GEST 011, Newton's Clock & Heisenberg's Dice, Fall 2013

The Conservation Laws

(Mass, Force, Work, Energy, and Momentum)

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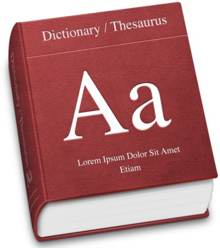
Mass and Force

"Mass"

(in dictionaries)



the quantity of matter that a body contains, as measured by its acceleration under a given "force" or by the force exerted on it by a gravitational field.

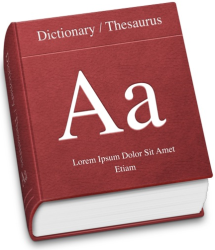
http://apple.com/

a quantitative measure of an object's resistance to acceleration.

http://en.wikipedia.org/

"Force"

(in dictionaries)



http://apple.com/

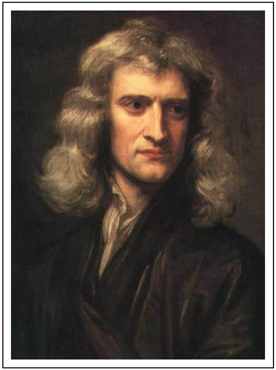
an influence tending to change the motion of a body or produce motion or stress in a stationary body. The magnitude of such an influence is often calculated by multiplying the "mass" of the body by its acceleration.

http://en.wikipedia.org/

In physics, a force is any influence that causes a free body to undergo an acceleration. Force can . . . cause an object with mass to change its velocity, i.e., to accelerate, or which can cause a flexible object to deform.

Gravitational Force

(Newton's Law)



Sir Isaac Newton
(1642–1727)

Image from Wikipedia

$$\boldsymbol{F} = G\,\frac{m_1 m_2}{r^2}\,\boldsymbol{e}_{12}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{N{\cdot}m^2/kg^2}$$

A mass generates an gravitational field, and the field acts force on the other mass.

Coulomb Force

(charge at rest or in motion)

Charles-Augustin de Coulomb (1736–1806)

Wikipedia

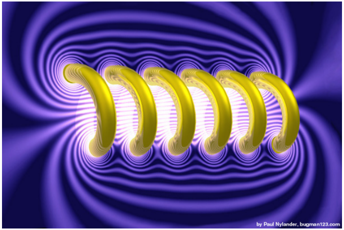
$$\boldsymbol{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\,\boldsymbol{e}_{12}$$

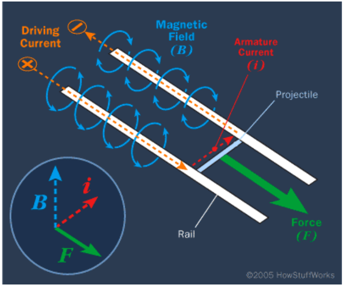
$$\frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9\ \mathrm{N{\cdot}m^2/C^2}$$

A charge generates an electric field, and the electric field acts force on the other charge.

Lorentz Force

(charge in motion)





$$\boxed{\boldsymbol{F} = q\,\boldsymbol{v} \times \boldsymbol{B}}$$

A moving charge generates a magnetic field, and the field acts force on other moving charges.

Images courtesy of `http://bugman123.com` and `http://HowStuffWorks.com/`, respectively.

Photo by Guy H. 2006 / Mountain Project, Inc.

El Matador at The Devil's Tower in Wyoming, USA

What About Frictional Force?

Photo by Guy H. 2006 / Mountain Project, Inc.
El Matador at The Devil's Tower in Wyoming, USA

Work and Energy

Work? What is It?

$$\boxed{\Delta W \equiv F \cdot \Delta X}$$

What Is Kinetic Energy?

$$\boxed{K \equiv \frac{1}{2}mv^2}$$

Let’s do some simple math!

- Consider a particle moving at velocity v at time t.

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- As $t \rightarrow t + dt$, the position will change $x \rightarrow x + dx$. Then:

$$dx = ?$$

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- As $t \to t + dt$,

$$x \to x + dx$$
$$5x \to 5x + \boxed{?}$$
$$x^2 \to x^2 + \boxed{?}$$

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- As $t \to t + dt$, the velocity will change $v \to v + dv$. Then:

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$$v \to v + dv$$
$$v^2 \to v^2 + \boxed{?}$$
$$v^3 \to v^3 + \boxed{?}$$

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- As $t \rightarrow t + dt$,

$$v \rightarrow v + dv$$
$$v^2 \rightarrow v^2 + \boxed{2v\,dv}$$
$$v^3 \rightarrow v^3 + \boxed{3v^2\,dv}$$

Another Face of Newton's 2nd Law

$$m\frac{dv}{dt} = F(x, v; t)$$

$$m\frac{dv}{dt}v = F \cdot v$$

$$m\,dv\,v = F \cdot v\,dt$$

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Kinetic Energy-Work Theorem

$$\boxed{dK = dW}$$
$$\boxed{\frac{dK}{dt} = \frac{dW}{dt} = F \cdot v}$$

The change in the kinetic energy equals to the work "done" to the system.

Kinetic Energy and Work

Kinetic Energy

$$K \equiv \frac{1}{2}mv^2$$

It is associated with the state of the particle in motion.

Work

$$dW = F \cdot dx\,, \quad W = F \cdot L$$

It is associated with the process that brings the change in the motion of the particle.

$$dK = dW$$

Kinetic Energy and Potential Energy

Kinetic Energy

$$K \equiv \frac{1}{2}mv^2$$

It is associated with the state of the particle in motion.

Potential Energy

$$dU = -dW = -F \cdot dx\,, \quad U = -F \cdot L$$

It is associated with the hypothetical process that brings the change in the motion of the particle.

$$dK + dU = 0\,, \quad \boxed{K + U = \text{constant}}$$

Force vs Potential Energy

$$\boxed{dU = -dx \cdot F}$$

$$U(x) = -\int_{x_0}^{x} dx' \cdot F(x')$$

Potential energy has the same information as force.

What is Energy?

- Some quantity associated with the state of the system.
- Some quantity that is conserved.
- Its expression takes many different forms:

$$K = \frac{1}{2}mv^2\,, \quad U = mgx\,, \quad U = \frac{1}{2}kx^2\,, \quad \cdots$$

- To be "interpreted" as a capacity to perform work.
- Mondern technology makes use of energy.

Linear Momentum

What is linear momentum?

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Wikipedia

In classical mechanics, [linear] momentum is the product of the mass and velocity of an object.

$$(\text{linear momentum}) = (\text{mass}) \times (\text{velocity})$$

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In classical mechanics, [linear] momentum is the product of the mass and velocity of an object.

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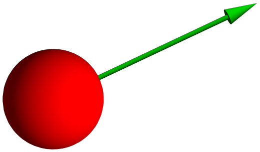
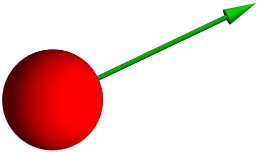
So what?

Conservation of Linear Momentum

(single particle)



Conservation of Linear Momentum
(single particle)

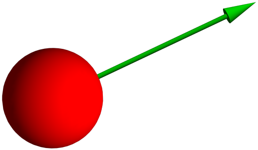


Newton's 1st Law

If no *net* force acts on a body, the body's velocity cannot change.

Conservation of Linear Momentum

(single particle)



(linear momentum) = (mass) $\times$ (velocity)

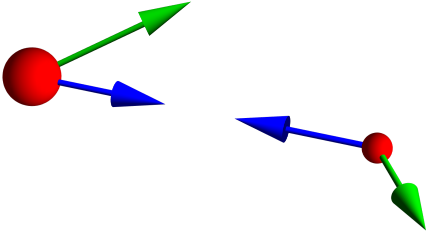
Linear momentum is conserved!

Newton's 1st Law

If no *net* force acts on a body, the body's velocity cannot change.

Conservation of Linear Momentum

(many particles)

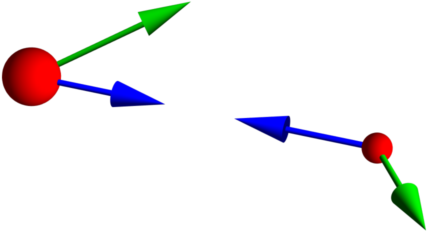


$$\frac{d}{dt}(m_1 \boldsymbol{v}_1) = \boldsymbol{F}_1 + \boldsymbol{G}_{12}$$

$$\frac{d}{dt}(m_2 \boldsymbol{v}_2) = \boldsymbol{F}_2 + \boldsymbol{G}_{21}$$

Conservation of Linear Momentum

(many particles)



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$$\frac{d}{dt}(m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2) = \boldsymbol{F}_1 + \boldsymbol{F}_2$$

$$\boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2 = m_1 \boldsymbol{v}_1 + m_2 \boldsymbol{v}_2$$

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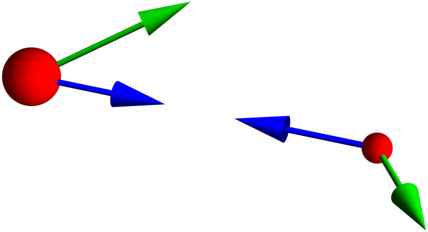
(many particles)



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Conservation of Linear Momentum

If no external net force acts on a system of particles, the total linear momentum of the system is conserved.

Angular Momentum

Densmore Shute bends the shaft (1938), Photograph by Harold Edgerton

A Glimpse of Geometry

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- Two nonidentical points define uniquely a (straight) line.

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- ...

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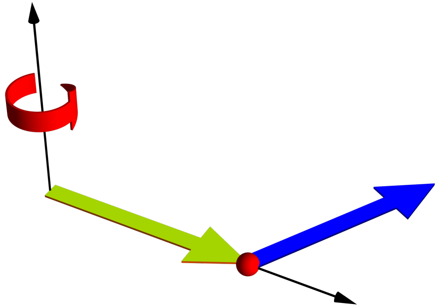
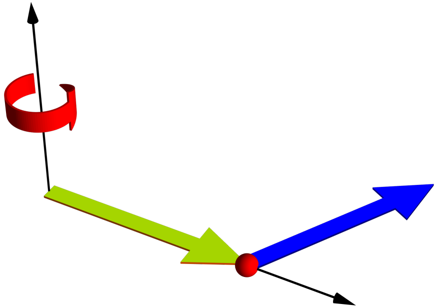
We want to describe a rotational motion.
What do we need?

Torque and Angular Momentum



Torque and Angular Momentum

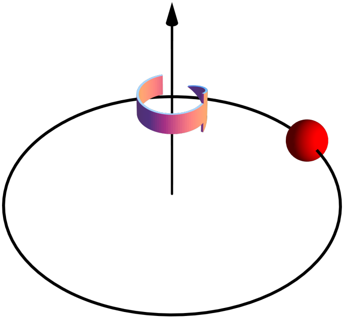


(torque) = (radius) × (force)

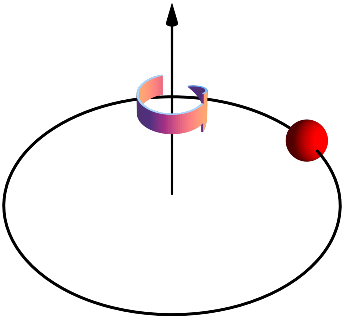
(angular momentum) = (radius) × (linear momentum)

Conservation of Angular Momentum

(single particle)



Conservation of Angular Momentum
(single particle)



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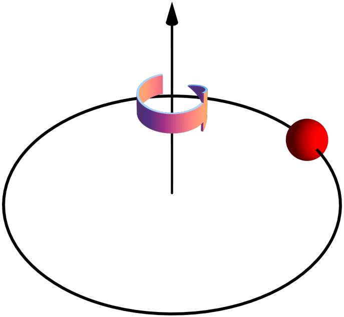

Newton's 1st Law

If no *net* force acts on a body, the body's velocity cannot change.

Angular Momentum Conservation

If no net torque acts on a body, the body's angular momentum cannot change.

Summary

- Force, introduced by means of an axiom.
- Energy and work, defined by means conservation laws.
- Linear and angular momentum, defined by means of conservation laws.

References