Copyright statement

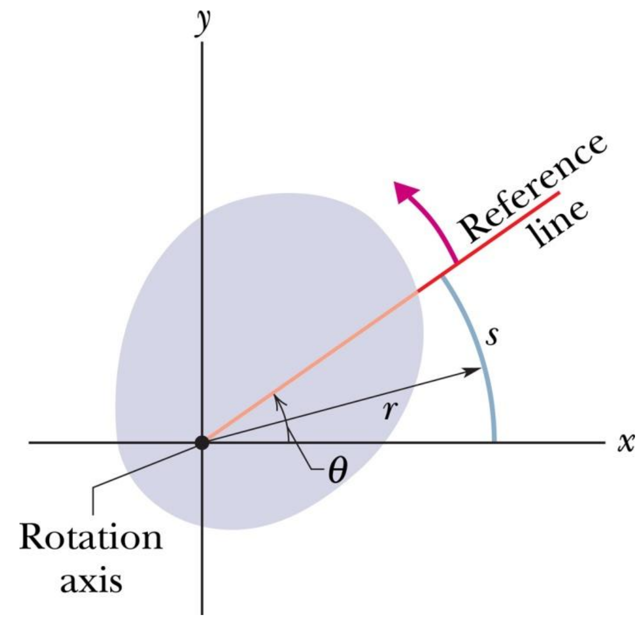
- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8^{th} and 9^{th} Ed.
- The rest is made by me.

Ch. 10 Rotation

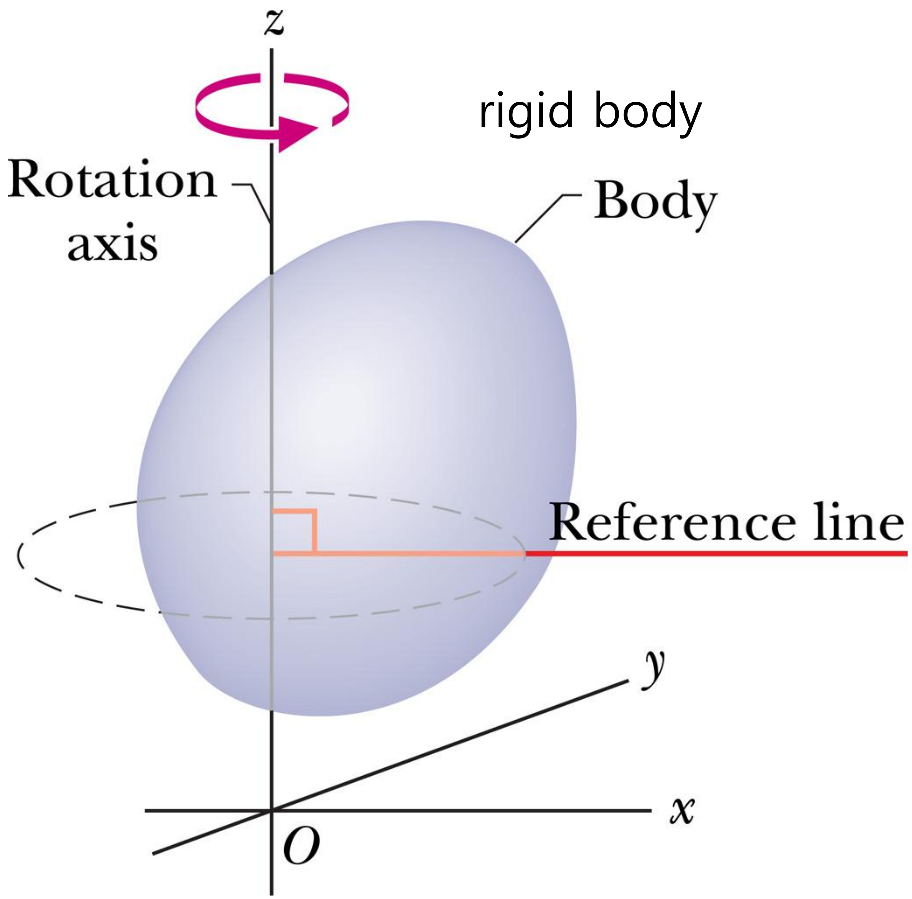
translational motion

rotational motion

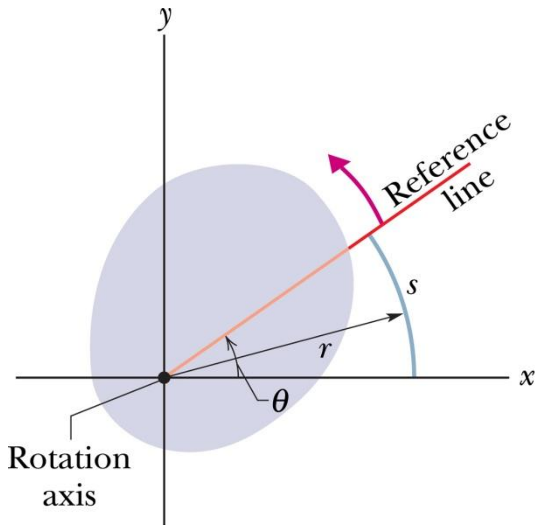
Rotational variables



rigid body



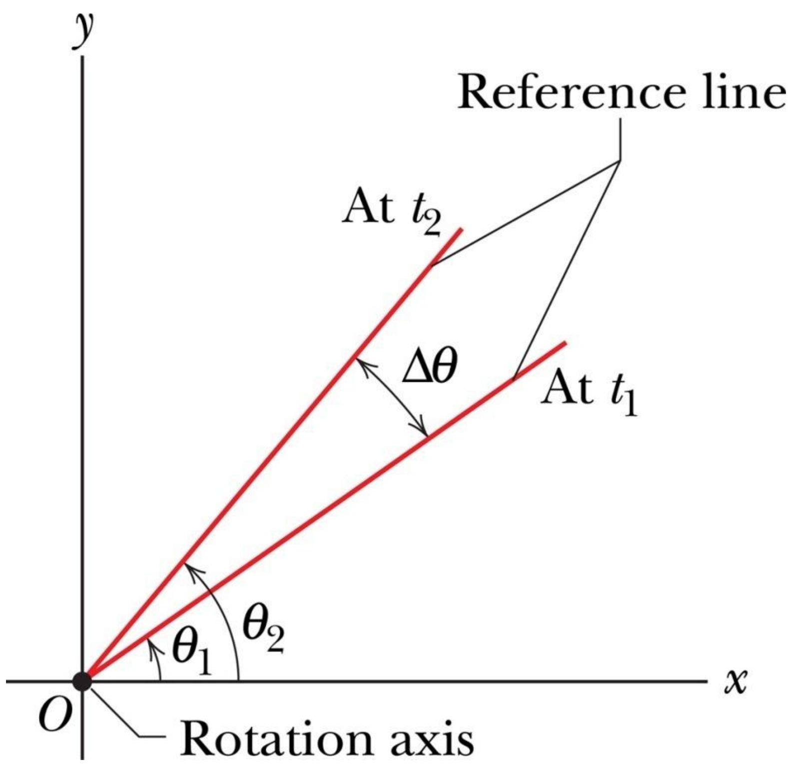
translation		rotation	
displacement	$\mathbf{r}$	angular D	θ
velocity	$\mathbf{v}$	angular V	ω
Accel.	$\mathbf{a}$	Angular A.	α



Angular position $\theta = \frac{s}{r}$ (in radian)

$$1 \text{ rev} = \frac{2\pi r}{r} = 2\pi \text{ radian}$$

$$1 \text{ rad} = 57.3^\circ$$

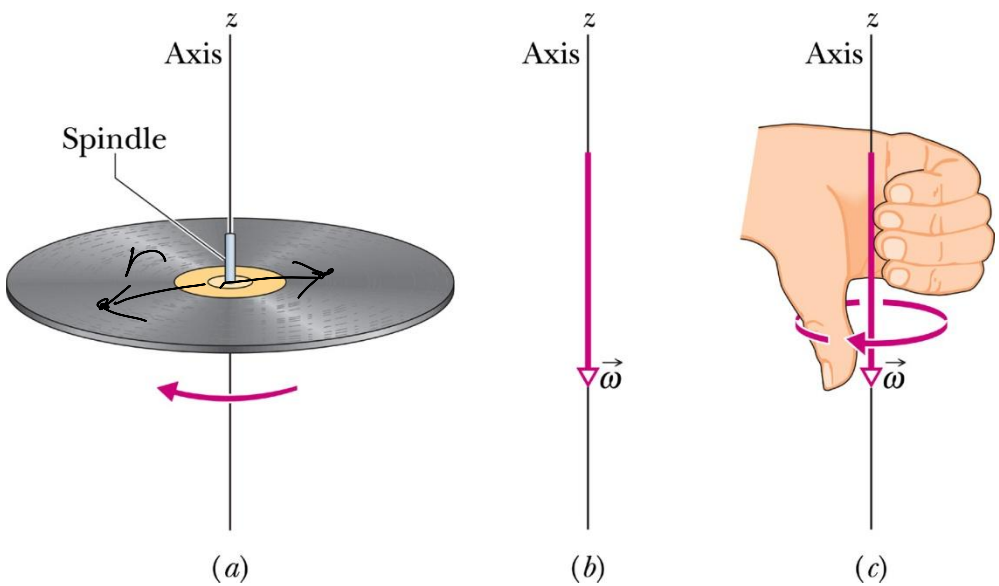


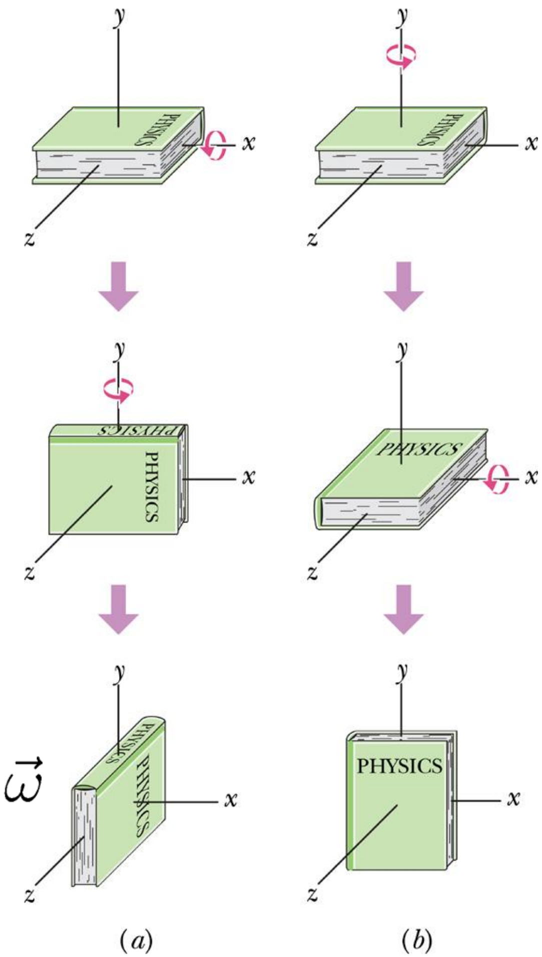
Angular displacement: $\Delta\theta = \theta_2 - \theta_1$

Angular velocity: $\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$

Angular acceleration: $\alpha = \lim_{\Delta t \to 0} \frac{\Delta\omega}{\Delta t} = \frac{d\omega}{dt}$

Ang. variables는 vector인가?





$$\mathbf{v} = \vec{\omega} \times \mathbf{r}$$

$$\mathbf{r} \times \mathbf{v} = \mathbf{r} \times (\vec{\omega} \times \mathbf{r}) = (\mathbf{r} \cdot \mathbf{r})\vec{\omega} - (\mathbf{r} \cdot \vec{\omega})\mathbf{r} = r^2\vec{\omega}$$

$x \to \frac{\pi}{2}$

$y \to \frac{\pi}{2}$

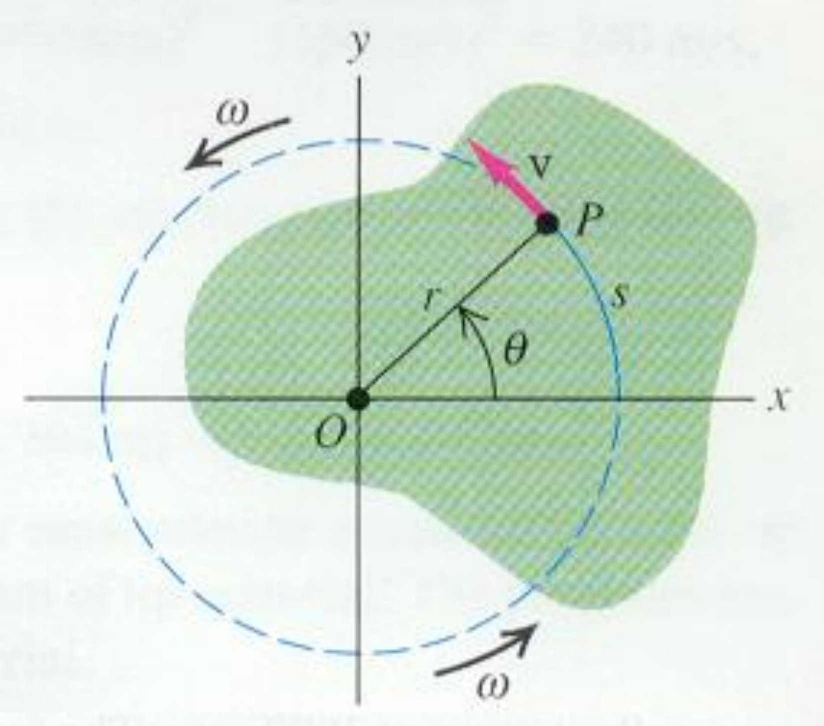
회전각도가 매우 작을 때 교환법칙이 성립

각운동에서의 변수들을 선운동에서의 변수들이 만족하는 방정식과 매우 비슷한 방정식을 만족한다.

$$x \to \theta,\ \ v \to \omega,\ \ a \to \alpha$$

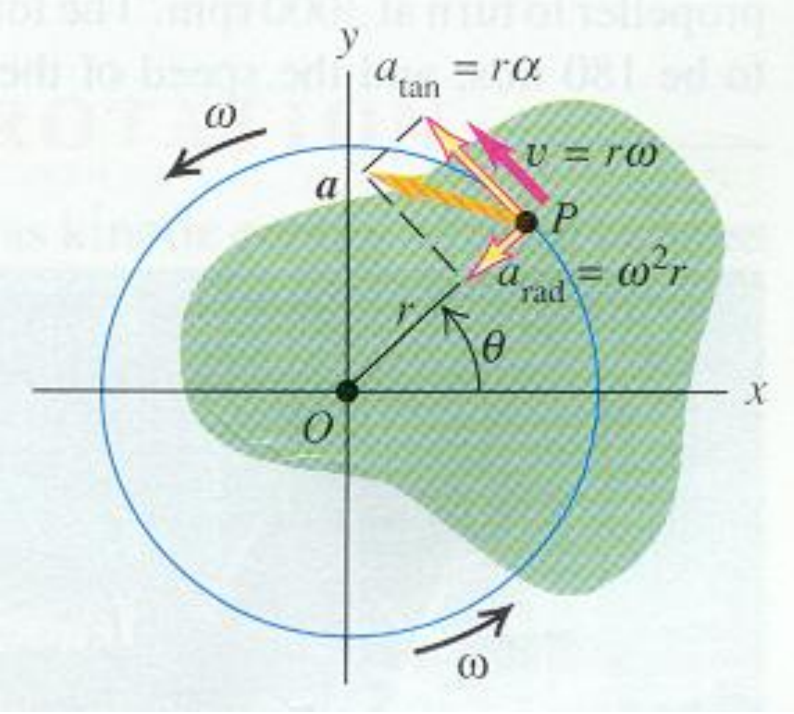
표 1: 선운동과 회전운동에서 변수들 사이의 관계

선운동 방정식	빠진 변수	빠진 변수	각운동 방정식
$v = v_0 + at$	$x - x_0$	$\theta - \theta_0$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_o t + \frac{1}{2}at^2$	v	ω	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	t	t	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a	α	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$
$x - x_0 = vt - \frac{1}{2}at^2$	v_0	ω_0	$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2$



$$s = r\theta$$

$$v = \frac{ds}{dt} = \frac{rd\theta}{dt} = r\omega$$

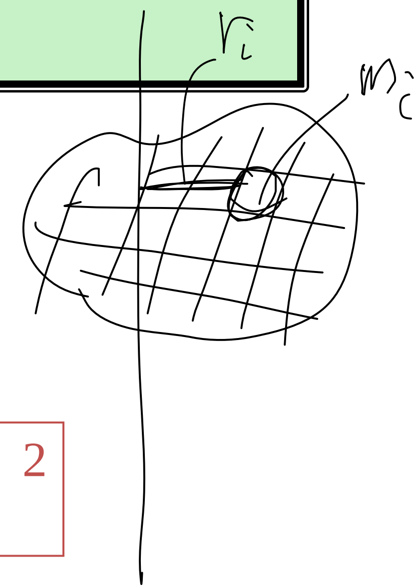


$$a_t = \frac{dv}{dt} = \frac{d(r\omega)}{dt} = r\alpha$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$\mathbf{a} = \mathbf{a}_t + \mathbf{a}_r$$

Moment of intertia



Rotational kinetic energy

$$K = \tfrac{1}{2}\sum_i m_i v_i^2$$

$$= \tfrac{1}{2}\sum_i m_i (r_i \omega_i)^2$$

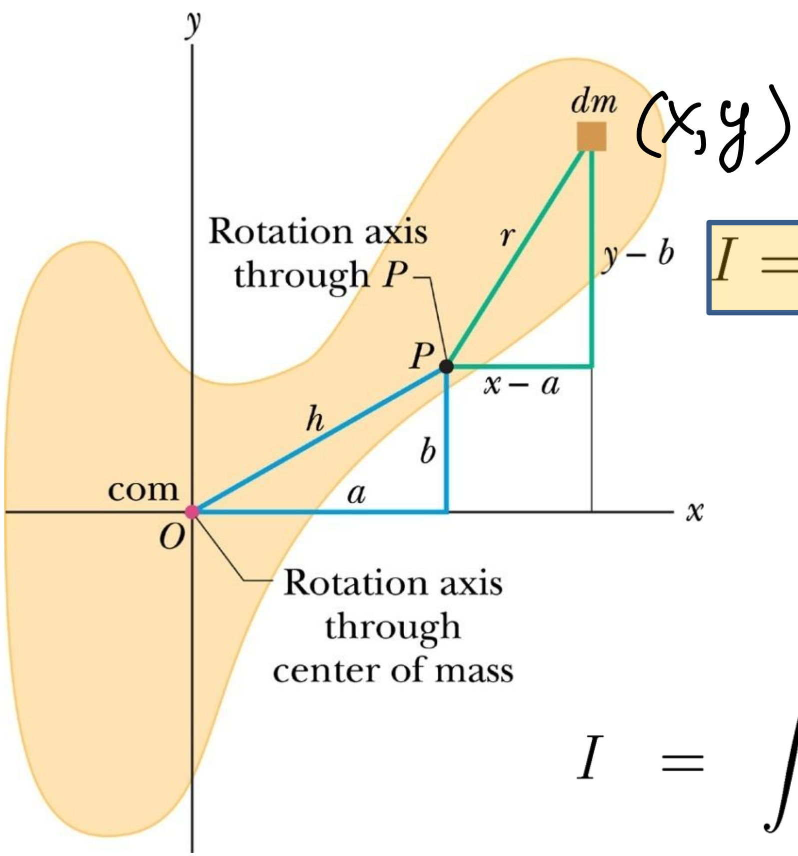
$$= \tfrac{1}{2}\sum_i m_i r_i^2 \omega^2$$

$$K = \tfrac{1}{2} I \omega^2$$

$$I \equiv \sum_i m_i r_i^2 = \int r^2 dm$$

moment of inertia

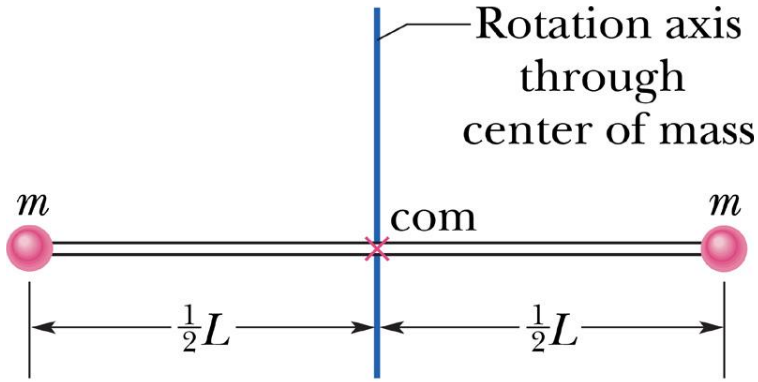
Parallel-axis theorem



$$I = I_{com} + Mh^2$$ (Parallel axis theorem)

$$\begin{aligned} I &= \int r^2 dm = \int \left[(x-a)^2 + (y-b)^2\right] dm \\ &= \int (x^2+y^2) dm - 2a\int x dm - 2b \int y dm \\ &\quad + \int (a^2+b^2) dm \end{aligned}$$

Sample prob.

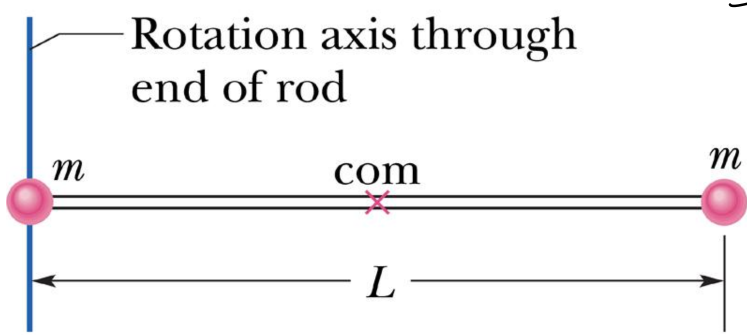


(a)

$$I = \frac{mL^2}{4} \times 2 = \frac{mL^2}{2}$$

P.A.T

$$I = \frac{mL^2}{2} + (2m)\frac{L^2}{4} = mL^2$$



(b)

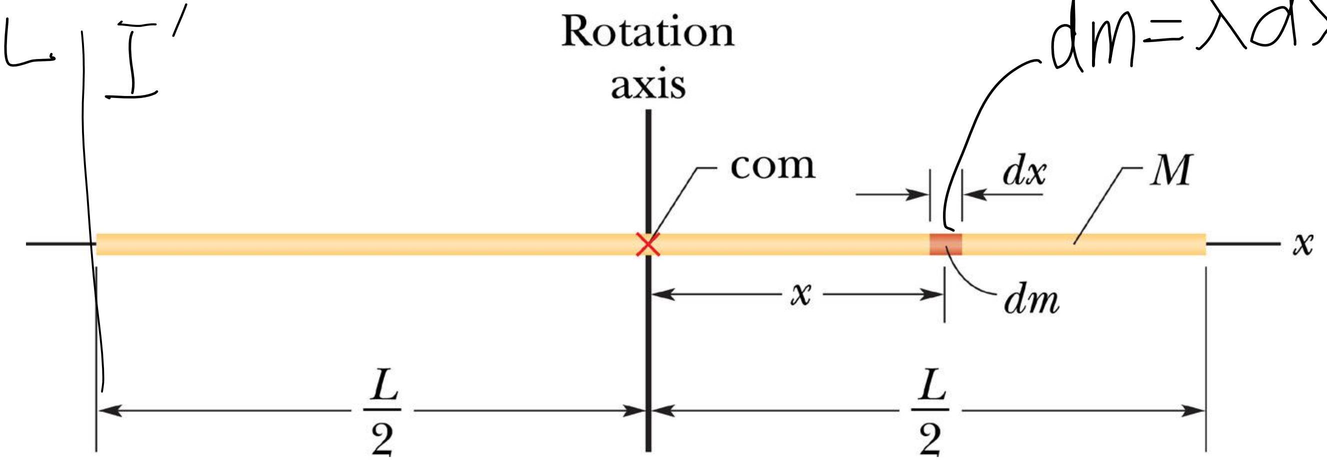
$$I = mL^2$$

Sample prob.

$\lambda = \frac{M}{L}$

$dm = \lambda dx$

I'



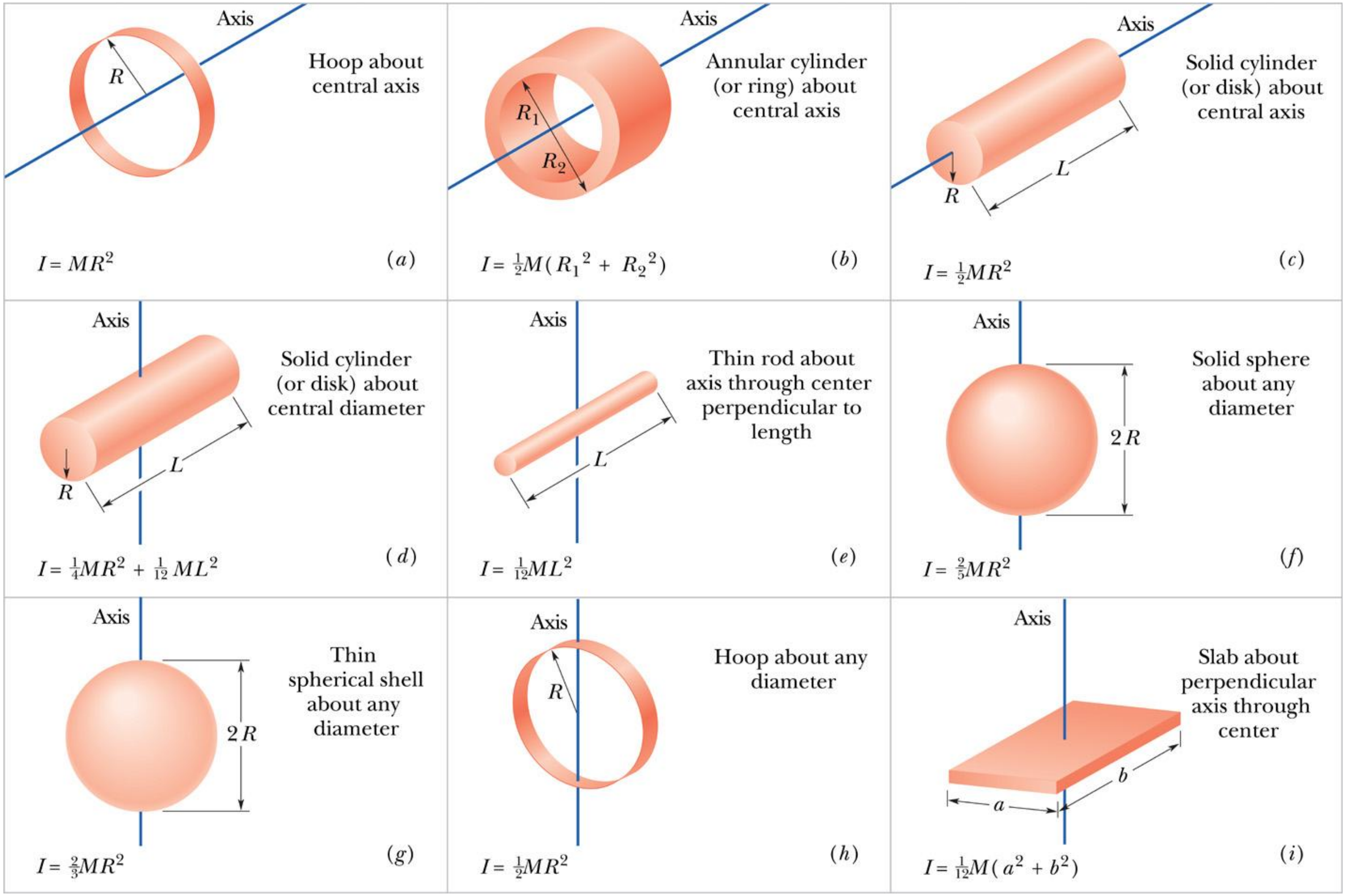
$$dI = dm\, x^2 = \lambda x^2 dx$$

$$I = \int_{-L/2}^{L/2} \lambda x^2 dx = 2\lambda \int_0^{L/2} x^2 dx = \frac{2\lambda}{3} \frac{L^3}{8}$$

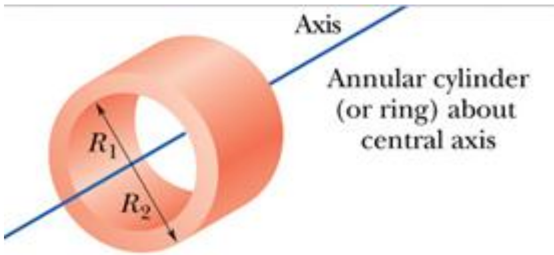
$$= \frac{\lambda L^3}{12} = \frac{1}{12} L^3 \frac{M}{L} = \frac{ML^2}{12}$$

$$I' = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{1}{3} ML^2$$

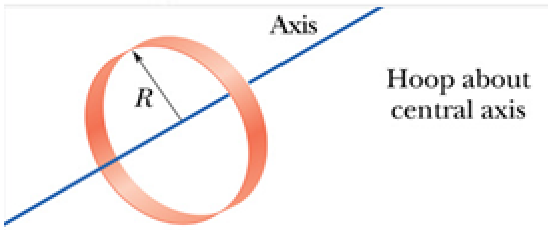
MOI 계산하기



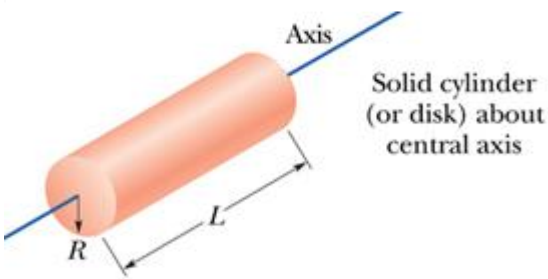
Axis
R
Hoop about central axis
$I = MR^2$
(a)
Axis
R_1
R_2
Annular cylinder (or ring) about central axis
$I = \frac{1}{2}M(R_1{}^2 + R_2{}^2)$
(b)
Axis
R
L
Solid cylinder (or disk) about central axis
$I = \frac{1}{2}MR^2$
(c)
Axis
R
L
Solid cylinder (or disk) about central diameter
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
L
Thin rod about axis through center perpendicular to length
$I = \frac{1}{12}ML^2$
(e)
Axis
2R
Solid sphere about any diameter
$I = \frac{2}{5}MR^2$
(f)
Axis
2R
Thin spherical shell about any diameter
$I = \frac{2}{3}MR^2$
(g)
Axis
R
Hoop about any diameter
$I = \frac{1}{2}MR^2$
(h)
Axis
a
b
Slab about perpendicular axis through center
$I = \frac{1}{12}M(a^2 + b^2)$
(i)



$I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)



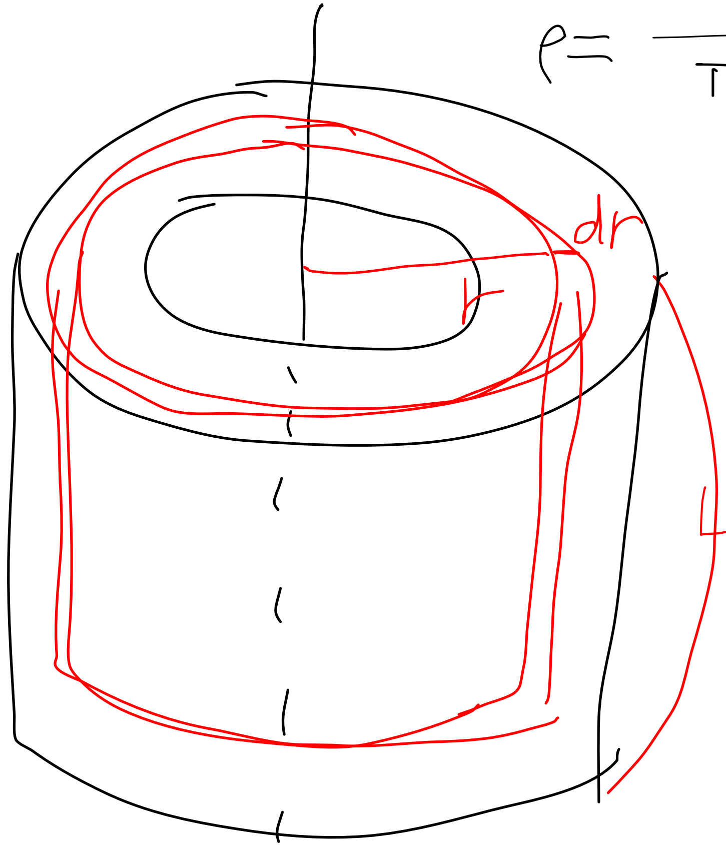
$I = MR^2$ (a)



$I = \frac{1}{2}MR^2$ (c)

$(r+dr)^2 - r^2$
$= 2r\,dr + (dr)^2$

$\rho = \frac{M}{\pi(R_2^2 - R_1^2)L}$

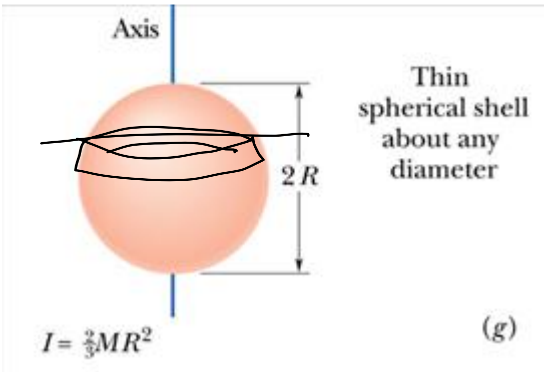


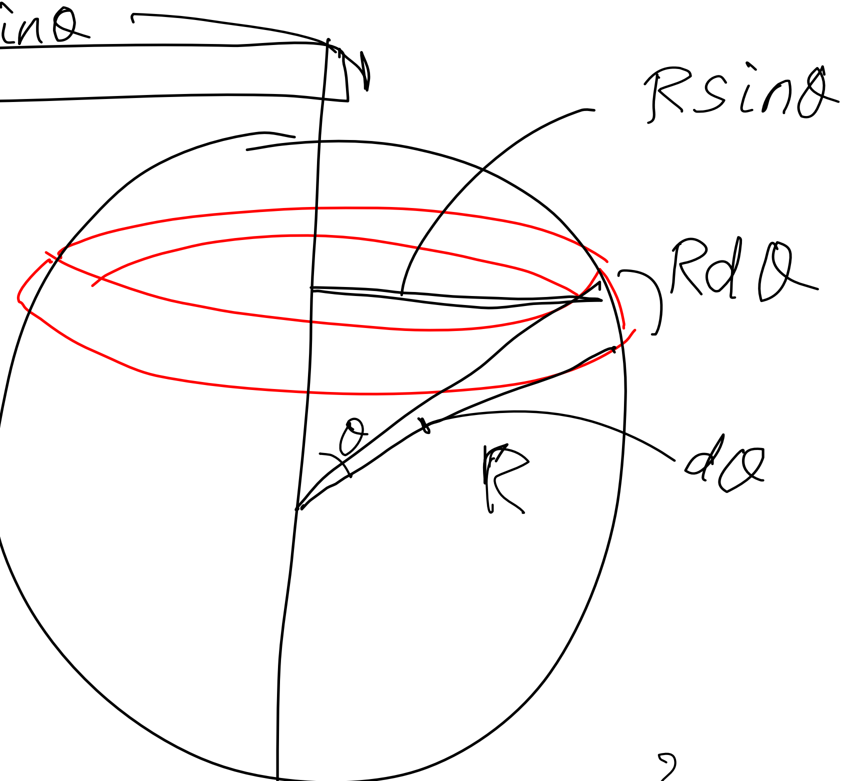
$I = \frac{1}{2}M(R_1^2 + R_2^2)$

$dm = \rho\, dV = 2\pi r\, dr\, L\rho$

$dI = r^2 dm = 2\pi L\rho\, r^3 dr$

$I = 2\pi L\rho \int_{R_1}^{R_2} r^3 dr = \frac{2\pi L\rho}{4}(R_2^2 - R_1^2)(R_2^2 + R_1^2)$





$$\sigma = \frac{M}{4\pi R^2}$$

$$dm = \sigma\, dA = 2\pi R^2 \sin\theta\, d\theta\, \sigma$$

$$dI = dm\, R^2 \sin^2\theta = 2\pi R^4 \sin^3\theta\, \sigma\, d\theta$$

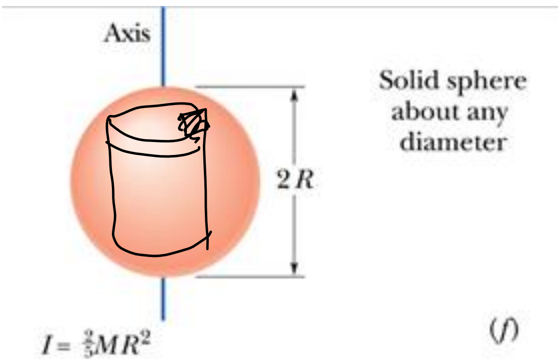
$$I = 4\pi R^4 \sigma \int_0^{\frac{\pi}{2}} \sin^3\theta\, d\theta = 4\pi R^4 \sigma \int_0^1 (1-x^2)\, dx$$

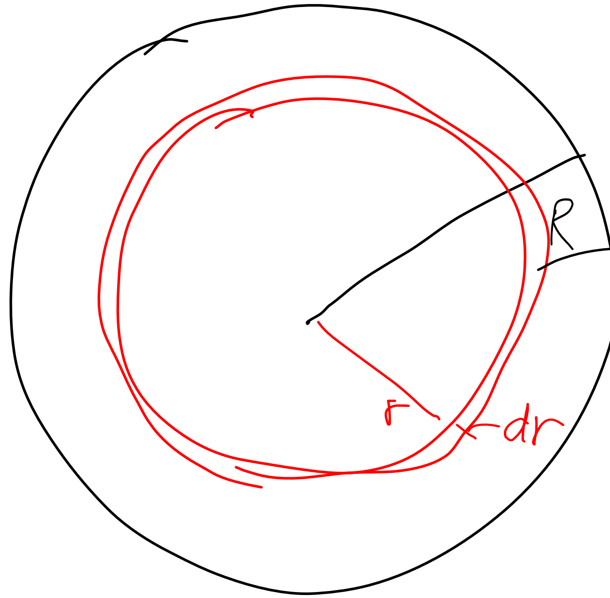
$$(1-\cos^2\theta)\frac{\sin\theta\, d\theta}{d\cos\theta}$$

$\cos\theta = x$

$-\sin\theta\, d\theta = dx$

$$= \frac{8\pi}{3} R^4 \frac{M}{4\pi R^2} = \frac{2}{3} M R^2$$





$$dV = 4\pi r^2 dr$$

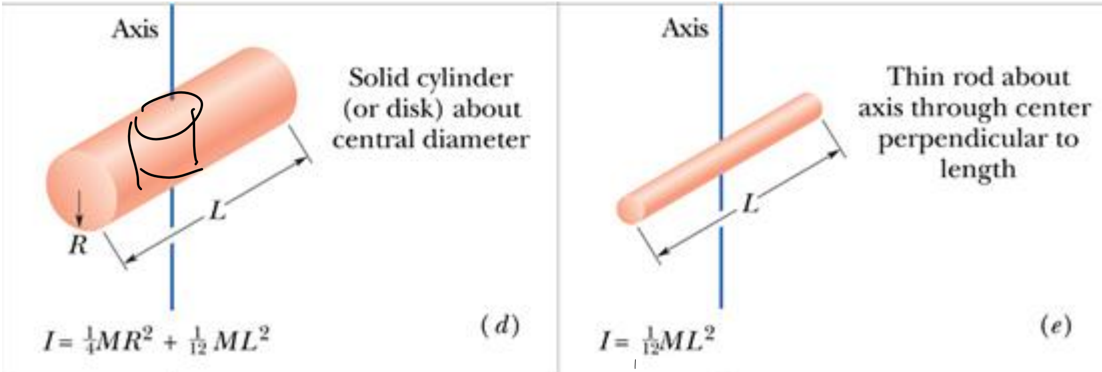
$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

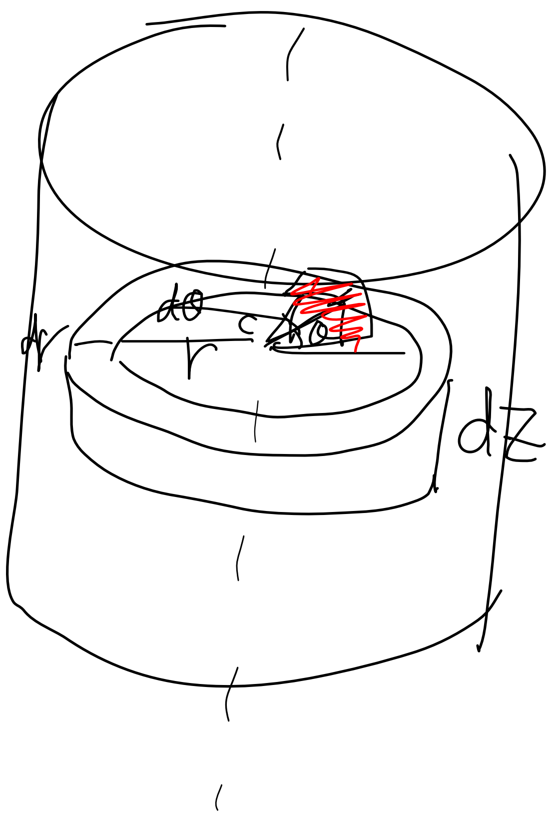
$$dI = \frac{2}{3} dm\, r^2$$

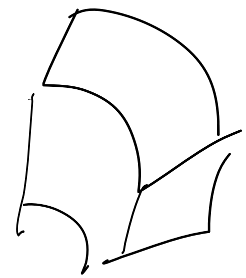
$$= \frac{2}{3}\rho r^2 dV$$

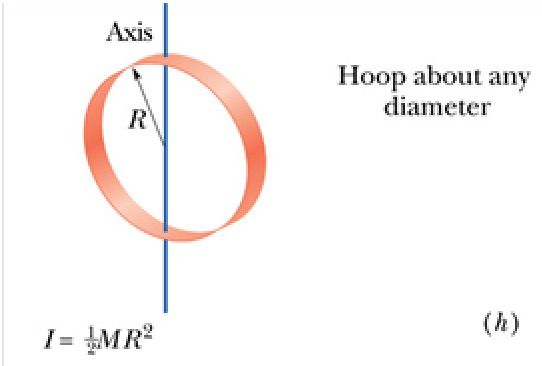
$$= \frac{2}{3}\rho 4\pi r^4 dr$$

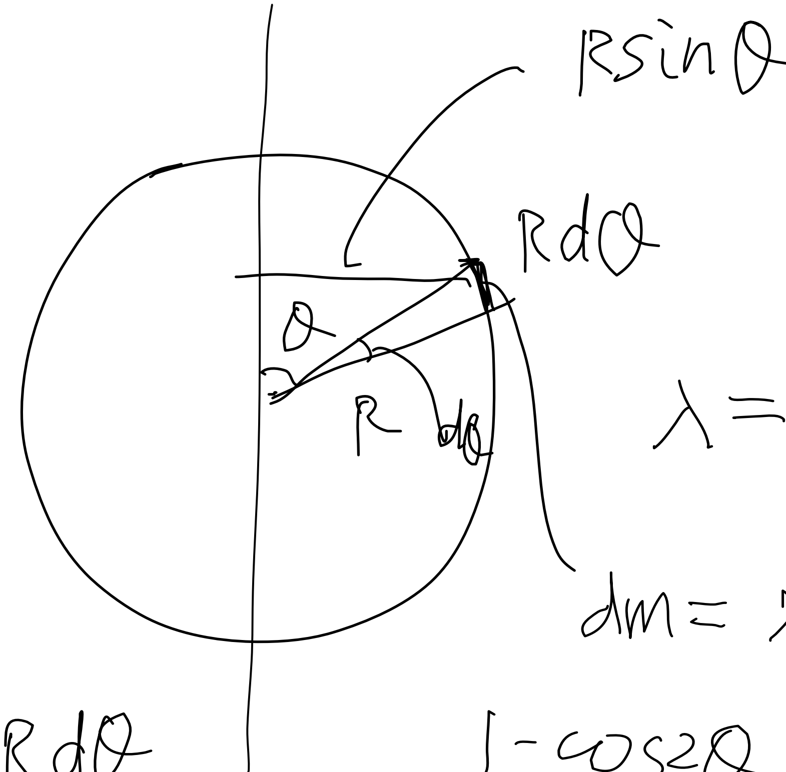
$$I = \frac{8\pi}{3}\rho \int_0^R r^4 dr = \frac{8\pi\rho}{15} R^5 = \frac{8\pi R^5}{15} \frac{M}{\frac{4}{3}\pi R^3} = \frac{2}{5} M R^2$$


Axis
Solid cylinder (or disk) about central diameter
R
L
$I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$
(d)
Axis
Thin rod about axis through center perpendicular to length
L
$I = \frac{1}{12}ML^2$
(e)


dθ
dr
r
dz







$$\lambda = \frac{M}{2\pi R}$$

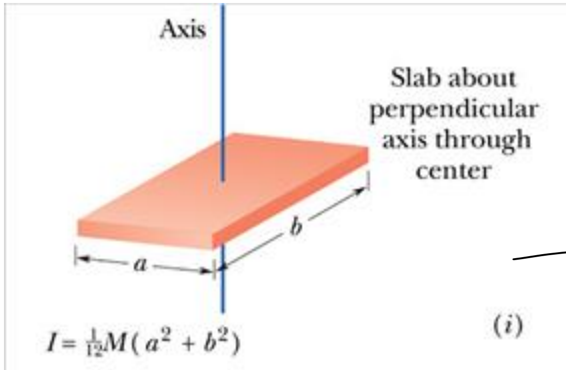
$$dm = \lambda R\,d\theta$$

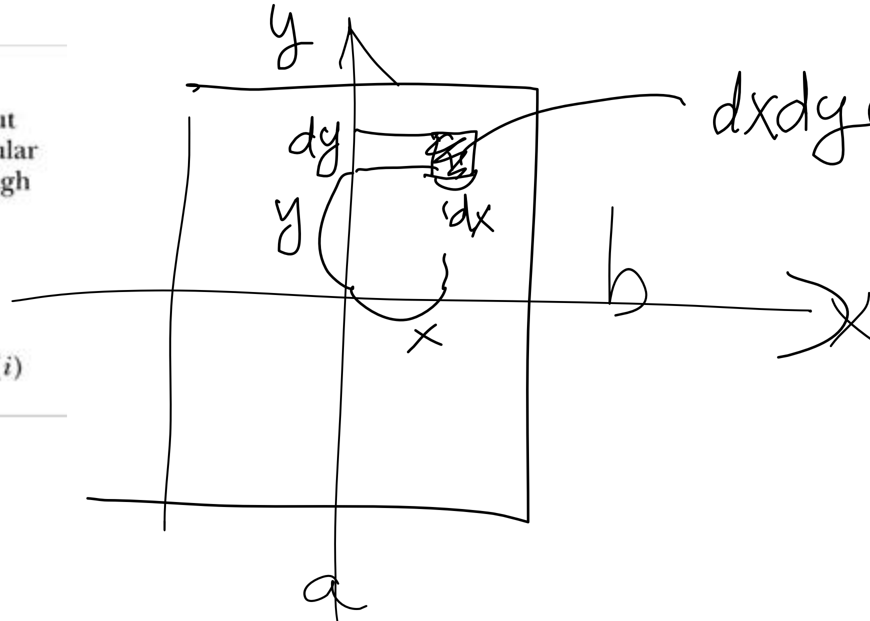
$$dI = R^2 \sin^2\theta\, \lambda R\, d\theta$$

$$\frac{1-\cos 2\theta}{2}$$

$$I = 2\lambda R^3 \int_0^{\frac{\pi}{2}} \sin^2\theta\, d\theta = \lambda \pi R^3$$

$$= \frac{M}{2\pi R}\pi R^3 = \frac{1}{2} M R^2$$





$$dxdy\sigma = dm$$

$$\sigma = \frac{M}{ab}$$

$$dI = (x^2+y^2)\,dm = \sigma(x^2+y^2)\,dx\,dy$$

$$I = 4\sigma\int_0^{\frac{b}{2}} dx \int_0^{\frac{a}{2}} dy\,(x^2+y^2)$$

$$= 4\sigma\int_0^{\frac{b}{2}} dx \left[x^2y + \frac{y^3}{3}\right]\Bigg|_0^{\frac{a}{2}} = 4\sigma\int_0^{\frac{b}{2}} dx\left[\frac{a}{2}x^2 + \frac{a^3}{24}\right]$$

$$= 4\sigma\left[\frac{ab^3}{48} + \frac{a^3b}{48}\right] = \frac{ab}{12}\sigma(a^2+b^2) = \frac{M}{12}(a^2+b^2)$$