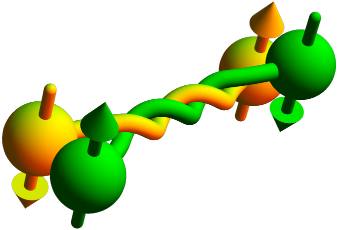
# War of the Titans

## (Einstein, Bohr, and Bell)

**Mahn-Soo Choi (Korea University)**

December 6, 2013 (v5.5)



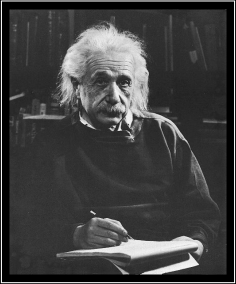
The Battle between the Gods and the Titans (J. Wtewael, 1566–1638)

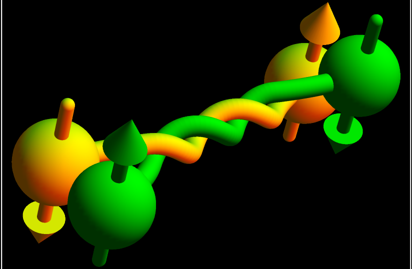
Zeus with thunderbolt (Musée du Louvre, Paris)

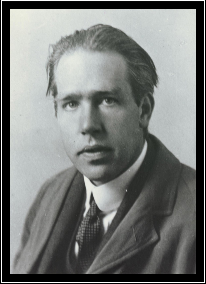
Image courtesy of http://www.theoi.com/

The Olympians (18c)

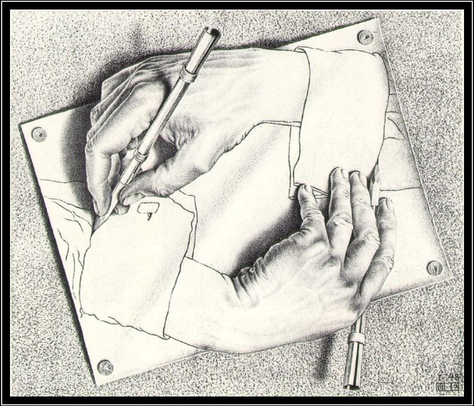
Image courtesy of http://www.crystalinks.com/



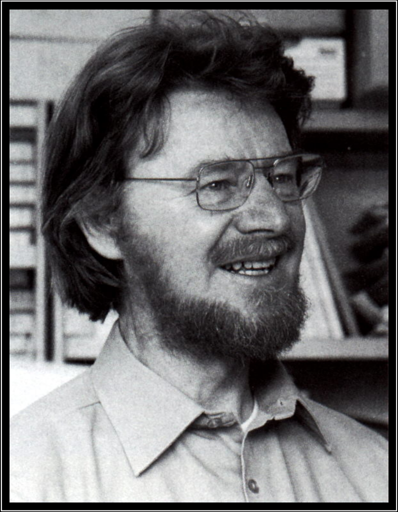




Left and right images courtesy of Wikipedia

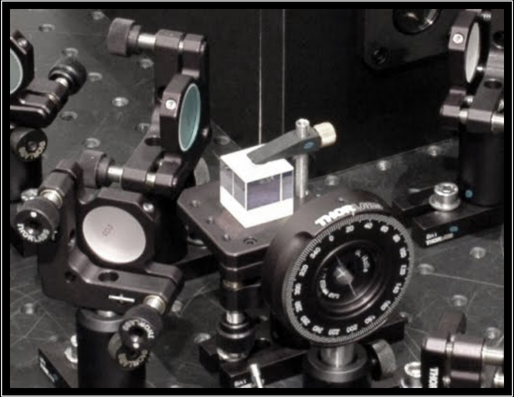


Drawing Hands (Escher, 1948)



John S. Bell (1928–1990)

Images courtesy of Wikipedia



Aspect, Grangier & Roger, Phys. Rev. Lett. (1982); Aspect, Dalibard & Roger, Phys. Rev. Lett. (1982)

# The New Era of Quantum Physics

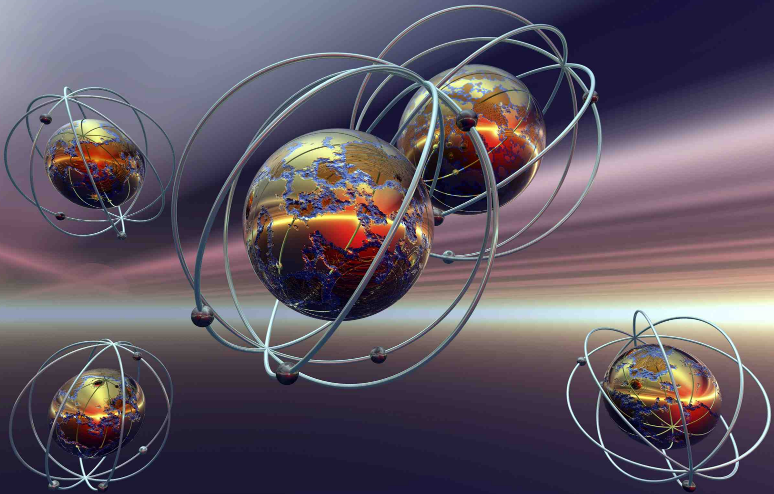


Image courtesy of Ryan Curtis (2012)

# The EPR Paradox

**(Einstein, Podolsky & Rosen, Phys. Rev., 1935; Bohm 1952)**

# Separable State

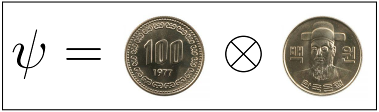
## (simple example)

$\psi =$ 

$\otimes$ 

# Separable State

## (simple example)



| $A$ | $B$ | probability |
|---|---|---|

# Separable State

## (simple example)

$\psi =$  $\otimes$ 

| $A$ | $B$ | probability |
|---|---|---|
|  |  | 0 |
|  |  | 1 |
|  |  | 0 |
|  |  | 0 |

# Separable State

## (another example)

$\psi = \Bigg($ 

$+$ 

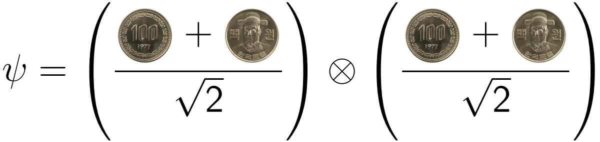
$\Bigg) / \sqrt{2} \;\otimes\; \Bigg($ 

$+$ 

$\Bigg) / \sqrt{2}$

# Separable State

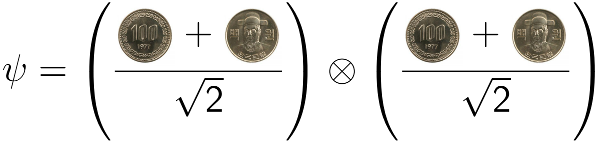
## (another example)



| $A$ | $B$ | probability |
|---|---|---|

# Separable State

## (another example)



| $A$ | $B$ | probability |
|---|---|---|
|  |  | $1/4$ |
|  |  | $1/4$ |
|  |  | $1/4$ |
|  |  | $1/4$ |

# Separable State

(another example)

$\psi =$ 

$2$

| $A$ | $B$ | probability |
|---|---|---|
|  |  | $1/4$ |
|  |  | $1/4$ |
|  |  | $1/4$ |
|  |  | $1/4$ |

# Entangled State

(Einstein, Podolsky & Rosen, Phys. Rev., 1935)

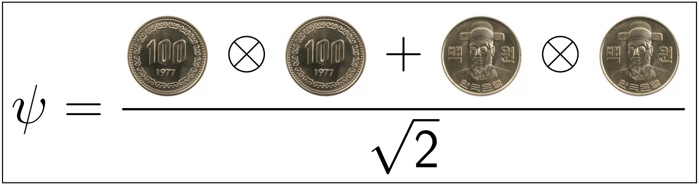
$\psi =$


$\otimes$


$+$


$\otimes$


$/\ \sqrt{2}$



| $A$ | $B$ | probability |
| --- | --- | --- |

# Entangled State

(Einstein, Podolsky & Rosen, Phys. Rev., 1935)

$\psi =$ ( 

$\otimes$ 

$+$ 

$\otimes$ 

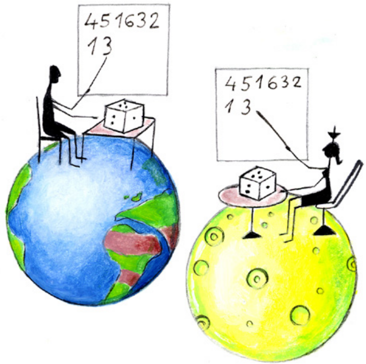
) / $\sqrt{2}$

| *A* | *B* | probability |
|---|---|---|
|  |  | $1/2$ |
|  |  | 0 |
|  |  | 0 |
|  |  | $1/2$ |

# Non-local?

## (The EPR Paradox)

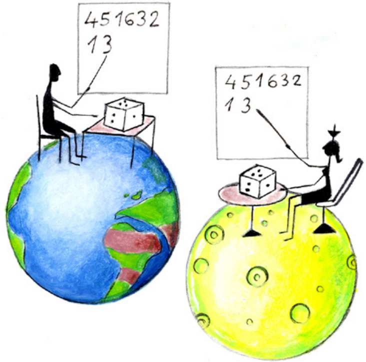
$$\psi_{\text{before}} = \frac{1}{\sqrt{6}}\left(\text{⚀⚀} + \text{⚁⚁} + \text{⚂⚂} + \text{⚃⚃} + \text{⚄⚄} + \text{⚅⚅}\right)$$



# Faster Than Light?

## (The EPR Paradox)

$\psi_{\text{before}} = \frac{1}{\sqrt{6}}\Big($ ⚀⚀ + ⚁⚁ + ⚂⚂ + ⚃⚃ + ⚄⚄ + ⚅⚅ $\Big)$




$\psi_{\text{after}} =$ ⚂⚂

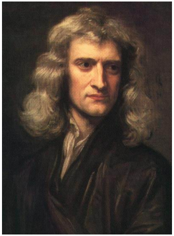
The “effect” of measurement by Alice propagates faster than light to Bob?

The “effect” of measurement by Alice propagates faster than light to Bob?

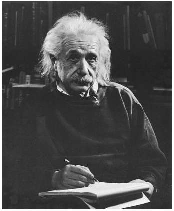
**Dieks, Phys. Lett. A (1982)**

[S]uperluminal communication (faster-than-light communication by means of an EPR-type experimental set-up) is not possible.

# The Concept of Time
## ("simultaneity")



Time is considered to be "absolute" and to flow "equably" for all observers. Events seen by two different observers in motion relative to each other produces a mathematical concept of time.



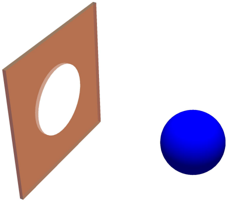
Invoking a method of synchronizing clocks using the constant, finite speed of light as the maximum signal velocity. This led directly to the result that observers in motion relative to one another will measure different elapsed times for the same event.

Source: Wikipedia
See also: Jammer (2006).

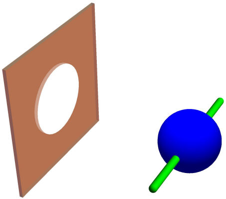
## Einstein's

# Hidden Variable Theory

# Hidden Variables?



vs



# Hidden Variable Theories

## (Einstein, Podolsky & Rosen, Phys. Rev., 1935)

"If, without in any way disturbing a system, we can predict with certainty the value of a physical quantity, then there exists an element of physical reality corresponding to this physical quantity."
— Einstein, Podolsky & Rosen, Phys. Rev. (1935)

# Hidden Variable Theories

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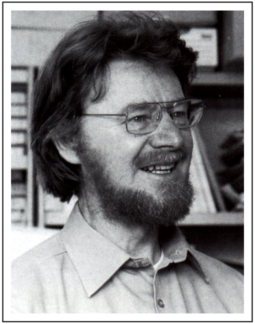
The outcomes of measurements on each system are predetermined and independent of the measurements carried out on the other system(s).
— Barrett, Kent & Pironio, Phys. Rev. Lett. (2006)

# Putting It to a Realistic Test

(Bell, Physics, 1964; Bell, Rev. Mod. Phys., 1966)



Bell's inequality

# Hardy's Test of Quantum Theory

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 9 | 8 | | | | | | 1 | |
| | | 2 | 6 | | | | | |
| 1 | | 4 | | | | | | 7 |
| | | | | 2 | 3 | 9 | 5 | |
| 6 | | 9 | | 5 | | 4 | 2 | 3 |
| | 2 | | 9 | 4 | | 1 | | |
| 2 | | | | | | 5 | | 1 |
| | | | 5 | | 7 | 6 | | |
| | 5 | | | | | | 8 | 9 |

# Key Distribution Problem

## Singh (1999)

1. There is a pair of padlock and key. Only the key can open the padlock.
2. Alice wants to send a love letter to Bob. She doesn't want Eve to intercept the letter before it reaches Bob.
3. Alice decides to put the letter in a box and lock it with the padlock.
4. Should Alice give the key to Bob before she sends her letter?

# Winning the Right Prize

**Smullyan (1985); Smullyan (1987)**

1. Suppose I offer two prizes, Prize 1 and Prize 2.
2. You are to make a statement. If the statement is true, then I am to give you one of the two prizes (not saying which one). If your statement is false, then you get no prize.
3. Obviously you can be sure of winning one of the two prizes by saying: "Two plus two is four," but suppose you have your heart set on Prize 1.
4. What statement could you make that would *guarantee* that you will get Prize 1?

# Hardy's Test


(http://www.uv.es/)

| | (1) *HH* | (2) *HD* | (3) *DH* | (4) *DD* |
|---|---|---|---|---|
| 00 | ■ | | | ?? |
| 01 | | | ■ | |
| 10 | | ■ | | |
| 11 | | | | |

# Hardy's Test

## (Hardy, Phys. Rev. Lett., 1992; Hardy, Phys. Rev. Lett., 1993)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| | (1) *HH* | (2) *HD* | (3) *DH* | (4) *DD* |
|---|---|---|---|---|
| 00 | ■ | | | ?? |
| 01 | | | ■ | |
| 10 | | ■ | | |
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|---|---|---|---|---|
| 00 | ■ | | | ?? |
| 01 | | | ■ | |
| 10 | | ■ | | |
| 11 | | | | |

- (1) implies that $H_1 = 1$ or $H_2 = 1$.

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|---|---|---|---|---|
| 00 | ■ | | | ?? |
| 01 | | | ■ | |
| 10 | | ■ | | |
| 11 | | | | |

- (1) implies that $H_1 = 1$ or $H_2 = 1$.
- If $H_1 = 1$, then necessarily $D_2 = 1$, because (2).
- If $H_2 = 1$, then necessarily $D_1 = 1$, because (3).

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| | (1) $HH$ | (2) $HD$ | (3) $DH$ | (4) $DD$ |
|---|---|---|---|---|
| 00 | ■ | | | 0 |
| 01 | | | ■ | |
| 10 | | ■ | | |
| 11 | | | | |

- (1) implies that $H_1 = 1$ or $H_2 = 1$.
- If $H_1 = 1$, then necessarily $D_2 = 1$, because (2).
- If $H_2 = 1$, then necessarily $D_1 = 1$, because (3).
- Therefore, (4) follows from (1), (2), and (3).

# Hardy's Test on Quantum States

**(Goldstein, Phys. Rev. Lett., 1994)**

$$|\psi\rangle = \frac{|H\rangle \otimes |H\rangle + |V\rangle \otimes |H\rangle + |H\rangle \otimes |V\rangle}{\sqrt{3}}$$

$$H : \begin{cases} |V\rangle \to 0 \\ |H\rangle \to 1 \end{cases} \qquad D : \begin{cases} |F\rangle \equiv (|V\rangle - |H\rangle)/\sqrt{2} \to 0 \\ |D\rangle \equiv (|V\rangle + |H\rangle)/\sqrt{2} \to 1 \end{cases}$$

- $P_\psi(00|HH) = 0.$

# Hardy's Test on Quantum States

(Goldstein, Phys. Rev. Lett., 1994)

$$|\psi\rangle = \frac{|H\rangle \otimes |H\rangle + |V\rangle \otimes |H\rangle + |H\rangle \otimes |V\rangle}{\sqrt{3}}$$

$$|\psi\rangle = \frac{\sqrt{2}}{\sqrt{3}} \left( \frac{|H\rangle + |V\rangle}{\sqrt{2}} \right) \otimes |H\rangle + \frac{1}{\sqrt{3}} |H\rangle \otimes |V\rangle$$

$$H : \begin{cases} |V\rangle \to 0 \\ |H\rangle \to 1 \end{cases} \qquad D : \begin{cases} |F\rangle \equiv (|V\rangle - |H\rangle)/\sqrt{2} \to 0 \\ |D\rangle \equiv (|V\rangle + |H\rangle)/\sqrt{2} \to 1 \end{cases}$$

- $P_\psi(00|HH) = 0.$
- $P_\psi(01|DH) = 0.$

# Hardy's Test on Quantum States

(Goldstein, Phys. Rev. Lett., 1994)

$$|\psi\rangle = \frac{|H\rangle \otimes |H\rangle + |V\rangle \otimes |H\rangle + |H\rangle \otimes |V\rangle}{\sqrt{3}}$$

$$|\psi\rangle = \frac{\sqrt{2}}{\sqrt{3}} |H\rangle \otimes \left( \frac{|H\rangle + |V\rangle}{\sqrt{2}} \right) + \frac{1}{\sqrt{3}} |V\rangle \otimes |H\rangle$$

$$H : \begin{cases} |V\rangle \to 0 \\ |H\rangle \to 1 \end{cases} \qquad D : \begin{cases} |F\rangle \equiv (|V\rangle - |H\rangle)/\sqrt{2} \to 0 \\ |D\rangle \equiv (|V\rangle + |H\rangle)/\sqrt{2} \to 1 \end{cases}$$

- $P_\psi(00|HH) = 0.$
- $P_\psi(01|DH) = 0.$
- $P_\psi(10|HD) = 0.$

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$$|\psi\rangle = \frac{|H\rangle \otimes |H\rangle + |V\rangle \otimes |H\rangle + |H\rangle \otimes |V\rangle}{\sqrt{3}}$$

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- $P_\psi(00|HH) = 0.$
- $P_\psi(01|DH) = 0.$
- $P_\psi(10|HD) = 0.$
- $P_\psi(00|DD) = \frac{1}{12} \neq 0.$
- Contradicting the LHV theory!!

# Bell Test Experiments

- Freedman & Clauser, Phys. Rev. Lett. (1972)
- Aspect, Grangier & Roger, Phys. Rev. Lett. (1981); Aspect, Grangier & Roger, Phys. Rev. Lett. (1982); Aspect, Dalibard & Roger, Phys. Rev. Lett. (1982)
- and many others

Most of the [...] experiments performed so far have "favored" quantum mechanics.

A. Shimony, "Bell's Theorem," Stanford Encyclopedia of Philosophy (2004, 2009)

# Hardy Test Experiment

- Di Giuseppe, De Martini & Boschi, Phys. Rev. A (1997)
- Irvine et al., Phys. Rev. Lett. (2005)
- Lundeen & Steinberg, Phys. Rev. Lett. (2009)
- and a few more ...

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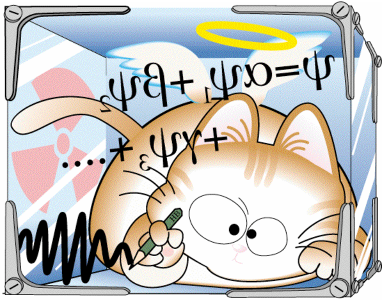
A. Shimony, "Bell's Theorem," Stanford Encyclopedia of Philosophy (2004, 2009)

# Further Remarks

# Entanglement & Decoherence



Entanglement plays a fundamental role in the interpretation of quantum theory!

Images courtesy of http://rprochoroff.blogspot.com/ and http://physicsworld.com/, respectively

# What is it useful for?

- Quantum computation – it is generally believed that entanglement is essential for the exponential speed-up.
- Cryptography (key distribution, etc.)
- Quantum teleportation

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