Communication Systems II

[KECE322_01]

<2012-2nd Semester>

Lecture #19

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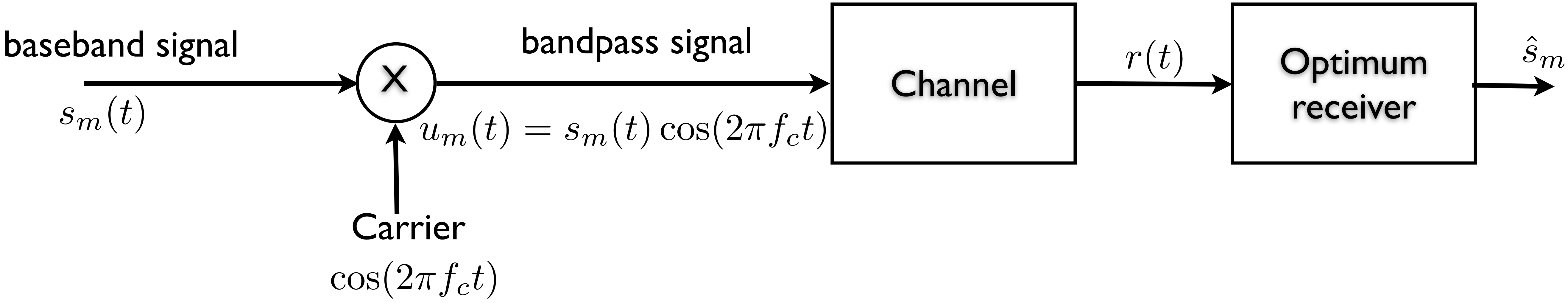
Outline

- PAM with carrier
- Equivalent lowpass representation of bandpass signals
- PSK (phase-shift keying) modulation

Transmission of Digital Information Via Carrier Modulation

- Signal transmission with carrier



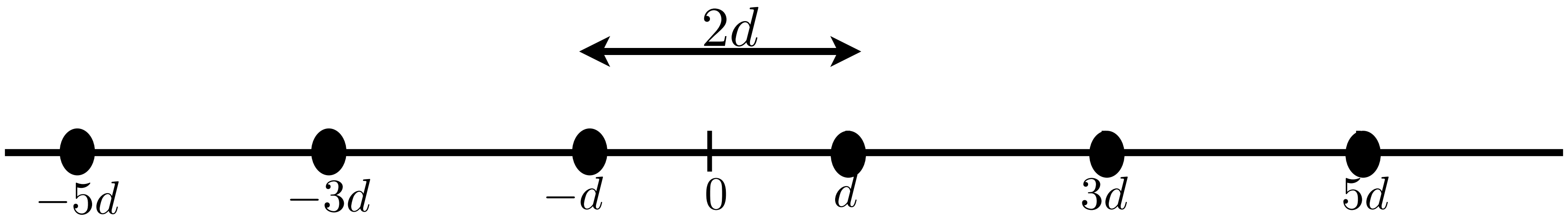
Baseband PAM

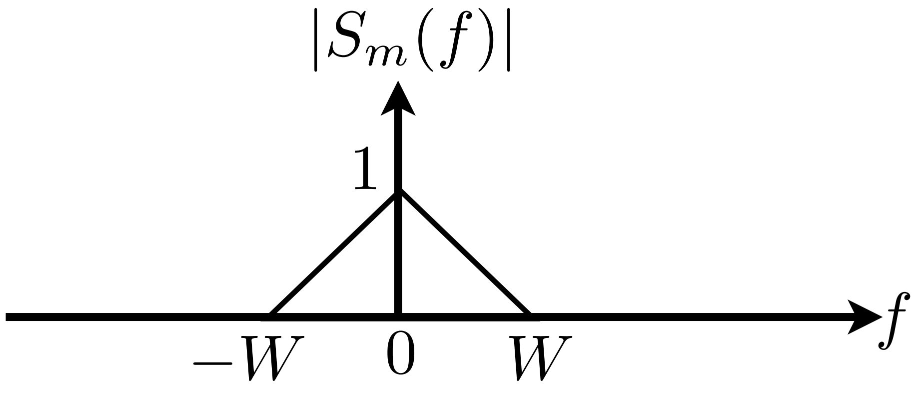
- PAM baseband signal

$$\begin{aligned} s_m(t) &= A_m g_T(t), \qquad 0 \leq t \leq T, \;\; m = 1, 2, \ldots, M \\ &= s_m \psi(t) \end{aligned}$$

where

$$s_m = (2m - 1 - M)d, \quad m = 1, 2, \ldots, M$$



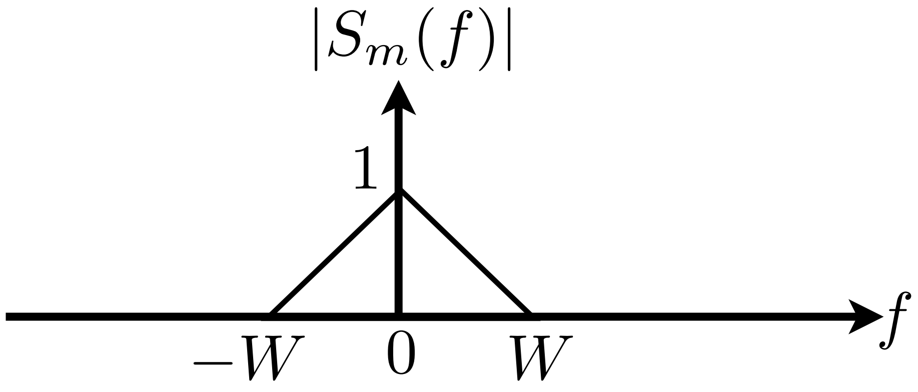


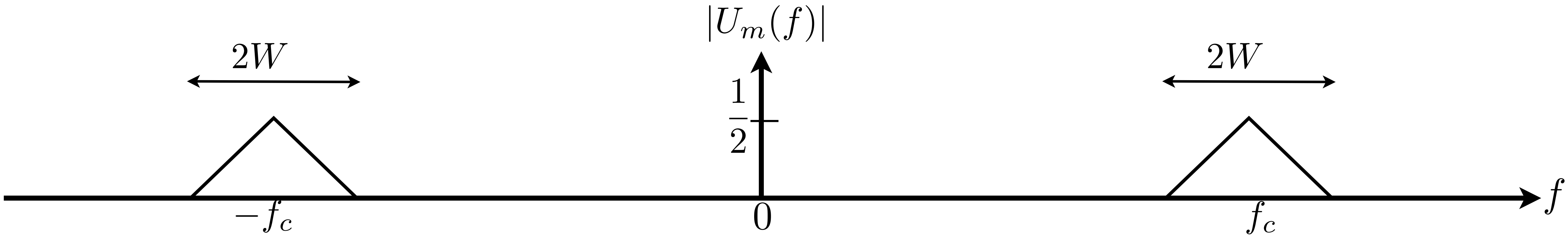
PAM with Carrier

- Transmitted signal waveform

$$u_m(t) = s_m(t)\cos(2\pi f_c t)$$

$$U_m(f) = \frac{1}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]$$

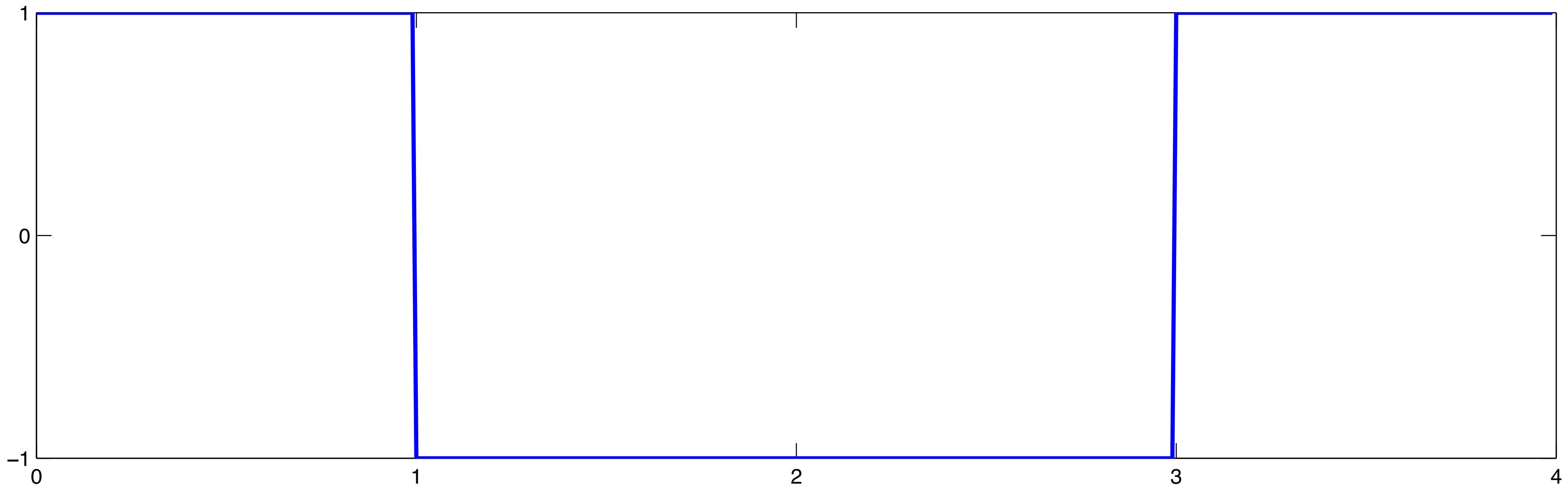




Energy of the bandpass signal waveforms

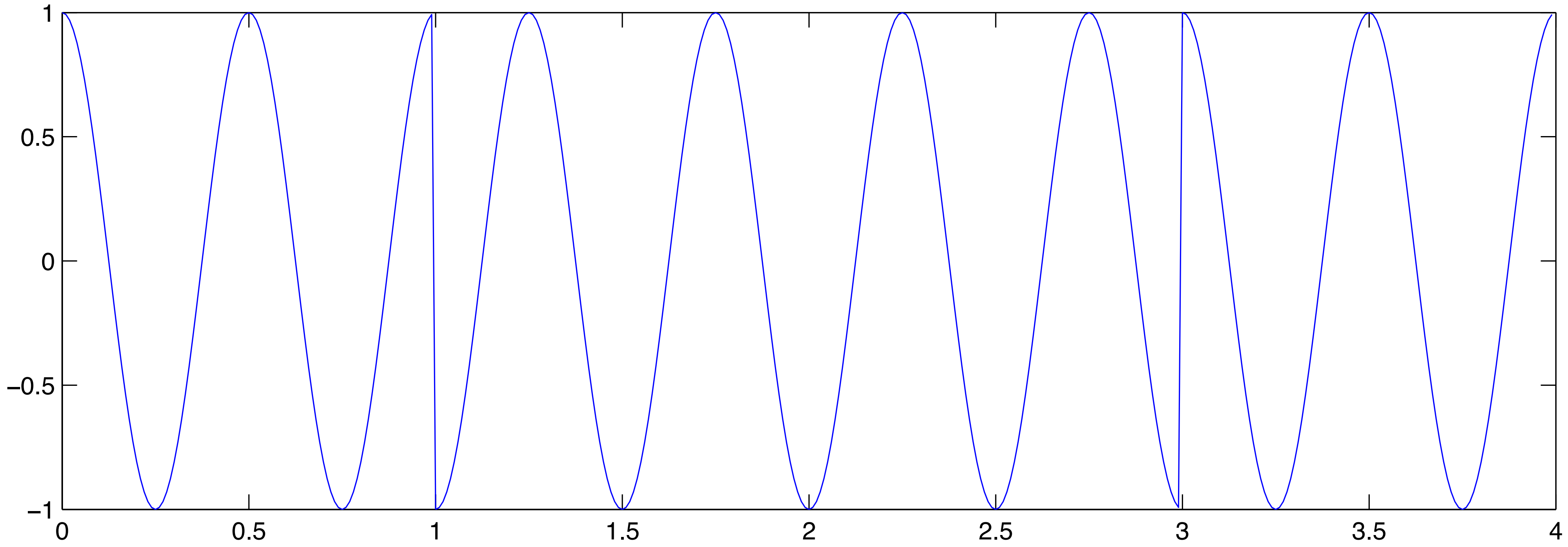
$$
\begin{aligned}
\mathcal{E}_m &= \int_{-\infty}^{\infty} u_m^2(t)\,dt = \int_{-\infty}^{\infty} s_m^2(t)\cos^2 2\pi f_c t\,dt \\
&= \frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\,dt + \underbrace{\frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\cos(4\pi f_c t)\,dt}_{=0} \\
&= \frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\,dt
\end{aligned}
$$

$a_1 = 1$ $a_2 = -1$ $a_3 = -1$ $a_4 = 1$

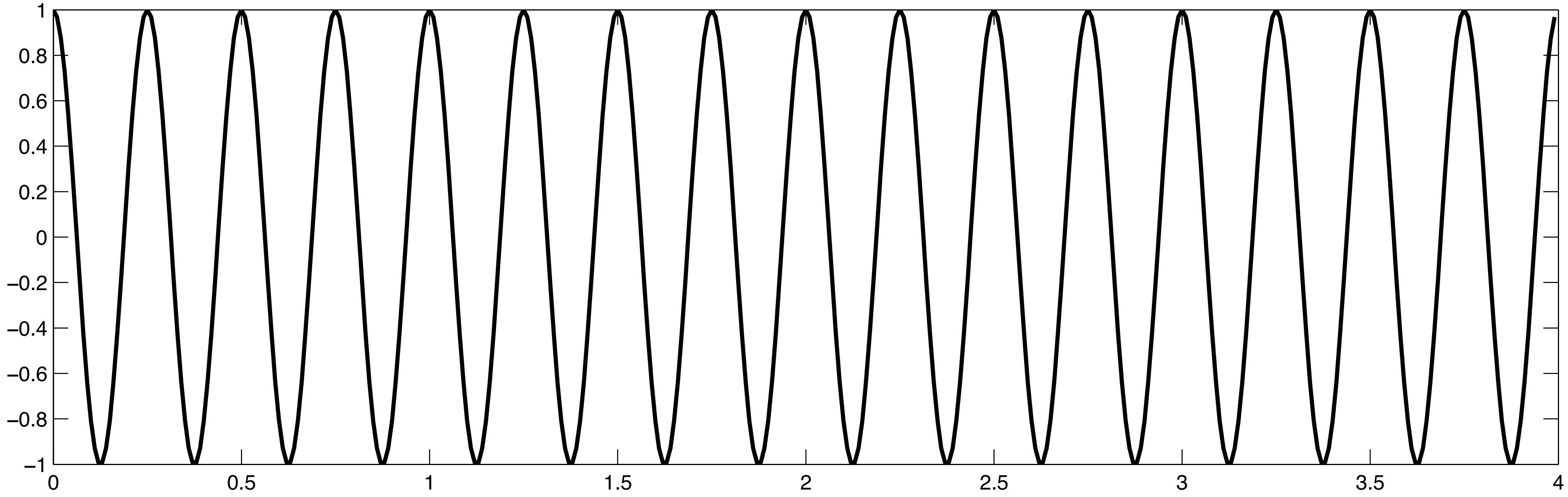


$$u_m(t) = s_m(t)\cos(2\pi f_c t)$$

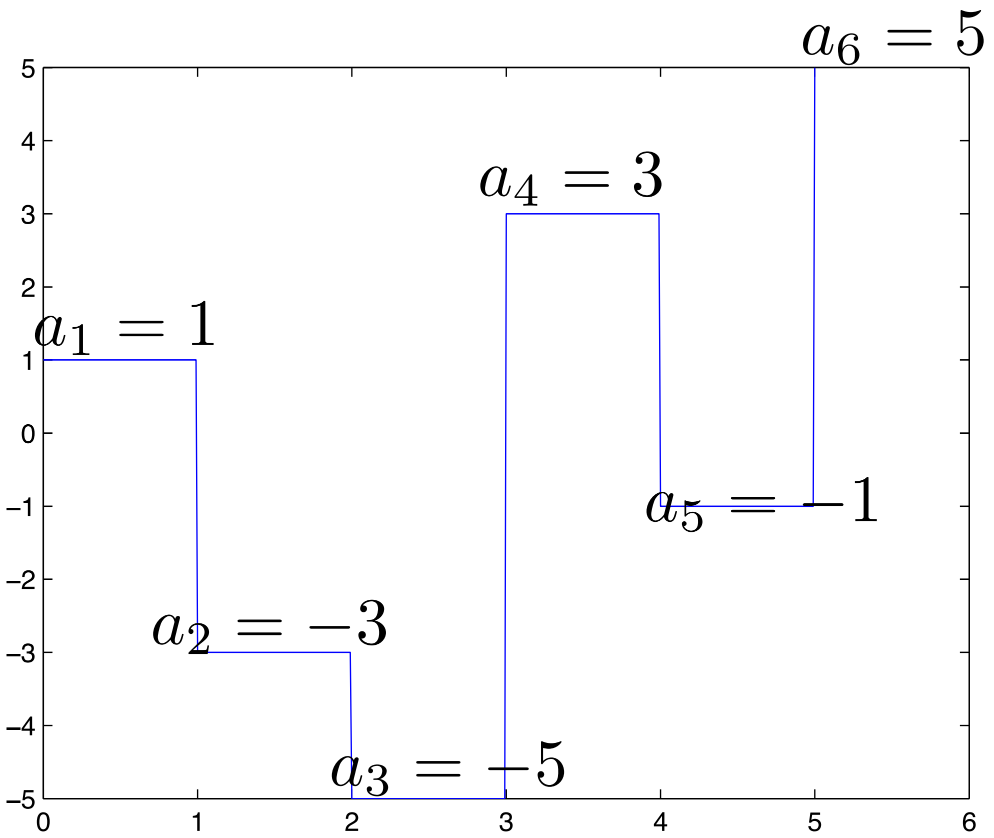
ex: $T = 1$ and $f_c = 2$

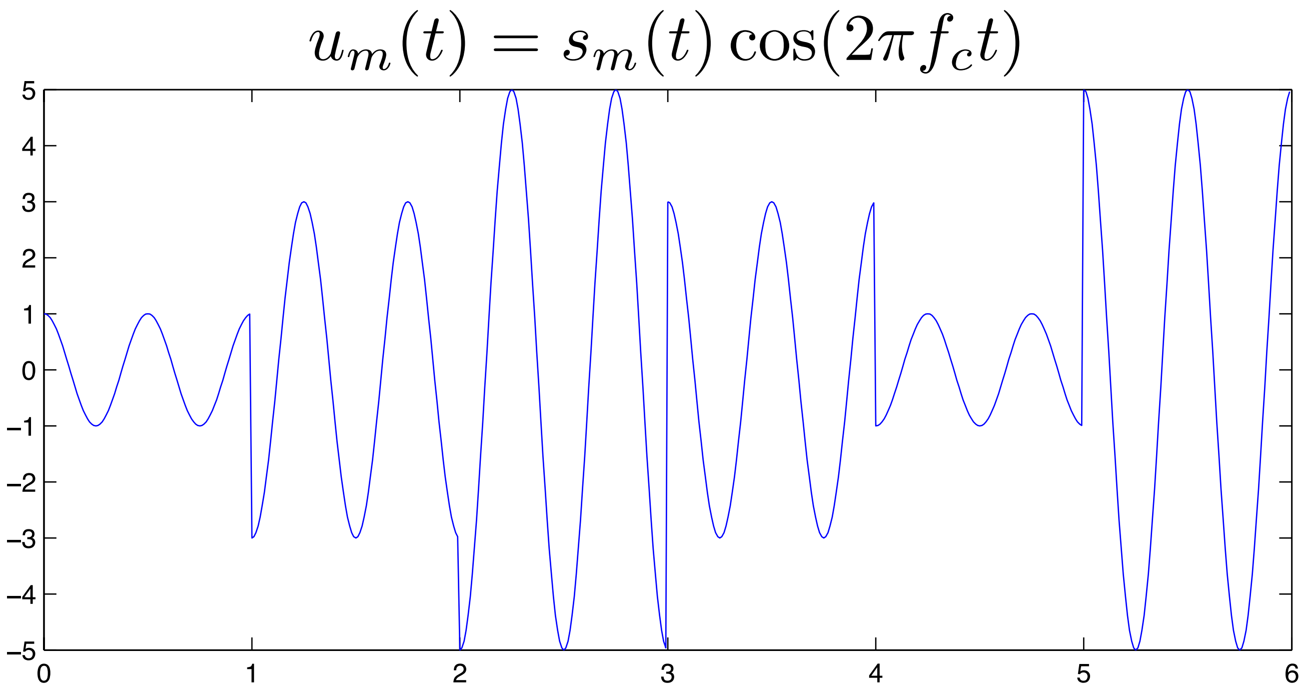


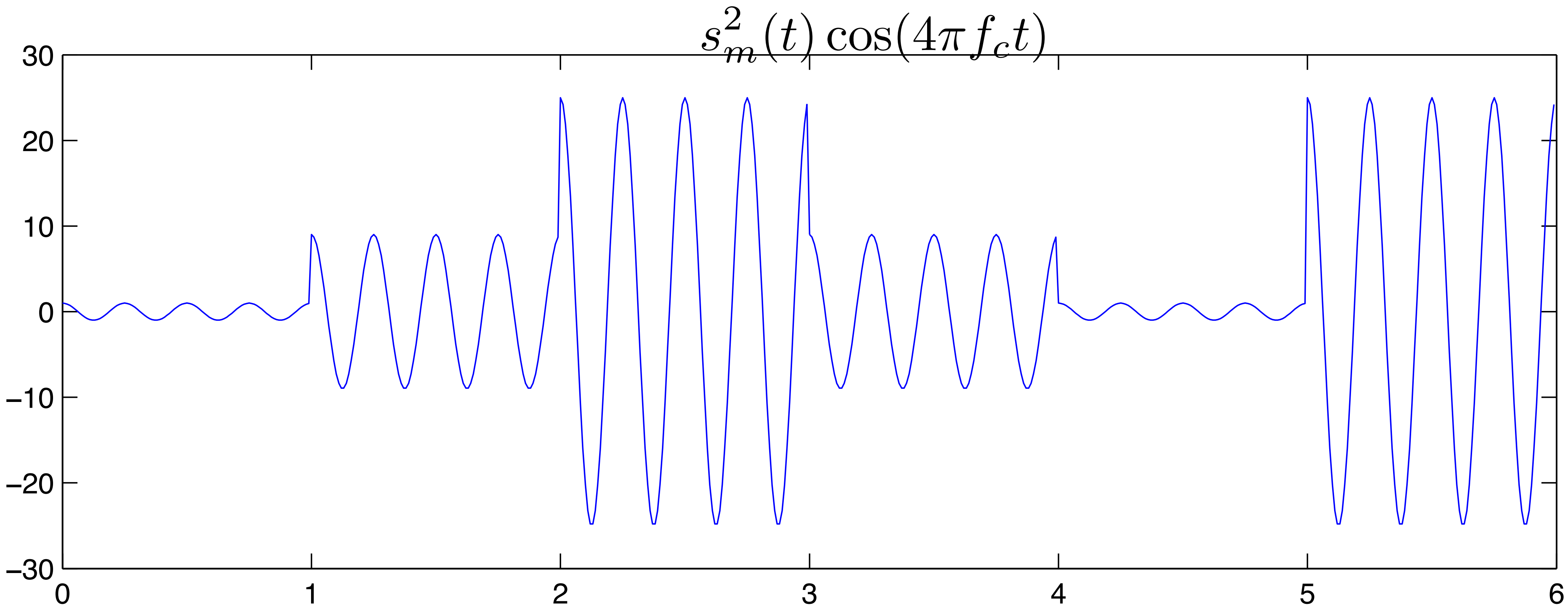
$$s_m^2(t)\cos(4\pi f_c t)$$



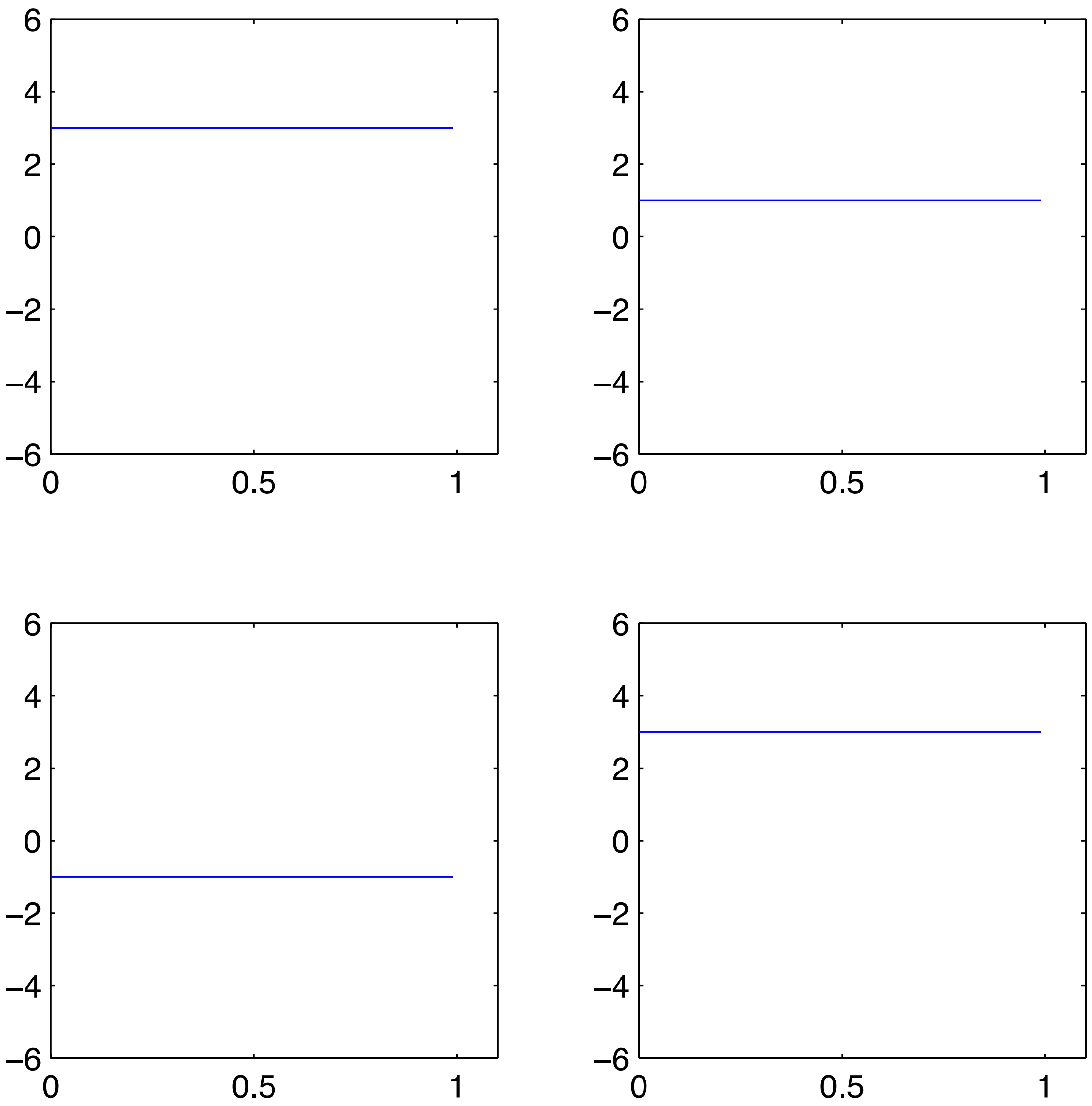
$$\int_{-\infty}^{\infty} s_m^2(t)\cos(4\pi f_c t)\,dt = 0$$


$a_6 = 5$
$a_4 = 3$
$a_1 = 1$
$a_5 = -1$
$a_2 = -3$
$a_3 = -5$
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
1
2
3
4
5
6

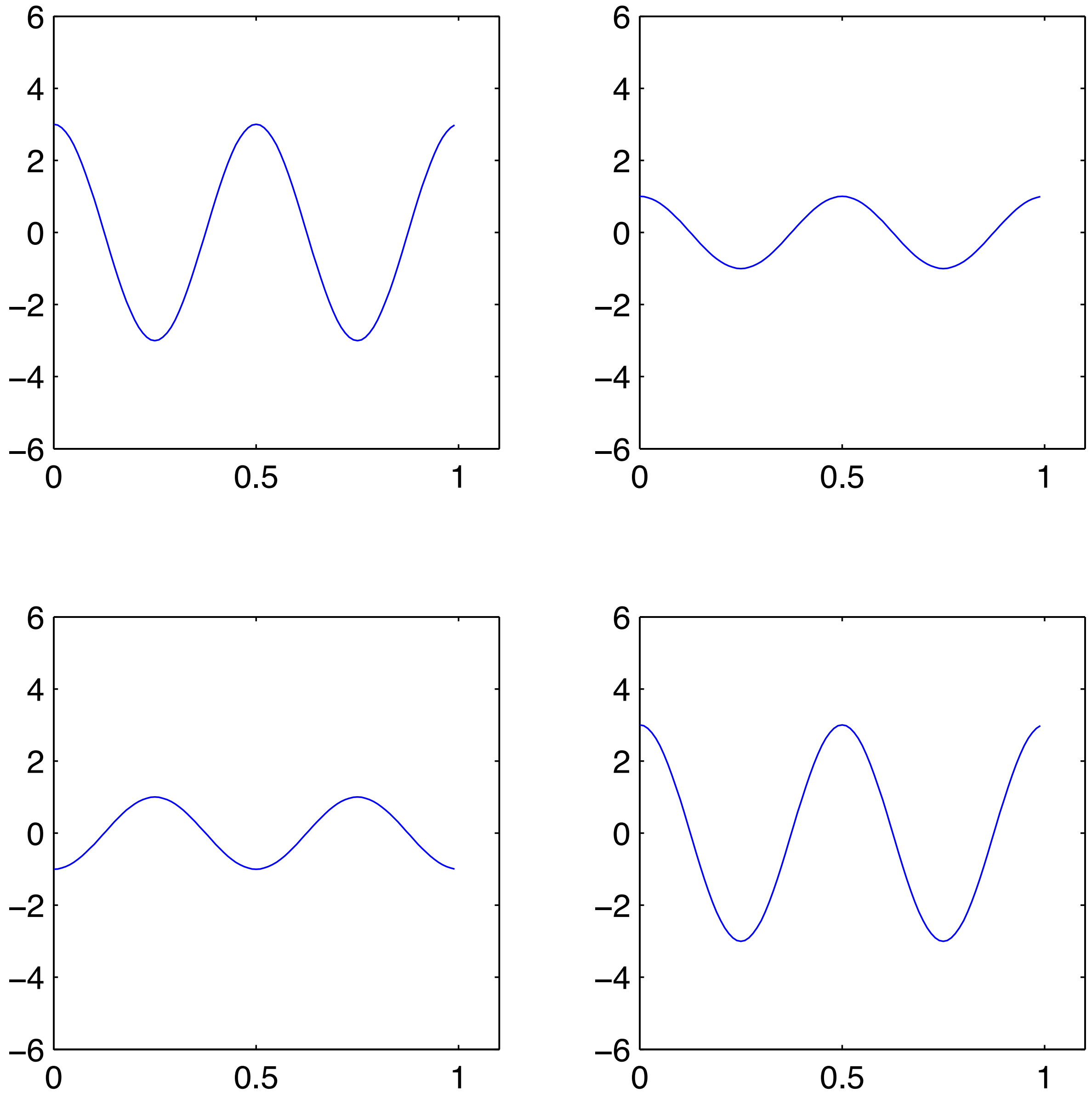

$u_m(t) = s_m(t)\cos(2\pi f_c t)$
5
4
3
2
1
0
−1
−2
−3
−4
−5
0
1
2
3
4
5
6


$s_m^2(t)\cos(4\pi f_c t)$
30
20
10
0
−10
−20
−30
0
1
2
3
4
5
6

$$s_m(t) = s_m \psi(t), \quad M = 4$$



$$u_m(t) = s_m(t)\cos(2\pi f_c t)$$



Geometric representation of PAM with carrier

- Baseband signal

$$s_m(t) = s_m\psi(t), \qquad m = 1, 2, \ldots, M$$

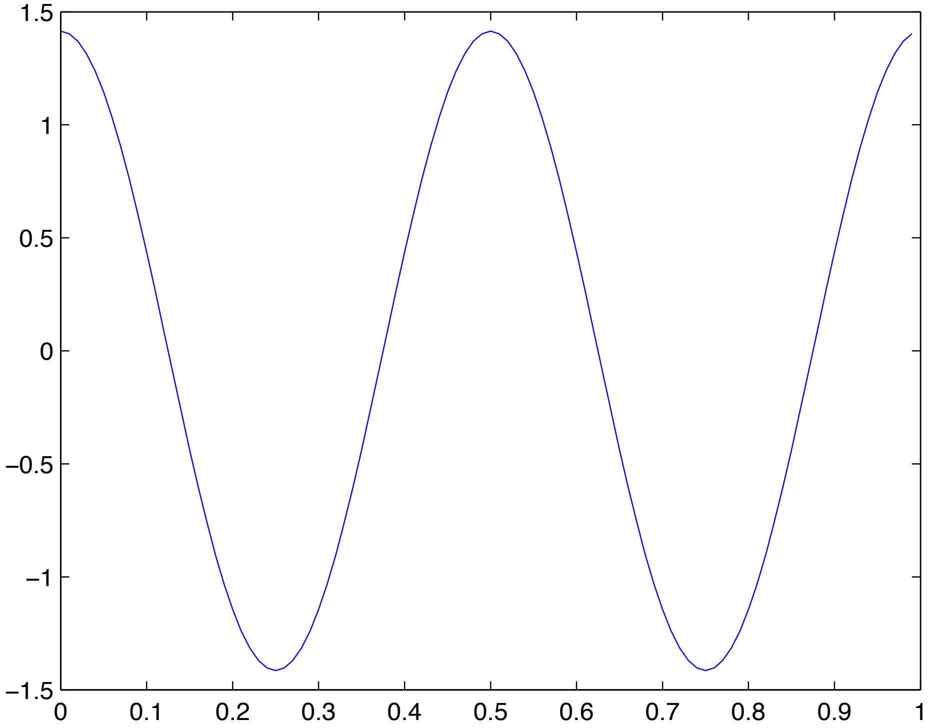
- Carrier modulated signal

$$u_m(t) = s_m(t)\cos(2\pi f_c t) = s_m\psi(t)\cos(2\pi f_c t), \qquad m = 1, 2, \ldots, M\ , \ \ 0 \le t \le T$$

$$= s_{cm}\psi_c(t)$$

where

$$\psi_c(t) = \sqrt{2}\psi(t)\cos(2\pi f_c t),$$



$$\begin{aligned}\int_{-\infty}^{\infty} \psi_c^2(t)\,dt &= 2\int_{-\infty}^{\infty} \psi^2(t)\cos^2(2\pi f_c t)\,dt \\ &= \int_{-\infty}^{\infty} \psi^2(t)\,dt + \int_{-\infty}^{\infty} \psi^2(t)\cos(4\pi f_c t)\,dt \\ &= 1\end{aligned}$$

- Bandpass waveforms by the carrier-modulated basis function

$$u_m(t) = \frac{s_m}{\sqrt{2}}\psi_c(t) = s_{cm}\psi_c(t), \qquad m = 1, 2, \ldots, M$$

$$\psi_c(t) = \sqrt{2}\psi(t)\cos(2\pi f_c t)$$

and

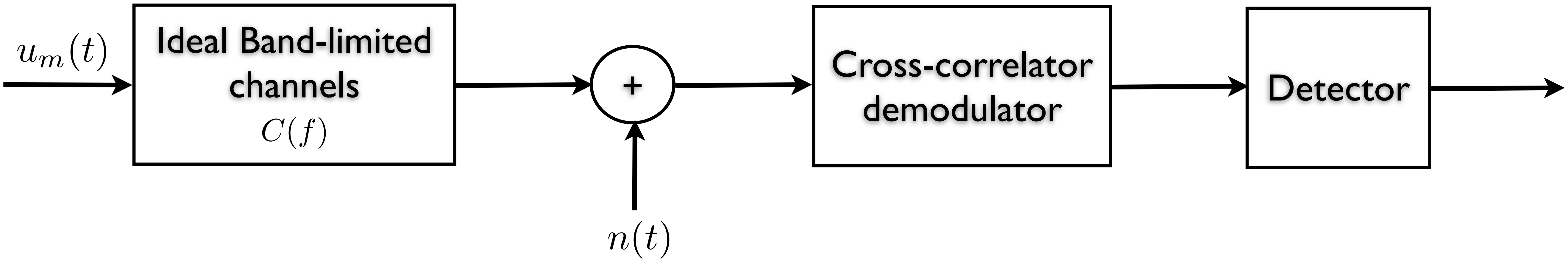
$$s_{cm} = \frac{s_m}{\sqrt{2}} = (2m - 1 - M)d/\sqrt{2}$$

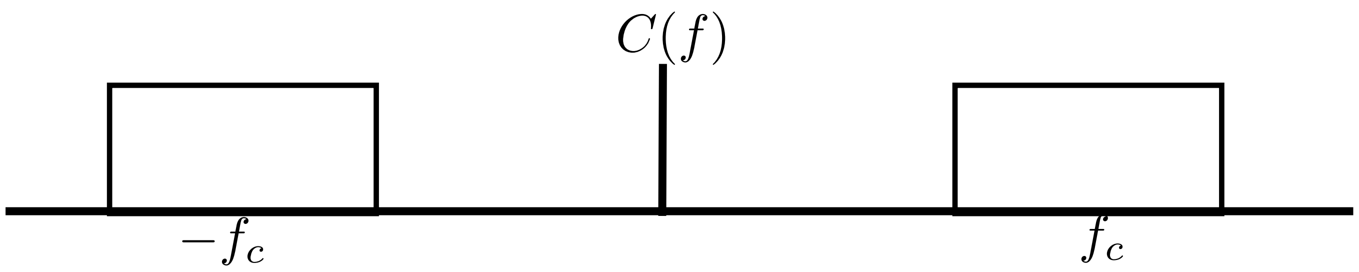
Demodulation and Detection of PAM

- Transmit signal

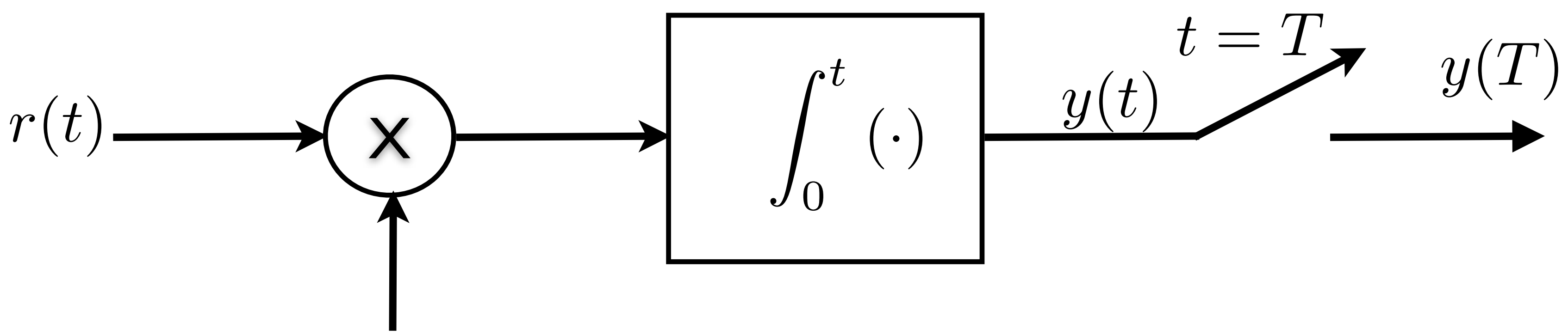
$$u_m(t) = s_m(t)\cos(2\pi f_c t) \qquad m = 1, 2, \ldots, M$$
$$= s_{cm}\psi_c(t)$$

- Received signal over band-limited AWGN channels





- Cross-correlator



$$\psi_c(t) = \sqrt{2}\psi(t)\cos(2\pi f_c t)$$

$$\begin{aligned} y(T) &= \int_0^T r(t)\psi_c(t)\,dt = \int_0^T \left[s_{cm}(t)\psi_c(t) + n(t)\right]\psi_c(t)\,dt \\ &= \int_0^T \left[\frac{s_m}{\sqrt{2}}\sqrt{2}\psi(t)\cos(2\pi f_c t) + n(t)\right]\sqrt{2}\psi(t)\cos(2\pi f_c t)\,dt \\ &= \sqrt{2}s_m \int_0^T \psi^2(t)\cos^2(2\pi f_c t)\,dt + \sqrt{2}\int_0^T n(t)\psi(t)\cos(2\pi f_c t)\,dt \\ &= \frac{s_m}{\sqrt{2}} + n = s_{cm} + n \end{aligned}$$

- Optimum detector

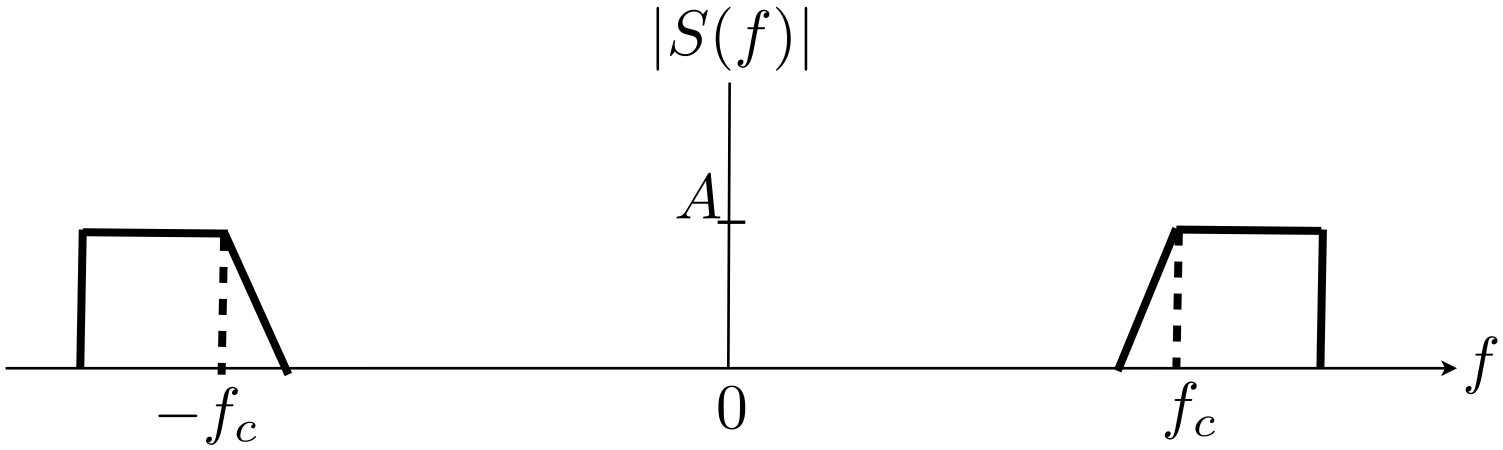
$$D(y, s_m) = (y - s_{cm})^2, \qquad m = 1, 2, \ldots, M$$

  - or equivalently

$$C(y, s_m) = 2ys_{cm} - s_{cm}^2$$

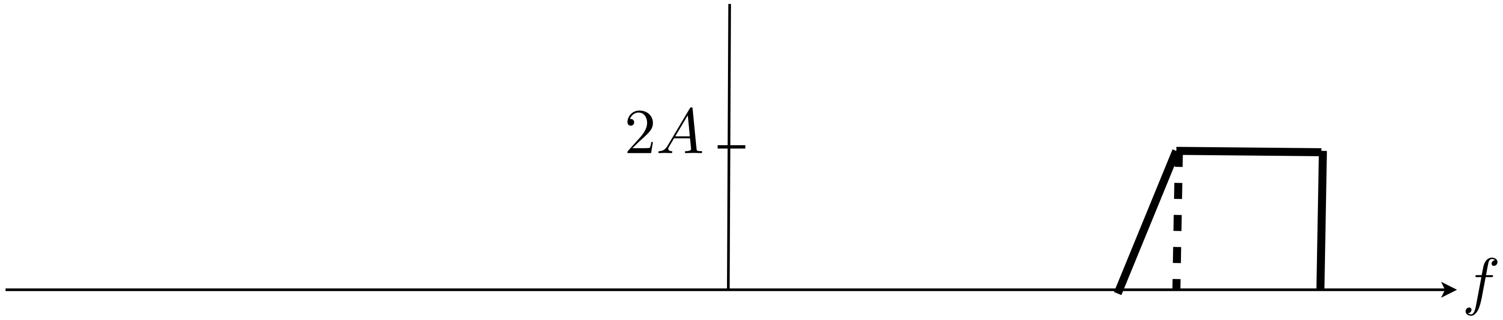
Representation of Band-Pass Signals

- Suppose a real-valued signal $s(t)$



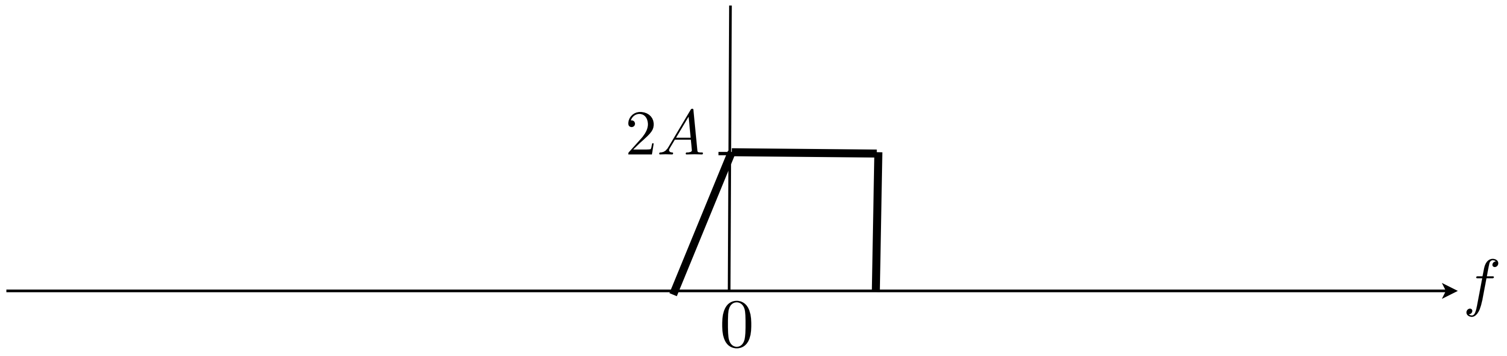
- Consider only the positive components with scaling by 2.

$$S_+(f) = 2u(f)S(f)$$



- Equivalent lowpass signal

$$S_l(f) = S_+(f + f_c)$$



$$s_l(t) = s_+(t)e^{-j2\pi f_c t}$$

Generally, $s_l(t)$ is complex signal so that we can write

$$s_l(t) = x(t) + jy(t)$$
$$= \quad a(t)e^{j\theta(t)}$$

where $a(t) = \sqrt{x^2(t) + y^2(t)}$

$$\theta(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

- Inverse Fourier transform of $S_+(f)$

$$s_+(t) = \mathcal{F}^{-1}[S_+(f)]$$

which is called "*analytic signal*".

$$s_+(t) = \int_{-\infty}^{\infty} S_+(f)e^{j2\pi ft}\,dt = \mathcal{F}^{-1}[2u(f)] * \mathcal{F}^{-1}[S(f)]$$

Note that

$\mathcal{F}^{-1}[2u(f)] \;=\; \delta(t) + \dfrac{j}{\pi t}$ and $\mathcal{F}^{-1}[S(f)] \;=\; s(t)$

- Then

$$s_+(t) \quad = \quad \left[\delta(t) + \frac{j}{\pi t}\right] * s(t) \quad = \quad s(t) * \delta(t) + \frac{j}{\pi t} * s(t)$$

$$= \quad s(t) + \underbrace{\frac{j}{\pi t} * s(t)}_{\hat{s}(t)}$$

Let us define

$$\hat{s}(t) = \frac{1}{\pi t} * s(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t - \tau}\, d\tau$$

$$= \quad h(t) * s(t)$$

where $h(t) = \dfrac{1}{\pi t}$, which is called *Hilbert transformer*.

- Hilbert transformer

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{1}{t}e^{-j2\pi ft} = \begin{cases} -j, & (f>0) \\ 0, & (f=0) \\ j, & (f<0) \end{cases}$$

Amplitude response

$$|H(f)| = 1$$

Phase response

$$\Theta(f) = \begin{cases} -\frac{1}{2}\pi, & f>0 \\ 0, & f=0 \\ \frac{1}{2}\pi, & f<0 \end{cases}$$

Hilbert transformer shifts the phase of the input signal by $\pi/2$, so called as phase shifter!

- Analytic signal

$$s_+(t) = s(t) + j\hat{s}(t)$$

- Equivalent lowpass signal

$$S_l(f) = S_+(f + f_c) \quad \longleftrightarrow \quad \begin{aligned} s_l(t) &= s_+(t)e^{-j2\pi f_c t} \\ &= [s(t) + j\hat{s}(t)]e^{-j2\pi f_c t} \end{aligned}$$

or equivalently

$$\begin{aligned} s(t) + j\hat{s}(t) = s_l(t)e^{j2\pi f_c t} \\ &= [x(t) + jy(t)] \cdot [\cos(2\pi f_c t) + j\sin(2\pi f_c t)] \\ &= [x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)] + j[x(t)sin(2\pi f_c t) + jy(t)\cos(2\pi f_c t)] \end{aligned}$$

$$s(t) + j\hat{s}(t) = s_l(t)e^{j2\pi f_c t}$$

$$\begin{aligned} s(t) &= \Re\left[s_l(t)e^{j2\pi f_c t}\right] \\ &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \\ &= \Re\left[a(t)e^{j\theta(t)}e^{j2\pi f_c t}\right] = a(t)\cos(2\pi f_c t + \theta(t)) \end{aligned}$$

where $a(t) = \sqrt{x^2(t) + y^2(t)}$

$$\theta(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

- Fourier transform of $s(t)$

$$S(f) = \int_{-\infty}^{\infty} s(t) e^{-j2\pi f t}\, dt = \int_{-\infty}^{\infty} \{\Re[s_l(t) e^{j2\pi f_c t}]\} e^{-j2\pi f t}\, dt$$

Note for complex value of ζ

$$\Re(\zeta) = \frac{1}{2}(\zeta + \zeta^*)$$

$$\begin{aligned} S(f) &= \frac{1}{2}\int_{-\infty}^{\infty} [s_l(t) e^{j2\pi f_c t} + s_l^*(t) e^{-j2\pi f_c t}] e^{-j2\pi f t}\, dt \\ &= \frac{1}{2}\left[\int_{-\infty}^{\infty} s_l(t) e^{-j2\pi(f-f_c)t}\, dt + \int_{-\infty}^{\infty} s_l^*(t) e^{-j2\pi(f+f_c)t}\, dt\right] \end{aligned}$$

$$\|$$

$$\left[\int_{-\infty}^{\infty} s_l(t) e^{-j2\pi(-f-f_c)t}\, dt\right]^*$$

$$= \frac{1}{2}[S_l(f - f_c) + S_l^*(-f - f_c)]$$

- Energy in the signal

$$E = \int_{-\infty}^{\infty} s^2(t)\, dt = \int_{-\infty}^{\infty} \{\Re[s_l(t) e^{j2\pi f_c t}]\}^2\, dt$$

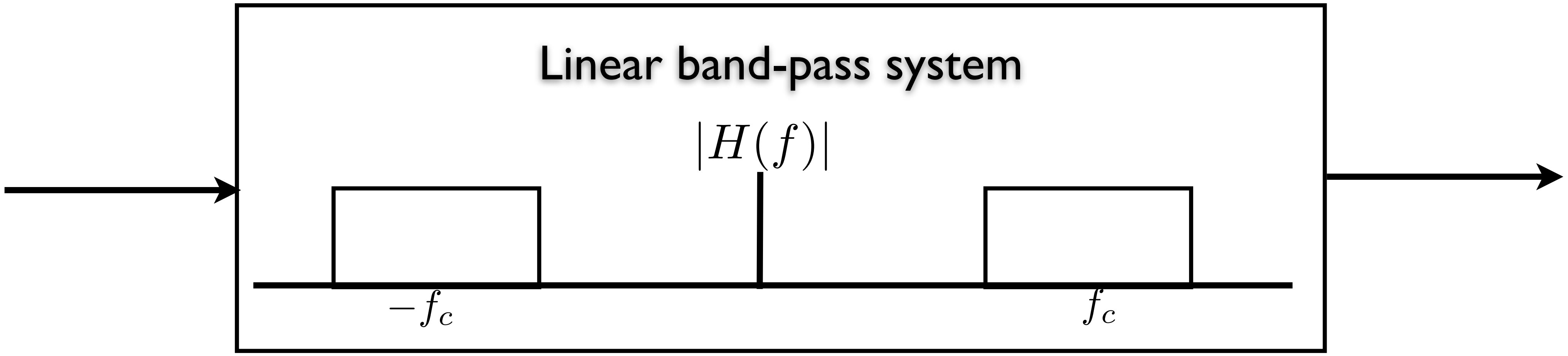
$$s_l(t) = a(t) e^{j\theta(t)}$$

$$= \int_{-\infty}^{\infty} [a(t) \cos(2\pi f_c t + \theta(t)]^2\ dt$$

Note $|s_l(t)| = a(t)$

$$\begin{aligned} E &= \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2\, dt + \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2 \cos[4\pi f_c t + 2\theta(t)]\, dt \\ &\approx \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2\, dt \end{aligned}$$

Representation of Linear Band-Pass Systems



- Impulse and frequency response of linear band-pass system

$$h(t) = \int_{-\infty}^{\infty} H(f)e^{j2\pi ft}\,df \qquad H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$$

- $h(t)$ is real-valued time function. Then, we have

$$H(f) = \int_{-\infty}^{\infty} h(t)e^{-j2\pi ft}\,dt$$
$$= \int_{-\infty}^{\infty} h(t)\cos(2\pi ft)\,dt - j\int_{-\infty}^{\infty} h(t)\sin(2\pi ft)\,dt$$

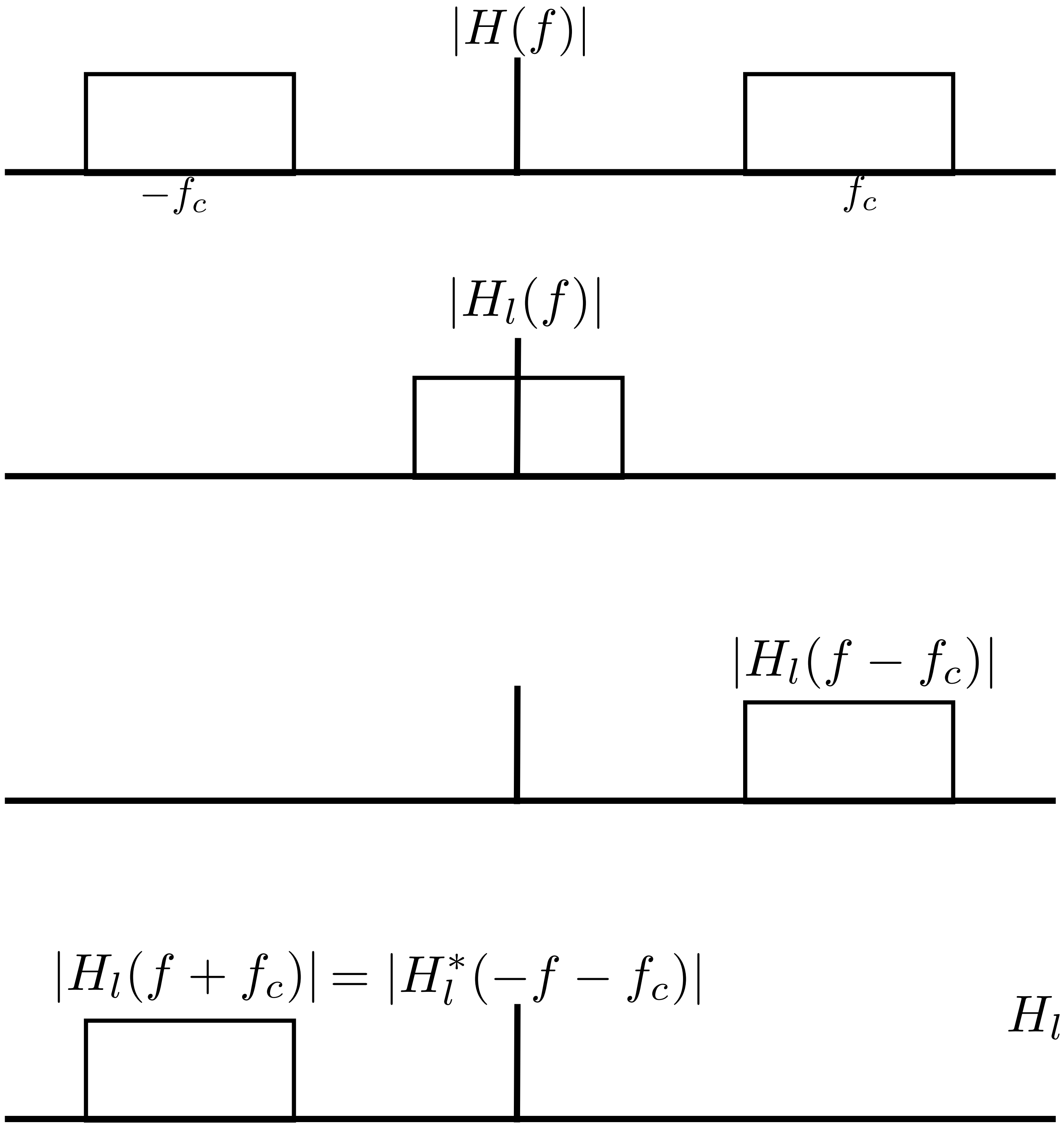
$$H(-f) = \int_{-\infty}^{\infty} h(t)e^{j2\pi ft}\,dt$$
$$= \int_{-\infty}^{\infty} h(t)\cos(2\pi ft)\,dt + j\int_{-\infty}^{\infty} h(t)\sin(2\pi ft)\,dt$$

$$H^*(f) = \int_{-\infty}^{\infty} h(t)\cos(2\pi ft)\,dt + j\int_{-\infty}^{\infty} h(t)\sin(2\pi ft)\,dt = H(-f)$$

or equivalently

$$\boxed{H^*(-f) = H(f)}$$

- Equivalent lowpass system model



$$H_l(f+f_c) = H_l^*(-f-f_c)$$

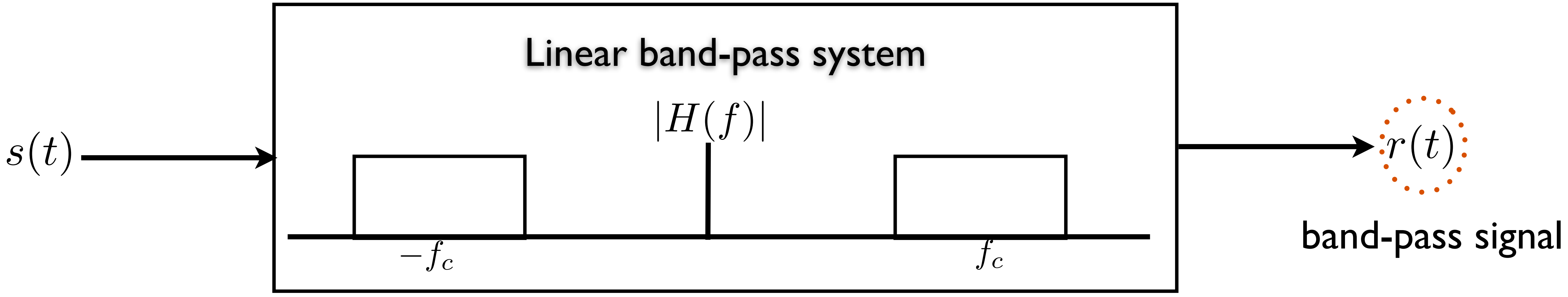
- Frequency response of band-pass system using equivalent low-pass system

$$H(f) = H_l(f - f_c) + H_l^*(-f - f_c)$$

- Impulse response of band-pass system using equivalent low-pass system

$$h(t) = h_l(t)e^{j2\pi f_c t} + h_l^*(t)e^{-j2\pi f_c t} = 2\Re[h_l(t)e^{j2\pi f_c t}]$$

Response of a Band-Pass System to a Band-Pass Signal



- Response

$$r(t) = \Re[r_l(t)e^{j2\pi f_c t}]$$

$$r(t) = s(t) * h(t) = \int_{-\infty}^{\infty} s(\tau)h(t-\tau)\,d\tau \quad \longleftrightarrow \quad R(f) = S(f)H(f)$$

- Response using equivalent low-pass signal representation

$$
\begin{aligned}
R(f) &= S(f)H(f) \\
&= \frac{1}{2}[S_l(f-f_c)+S_l^*(-f-f_c)][H_l(f-f_c)+H_l^*(-f-f_c)] \\
&= \frac{1}{2}\Big[S_l(f-f_c)H_l(f-f_c)+\overbrace{S_l(f-f_c)H_l^*(-f-f_c)}^{=0} \\
&\quad +\underbrace{S_l^*(-f-f_c)H(f-f_c)}_{=0}+S_l^*(-f-f_c)H_l^*(-f-f_c)\Big] \\
&= \frac{1}{2}[S_l(f-f_c)H_l(f-f_c)+S_l^*(-f-f_c)H_l^*(-f-f_c)] \\
&= \frac{1}{2}[R_l(f-f_c)+R_l^*(-f-f_c)]
\end{aligned}
$$

where $R_l(f) = S_l(f)H_l(f)$

- PAM signal

$$u_m(t) = s_m(t)\cos(2\pi f_c t) \quad = \Re[s_m(t)e^{j2\pi f_c t}]$$

where $s_l(t) = s_m(t) = s_m\psi(t)$

Phase-Shifted Keying (PSK) Modulation

- PSK signal is in the form of

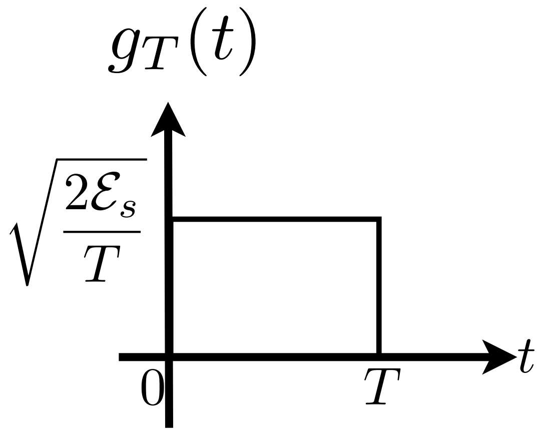
$$u_m(t) = g_T(t)\cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \quad m = 0, 1, \ldots, M-1, \;\; 0 \le t \le T$$

$$= \quad \Re[s_{ml}(t)e^{j2\pi f_c t}]$$

- PSK signal has its equivalent low-pass signal

$$s_{ml}(t) = g_T(t)e^{j\theta_m}, \quad m = 0, 1, \ldots, M-1$$

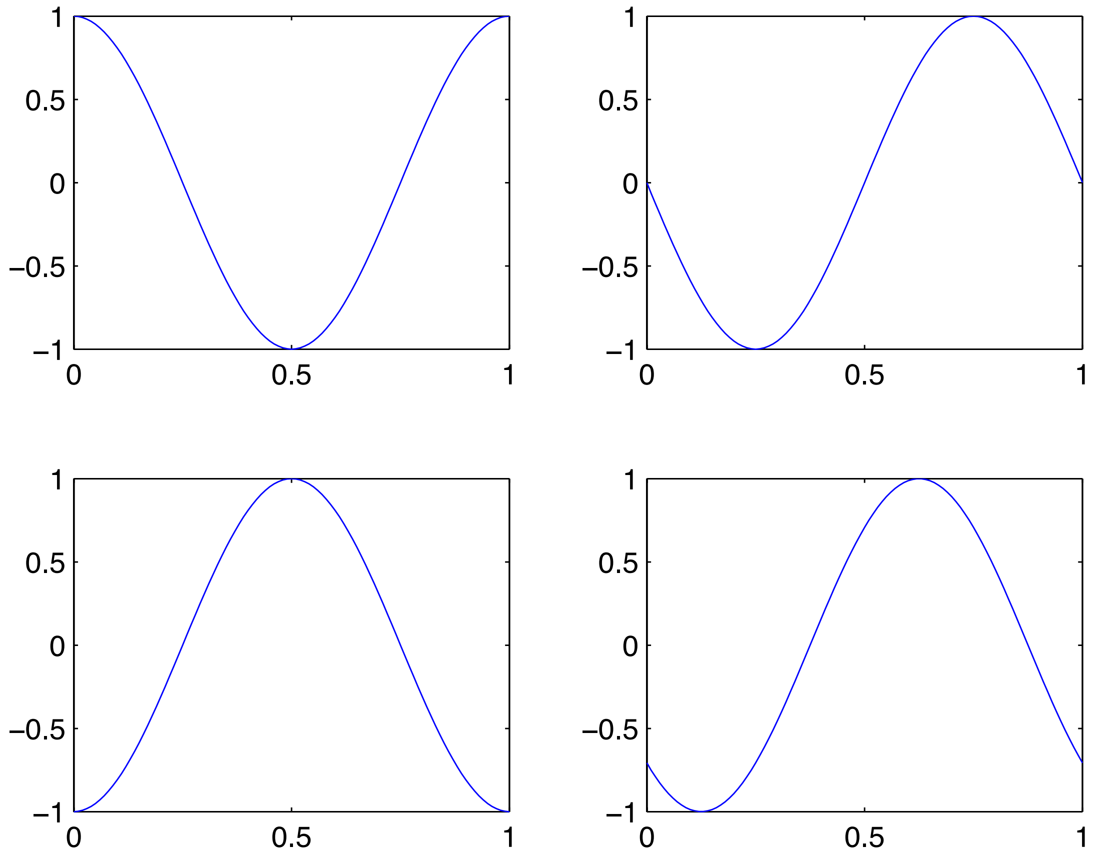
where $\theta_m = \frac{2\pi m}{M}$



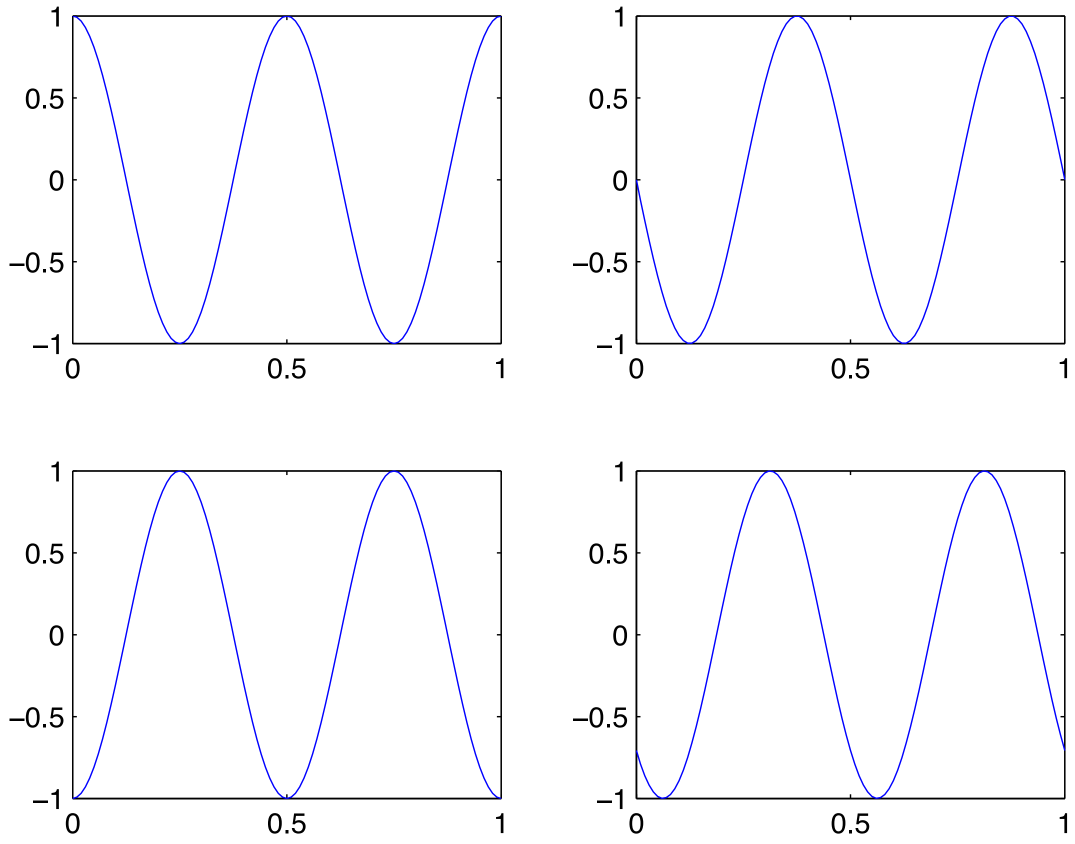
- 4-ary PSK or (QPSK)

Bit	$\theta_m = \frac{2\pi m}{M}$
0 0	0
0 1	$\pi/2$
1 1	π
1 0	$3\pi/2$

$f_c = 1,\ T = 1$



$f_c = 2,\ T = 1$



$$u_m(t) = \sqrt{\frac{2\mathcal{E}_s}{T}} \cos\left(2\pi f_c t + \frac{2\pi m}{M}\right), \quad m = 0, 1, \ldots, M-1, \quad 0 \le t \le T$$

Bit stream:

1 0	0 0	1 1	1 0	0 1	0 0	0 1
s_4	s_1	s_3	s_4	s_2	s_1	s_2