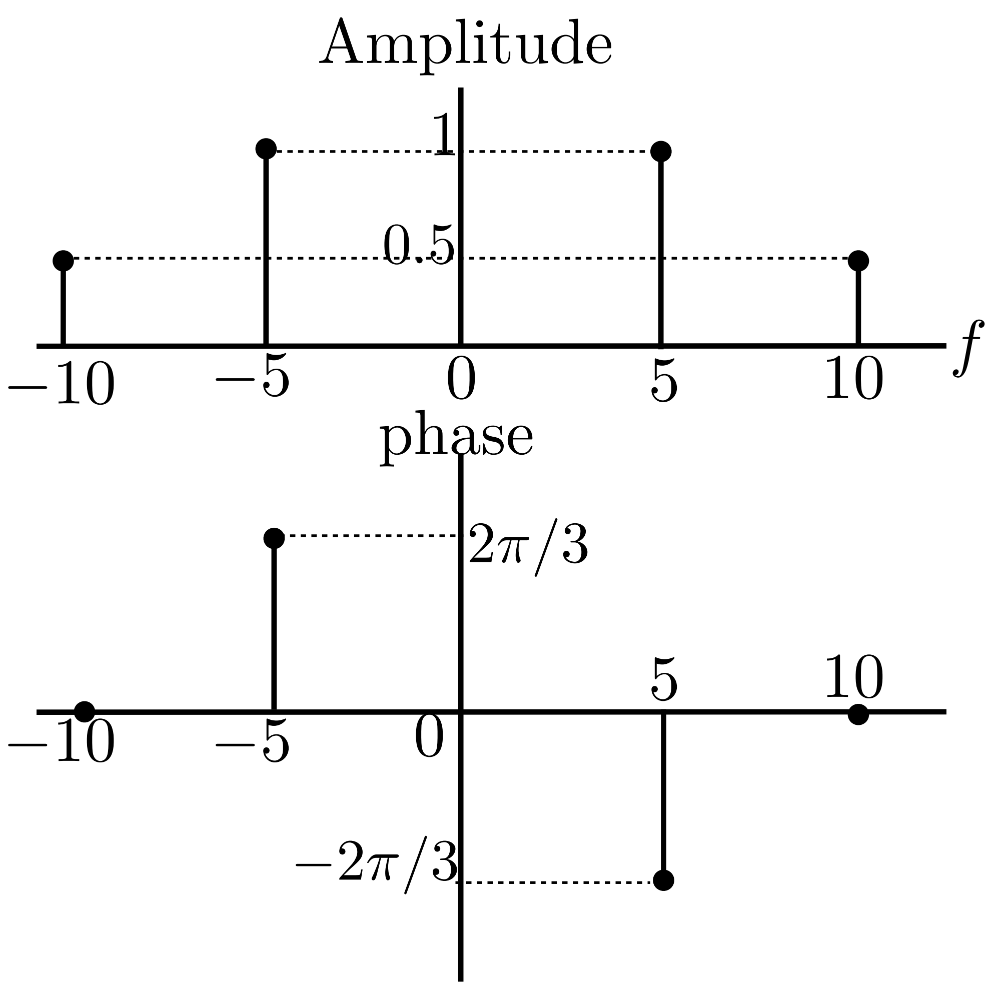


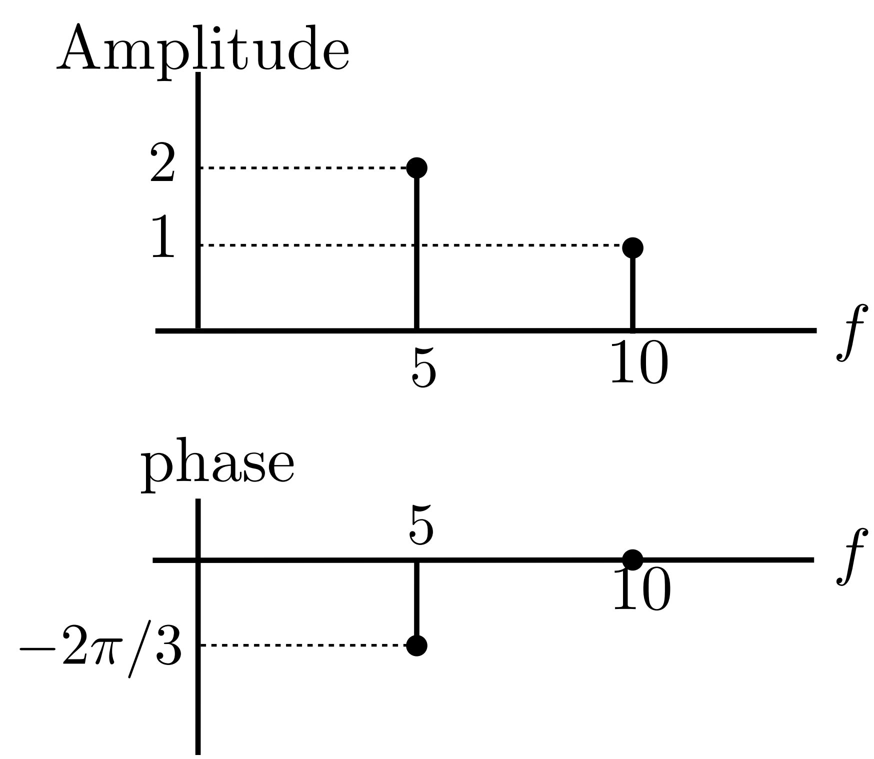
- *Example 2*, If more than one sinusoidal component is present in a signal, its spectra consists of multiple lines. For example, the signal

$$y(t) = 2\sin\left(10\pi t - \frac{1}{6}\pi\right) + \cos(20\pi t)$$

can be written as

$$\begin{aligned} y(t) &= 2\cos\left(10\pi t - \frac{2}{3}\pi\right) + \cos(20\pi t) \\ &= \Re\left[2e^{j(10\pi t - 2\pi/3)} + e^{j20\pi t}\right] \\ &= e^{j(10\pi t-2\pi/3)} + e^{-j(10\pi t - 2\pi/3)} + \frac{1}{2}e^{j20\pi t} + \frac{1}{2}e^{-j20\pi t} \end{aligned}$$





Singular Functions

- Unit step function
- Unit impulse function (Dirac delta function)
- Signum function (which will be discussed later on)

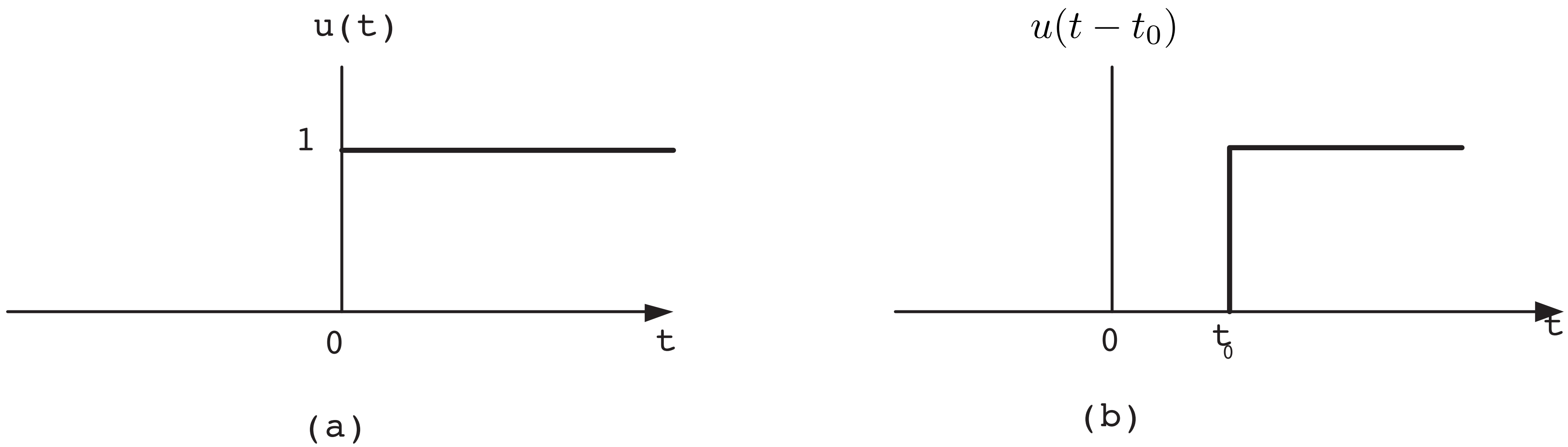
Unit Step Function

- **Definition**

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$

- **Shifted unit step function**

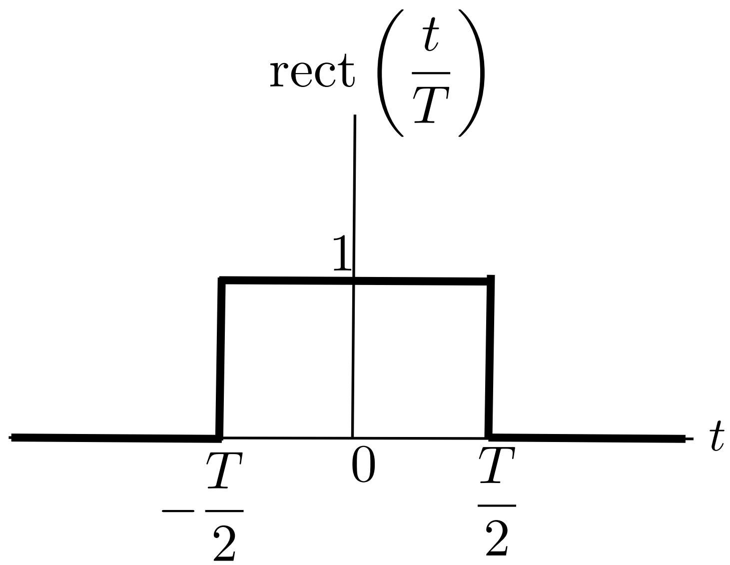
$$u(t - t_0) = \begin{cases} 1, & t > t_0 \\ 0, & t < t_0 \end{cases}$$



Unit Impulse Function (Dirac Delta Function)

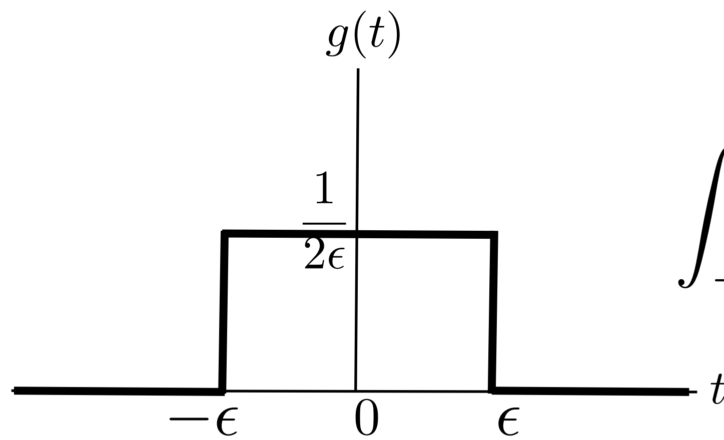
- Rectangular pulse

$$\mathrm{rect}\left(\frac{t}{T}\right) = \begin{cases} 1, & -\frac{T}{2} < t < \frac{T}{2} \\ 0, & \text{elsewhere} \end{cases}$$



- Consider the rectangular pulse given as

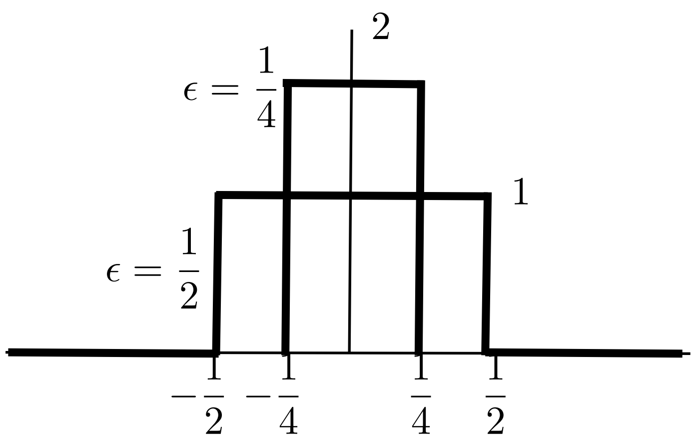
$$g(t) = \frac{1}{2\epsilon}\mathrm{rect}\left(\frac{t}{2\epsilon}\right)$$



Area=1

$$\int_{-\infty}^{\infty} g(t)\, dt = 1$$

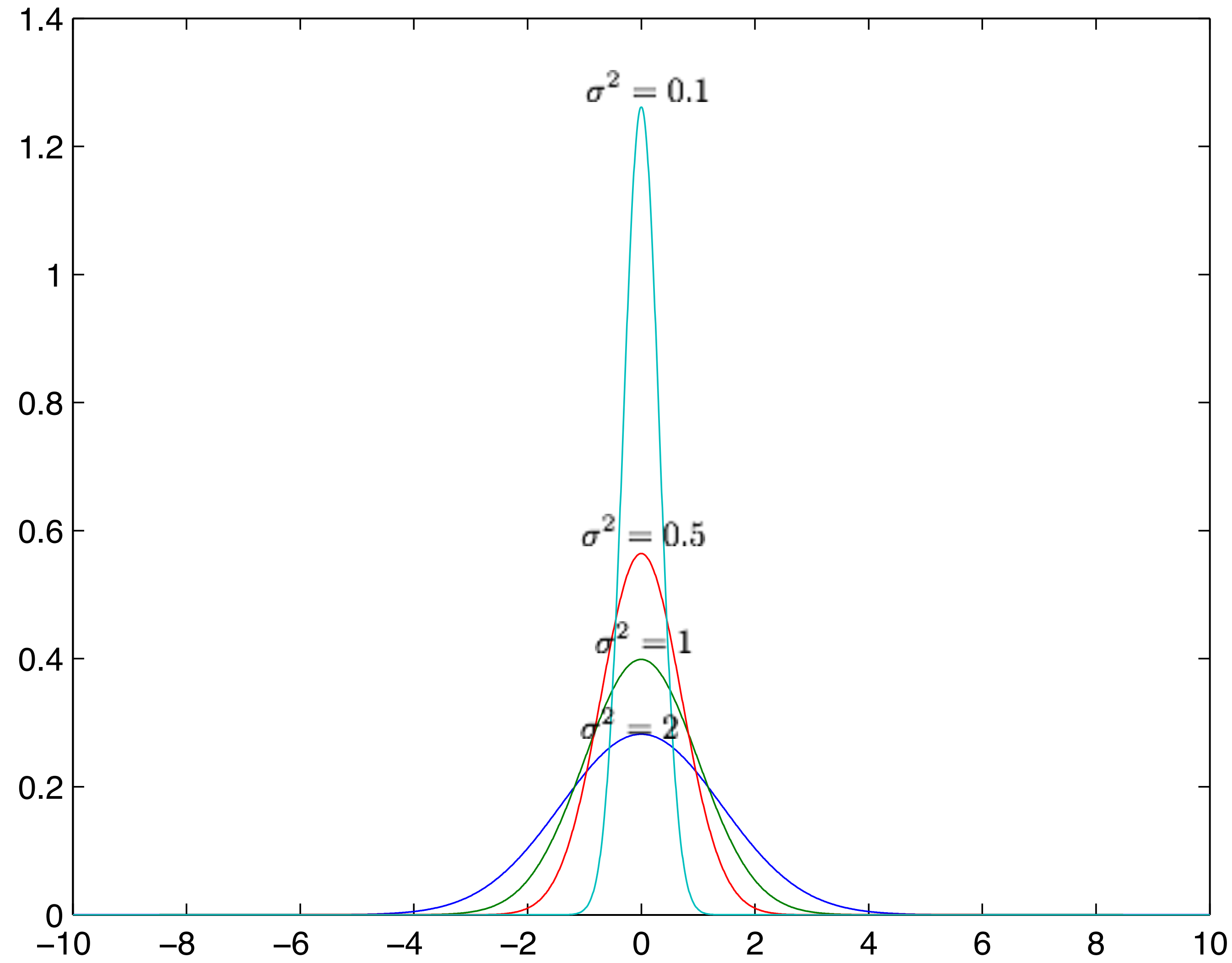
- Now consider $\lim_{\epsilon \to 0} g(t)$ in which case the area is still 1.



- Also consider the Gaussian pulse given as

$$g(t) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{t^2}{2\sigma^2}\right)$$

  - We can prove that $g(t)$ has a unit area, that is, $\int_{t=-\infty}^{\infty} g(t)\, dt = 1$
  - Now if we take $\sigma^2 \to 0$, $g(t)$ is in narrower gaussian pulse shape



- We define *Dirac delta function* as a function which has the property of $\lim_{\epsilon \to 0} g(t)$ (or $\lim_{\sigma^2 \to 0} g(t)$ in the Gaussian pulse) and denote it as $\delta(t)$.

- Definition of Dirac delta (or unit impulse) function

$$\int_{-\infty}^{\infty} x(t)\delta(t)\, dt = x(0)$$

where $x(t)$ is any continuous function at time $t = 0$

or

$$\int_{-\infty}^{\infty} x(t)\delta(t - t_0)\, dt = x(t_0)$$

where $x(t)$ is any continuous function at time $t = t_0$

- By considering the special case $x(t) = 1$ and $x(t) = 0$ for $t < t_1$ and $t > t_2$, the following two properties are obtained:

$$\int_{t_1}^{t_2} \delta(t - t_0)\, dt = 1, \qquad t_1 < t < t_2$$

and

$$\delta(t - t_0) = 0, \qquad t \neq t_0$$

- Some properties of the delta function

1. $\delta(at) = \frac{1}{|a|}\delta(t)$

2. $\delta(-t) = \delta(t)$

3.
$$\int_{t_1}^{t_2} x(t)\delta(t-t_0)\,dt = \begin{cases} x(t_0), & t_1 < t < t_0 \\ 0, & \text{otherwise} \\ \text{undefined} & \text{for } t_0 = t_1 \text{ or } t_2 \end{cases}$$

4. $x(t)\delta(t-t_0) = x(t_0)\delta(t-t_0)$, $x(t)$ continuous at $t = t_0$