Wireless Communications (ITC731)

Lecture Note 2

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Summary

- Mobile radio propagation
- Link budget, receiver sensitivity, link margin
- Co-channel interference
- Flat fading channel

Mobile Radio Propagation Environment

- Radio signals generally propagate according to three mechanisms
  - Reflection
  - Diffraction
  - Scattering
- Three independent phenomenon of radio propagated signal due to above three mechanisms
  - ***Path loss*** variation with distance
  - Slow log-normal ***shadowing***
  - Fast ***multipath fading***

Path Loss in Free Space

- Received power at distance d in free space

$$\Omega_p(d) = \Omega_t k \left(\frac{\lambda_c}{4\pi d}\right)^2$$

where

$$\begin{array}{lll} \Omega_t & : & \text{transmitted power} \\ \lambda_c & : & \text{wavelength} \\ k & : & \text{constant of proportionality} \end{array}$$

$$\begin{aligned} \Omega_{p(\mathrm{dBm})}(d) &= 10\log_{10}(1000 \cdot \Omega_p(d)) = 30 + 10\log_{10}\Omega_p(d) \\ &= \underbrace{30 + 10\log_{10}\Omega_t}_{} + \underbrace{10\log_{10}(k\lambda_c^2/16\pi^2)}_{\text{constant}} - 20\log_{10} d \\ &= \Omega_{t(\mathrm{dBm})} + K - 20\log_{10} d \end{aligned}$$

Path Loss in Mobile Environment

- Received power at distance d in mobile environment

$$\Omega_p(d) = \Omega_t k \left(\frac{\lambda_c}{4\pi d}\right)^{\beta} + \epsilon$$

β : path loss exponent from 2 to 8

$\epsilon \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$

- Received power in decibel at distance d in mobile environment

$$\Omega_{p(\mathrm{dBm})}(d) = \Omega_{t(\mathrm{dBm})} + K - 10\beta \log_{10} d + \epsilon_{(\mathrm{dB})}$$

- Received power at distance d_0 in mobile environment

$$\Omega_{p(\mathrm{dBm})}(d_0) = \Omega_{t(\mathrm{dBm})} + K - 10\beta \log_{10} d_0 + \epsilon''_{(\mathrm{dB})}$$

  - Mean of received power

$$E[\Omega_{p(\mathrm{dBm})}(d_0)] = \Omega_{t(\mathrm{dBm})} + K - 10\beta \log_{10} d_0 = \mu_{\Omega_{p(\mathrm{dBm})}}(d_0)$$

- Received power at distance d in mobile environment

$$\begin{aligned}\Omega_{p(\mathrm{dBm})}(d) &= \Omega_{t(\mathrm{dBm})} + K - 10\beta\log_{10} d + \epsilon_{(\mathrm{dB})} \\ &= \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) + 10\beta\log_{10} d_0 - 10\beta\log_{10} d + \epsilon_{(\mathrm{dB})} \\ &= \underbrace{\mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta\log_{10}(d/d_0)}_{\mu_{\Omega_{p(\mathrm{dBm})}}(d)} + \epsilon_{(\mathrm{dB})} \\ &= \mu_{\Omega_{p(\mathrm{dBm})}}(d) + \epsilon_{(\mathrm{dB})}\end{aligned}$$

- PDF of $\Omega_{p(\mathrm{dBm})}(d)$

$$p_{\Omega_{p(\mathrm{dBm})}(d)}(x) = \frac{1}{\sqrt{2\pi}\sigma_\Omega}\exp\left[-\frac{(x-\mu_{\Omega_{p(\mathrm{dBm})}}(d))^2}{2\sigma_\Omega^2}\right]$$

where $\mu_{\Omega_{p(\mathrm{dBm})}}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta\log_{10}(d/d_0) \quad (\mathrm{dBm})$

σ_Ω : shadow standard deviation ranged from 5 to 12 dB

Remarks

- Shadow standard deviation
  - In macro-cellular, $\sigma_\Omega = 8\,\mathrm{dB}$ is a typical value.
  - Nearly independent of the radio path length d
- Area mean

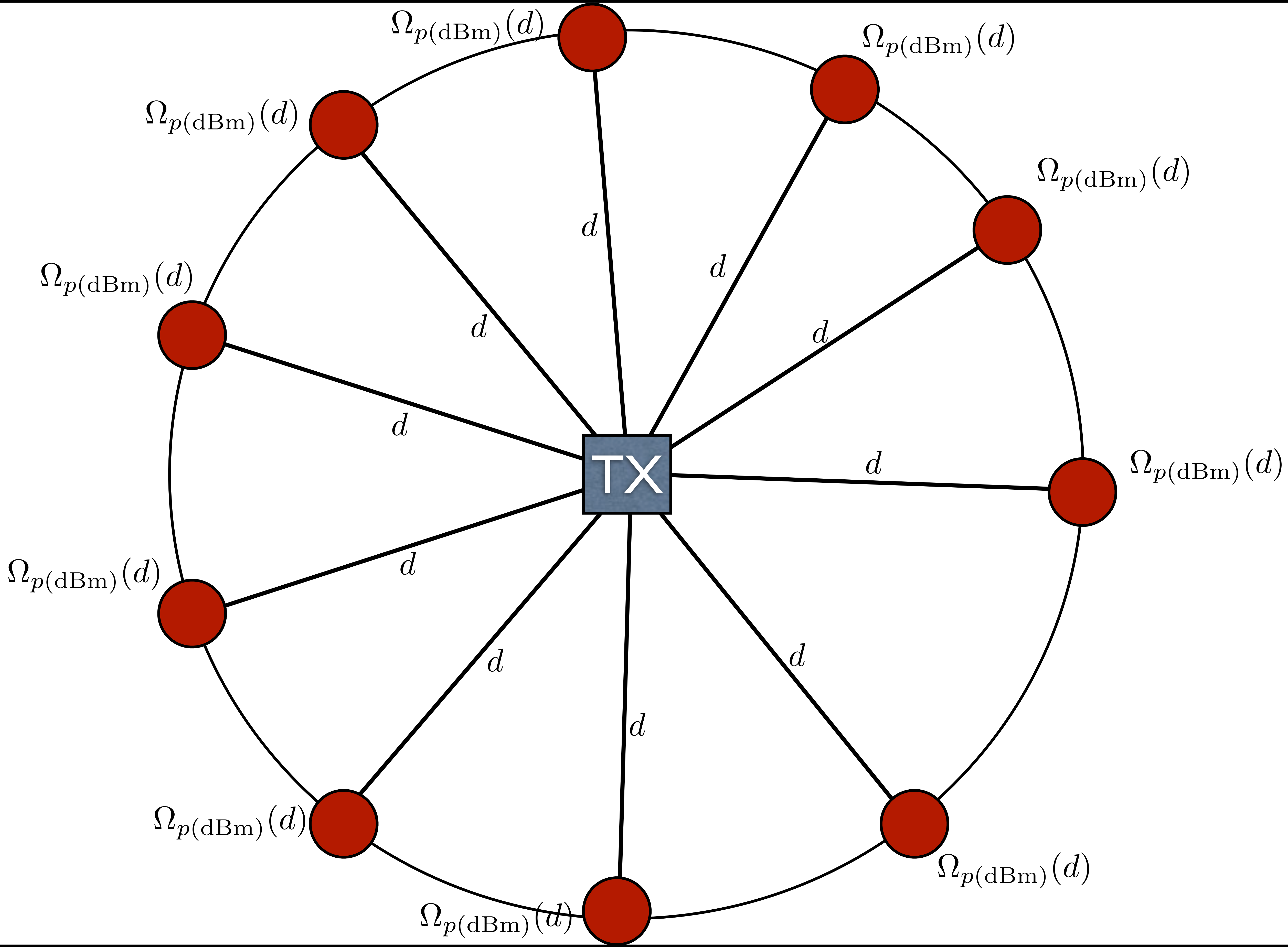
$$\mu_{\Omega_{p(\mathrm{dBm})}}(d) = \mu_{\Omega_{p(\mathrm{dBm})}}(d_0) - 10\beta \log_{10}(d/d_0) \quad (\mathrm{dBm})$$

- Local mean

Gaussian RV with zero mean and variance σ_Ω

$$\Omega_{p(\mathrm{dBm})}(d) \quad = \quad \mu_{\Omega_{p(\mathrm{dBm})}}(d) + \epsilon_{(\mathrm{dB})}$$

  - Local mean is the received power with shadowing.



TX
d
$\Omega_{p(\mathrm{dBm})}(d)$

Noise and Received Power

- Total input noise power to the receiver

$$N = kT_0 B_w F$$

- Effective received carrier power

$$\Omega_p = \frac{\Omega_t G_T G_R}{L_{R_X} L_P}$$

- Received carrier-to-noise ratio

$$\Gamma = \frac{\Omega_p}{N} = \frac{\Omega_t G_T G_R}{k T_0 B_w F L_{R_X} L_p}$$

$$\begin{aligned}
\Omega_t &= \text{transmitted carrier power} \\
G_T &= \text{transmitted power} \\
L_p &= \text{path loss} \\
G_R &= \text{receiver antenna gain} \\
\Omega_p &= \text{received signal power} \\
T_0 &= \text{receiving system noise temperature in degrees Kelvin} \\
B_w &= \text{receiver noise bandwidth} \\
N_0 &= \text{white noise power spectral density} \\
R_c &= \text{modulated symbol rate} \\
k &= 1.38 \times 10^{-23} Ws/K \text{ Boltzmann's constant} \\
F &= \text{Noise figure, typically to 5 to 6dB} \\
L_{R_X} &= \text{receiver implementation loss}
\end{aligned}$$

Link Budget

- Modulated symbol energy-to-noise ratio

$$\frac{E_c}{N_0} = \Gamma \times \frac{B_w}{R_c}$$

- Link budget is defined as the symbol energy-to-noise ratio such as

$$\frac{E_c}{N_0} = \frac{\Omega_t G_T G_R}{k T_0 R_c F L_{R_x} L_p}$$

or in decibel unit as

$$\begin{aligned}(E_c/N_0)_{(\mathrm{dB})} &= \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} \\ &- kT_{0(\mathrm{dBm})/\mathrm{Hz}} - R_{c(\mathrm{dBHz})} - F_{(\mathrm{dB})} - L_{R_x(\mathrm{dB})} - L_{p(\mathrm{dB})}\end{aligned}$$

Receiver Sensitivity

- Definition of receiver sensitivity

$$S_{R_x} = L_{R_x} k T_0 F (E_c/N_0) R_c$$

or in decibel unit as

$$S_{R_x(\mathrm{dBm})} = L_{R_x(\mathrm{dB})} + kT_{0(\mathrm{dBm})/\mathrm{Hz}} + F_{(\mathrm{dB})} + (E_c/N_0)_{(\mathrm{dB})} + R_{c(\mathrm{dBHz})}$$

- Example
  - Acceptable link quality (minimum required E_c/N_0) is given (eg. 17 dB)
  - Substitute this value into the receiver sensitivity equation.
  - Solving for $L_{p(\mathrm{dB})}$ will give maximum allowable path loss.

Maximum Allowable Path Loss

- Maximum allowable path loss

$$L_{\max\,(\mathrm{dB})} = \Omega_{t(\mathrm{dBm})} + G_{T(\mathrm{dB})} + G_{R(\mathrm{dB})} - S_{R_X\,(\mathrm{dBm})}.$$

Co-Channel Interference and Noise

- Carrier-to-noise ratio

$$\Gamma = \frac{\text{Carrier power}}{\text{Noise power}}$$

  - Thermal noise outage

$$O_N = P(\Gamma < \Gamma_{\text{th}})$$

- Carrier-to-interference ratio

$$\Lambda = \frac{\text{Carrier power}}{\text{Interference power}}$$

  - Co-channel interference outage

$$O_I = P(\Lambda < \Lambda_{\text{th}})$$

- Overall outage due to both thermal noise and co-channel interference

$$O = P(\Gamma < \Gamma_{\mathrm{th}} \ \text{or} \ \Lambda < \Lambda_{\mathrm{th}})$$

Shadow Margin

- Event of noise outage

$$\Gamma = \frac{\Omega_p(d)}{N} < \Gamma_{\text{th}}$$

or equivalently

$$\Omega_{p(\text{dBm})(d)} < \Omega_{\text{th}(\text{dBm})}$$

- Cell edge noise outage event

$$\Omega_{p(\text{dBm})(R)} < \Omega_{\text{th}(\text{dBm})}$$

- The edge noise outage probability

$$
\begin{aligned}
O_N(R) &= P(\Omega_{p(\mathrm{dBm})(R)} < \Omega_{\mathrm{th(dBm)}}) \\
&= \int_{-\infty}^{\Omega_{\mathrm{th(dBm)}}} \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ - \frac{(x - \mu_{\Omega_{p(\mathrm{dBm})}}(R)^2}{2\sigma_\Omega^2} \right\} dx
\end{aligned}
$$

$$
= Q\left(\frac{M_{\mathrm{shad}}}{\sigma_\Omega}\right)
$$

where

$M_{\mathrm{shad}} = \mu_{\Omega_{p(\mathrm{dBm})}} - \Omega_{\mathrm{th(dBm)}}$ is the Shadow margin,

and

$$
Q(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-y^2/2}\, dy.
$$

Example

Suppose that we wish to have $O_N(R) = 0.1$.
Determine the Shadow margin M_{shad}.

We solve

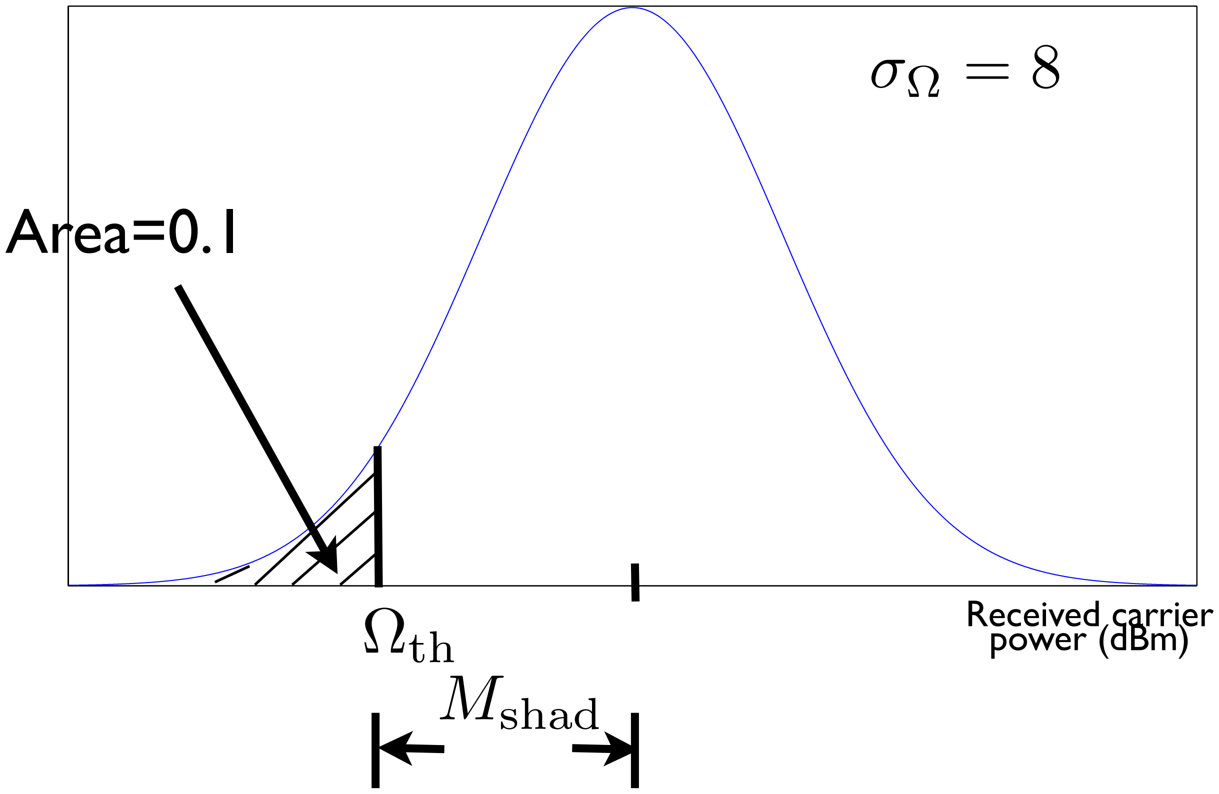
$$0.1 = Q\left(\frac{M_{\text{shad}}}{\sigma_\Omega}\right)$$

We have

$$\frac{M_{\text{shad}}}{\sigma_\Omega} = Q^{-1}(0.1) = 1.28$$

For $\sigma_\Omega = 8\,\text{dB}$, the required shadow margin i

$$M_{\text{shad}} = 1.28 \times 8 = 10.24\,\text{dB}$$



Area Outage Probability

- Area outage probability averaged over area of a cell

$$O_N = \frac{1}{\pi R^2} \int_0^R O(r) 2\pi r \, dr$$

$$= Q(X) - \exp\left\{XY + Y^2/2\right\} Q(X+Y)$$

edge noise outage probability

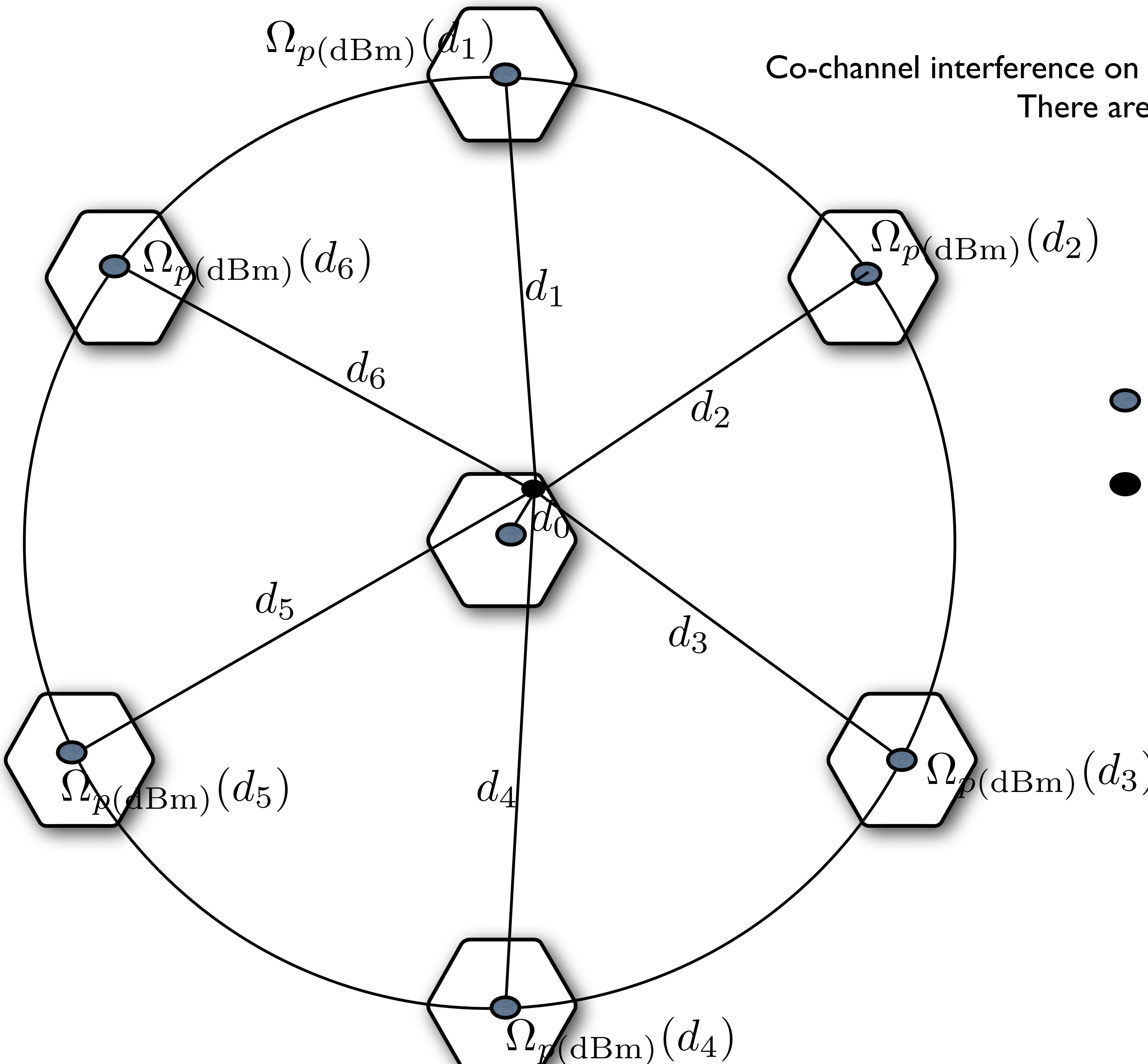
where

$$X = \frac{M_{\text{shad}}}{\sigma_\Omega},$$

$$Y = \frac{2\sigma_\Omega}{\beta\zeta}$$

where $\zeta = \frac{10}{\ln 10}$

Co-Channel Interference on the Downlink Channel



Co-channel interference on the downlink channel at a desired MS. There are six interfering BSs.

: BS

: MS

Total interference power in dBm

$$10 \log_{10} \left\{ \sum_{k=1}^{N_I} 10^{\Omega_{p(\mathrm{dBm})}(d_k)/10} \right\} .$$

Downlink Carrier-to-Interference Ratio

- Downlink carrier-to-interference ratio

$$\Lambda_{(\mathrm{dB})}(\mathbf{d}) = \Omega_{p(\mathrm{dBm})}(d_0) - 10\log_{10}\left\{\sum_{k=1}^{N_I} 10^{\Omega_{p(\mathrm{dBm})}(d_k)/10}\right\}.$$

where $\mathbf{d} = (d_0, d_1, \cdots, d_N)$

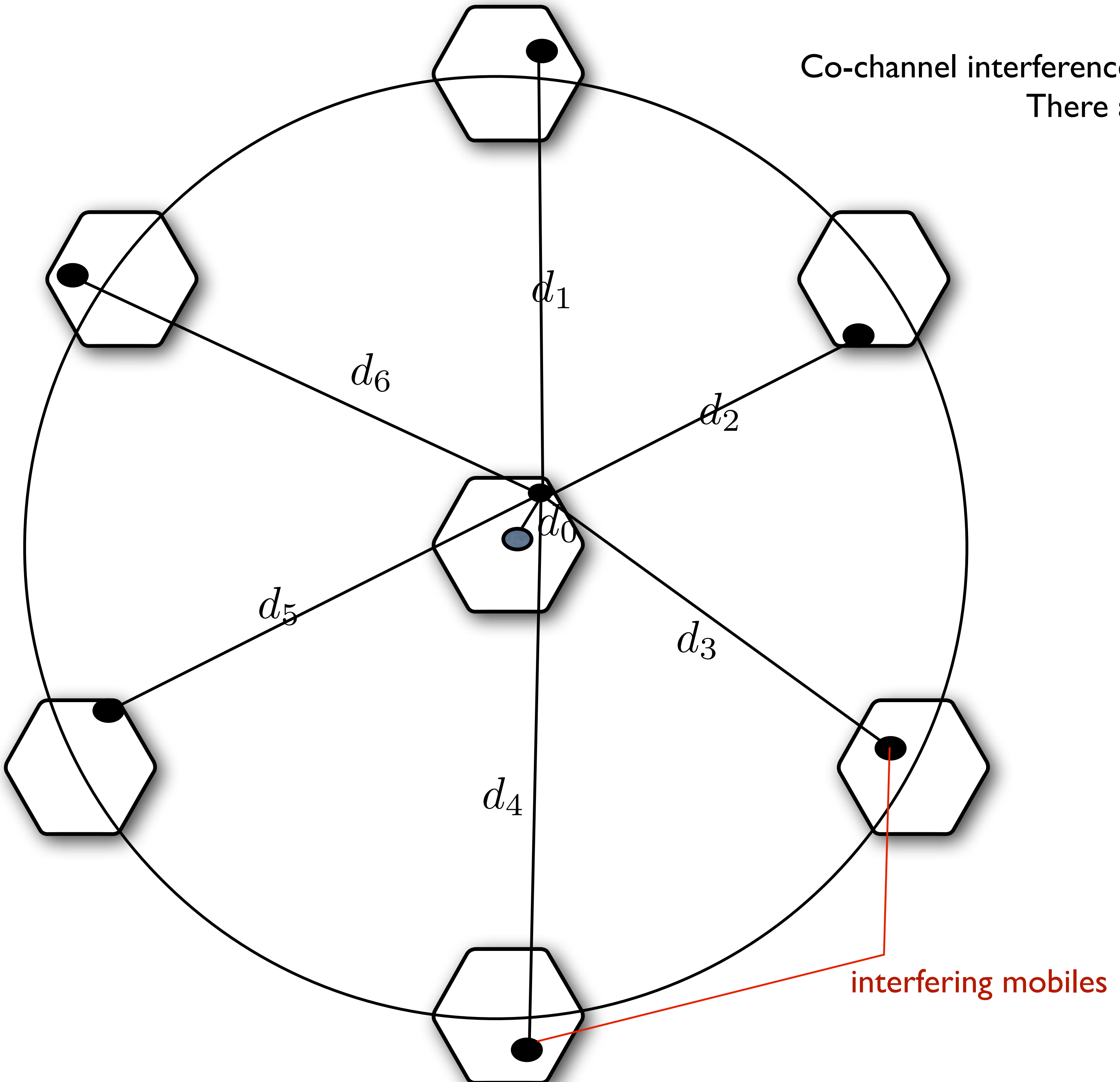
- CIR with soft handoff

$$\Lambda_{(\mathrm{dB})} = \max\left\{\Lambda_{0(\mathrm{dB})}(\mathbf{d}),\ \Lambda_{1(\mathrm{dB})}(\mathbf{d}), \cdots, \Lambda_{M(\mathrm{dB})}(\mathbf{d})\right\}$$

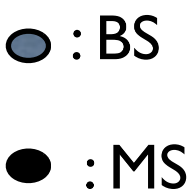
- Area averaged probability co-channel interference outage

$$O_I = P\left(\Lambda_{(\mathrm{dB})} < \Lambda_{\mathrm{th}(\mathrm{dB})}\right)$$

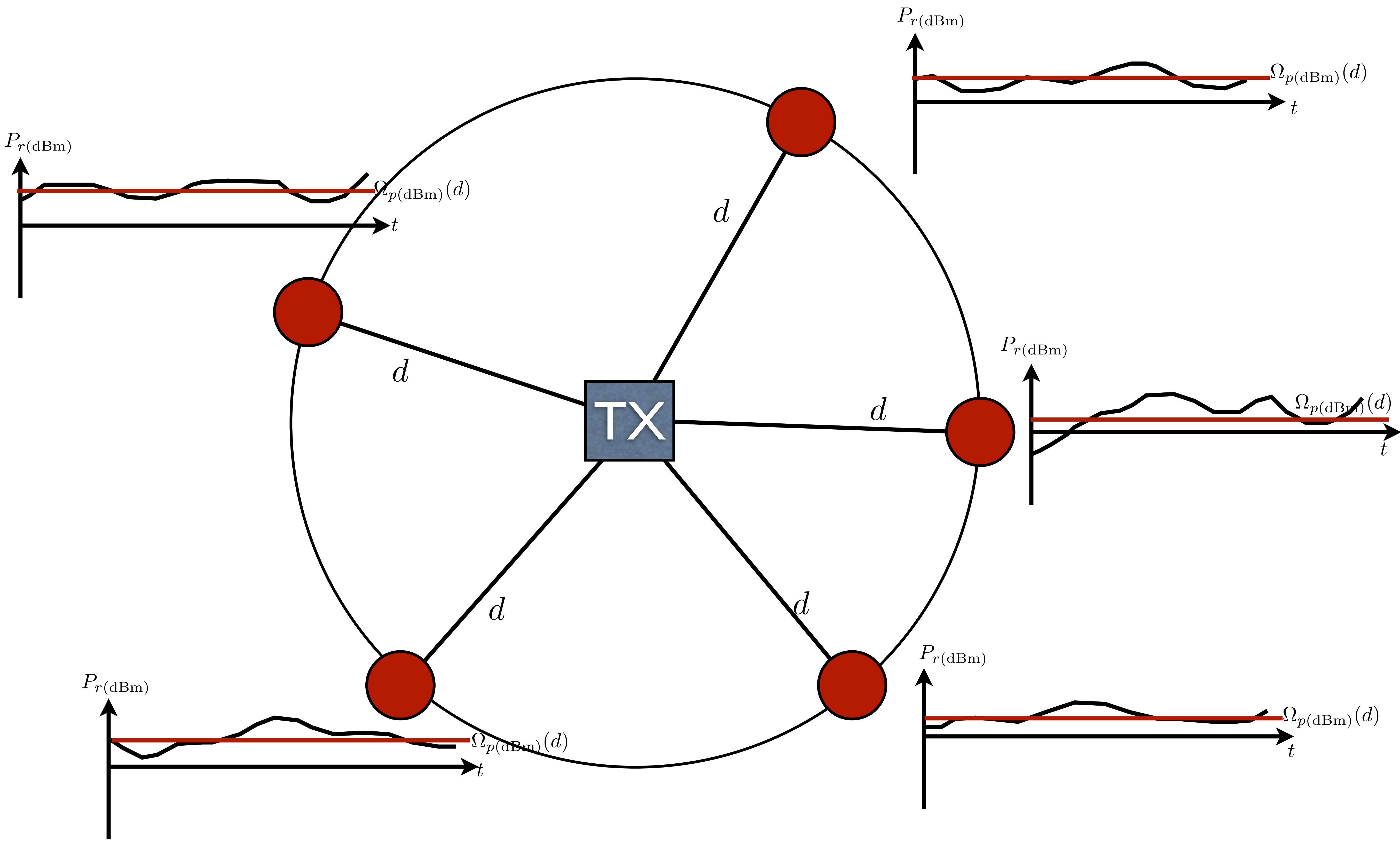
Co-Channel Interference on the Uplink Channel

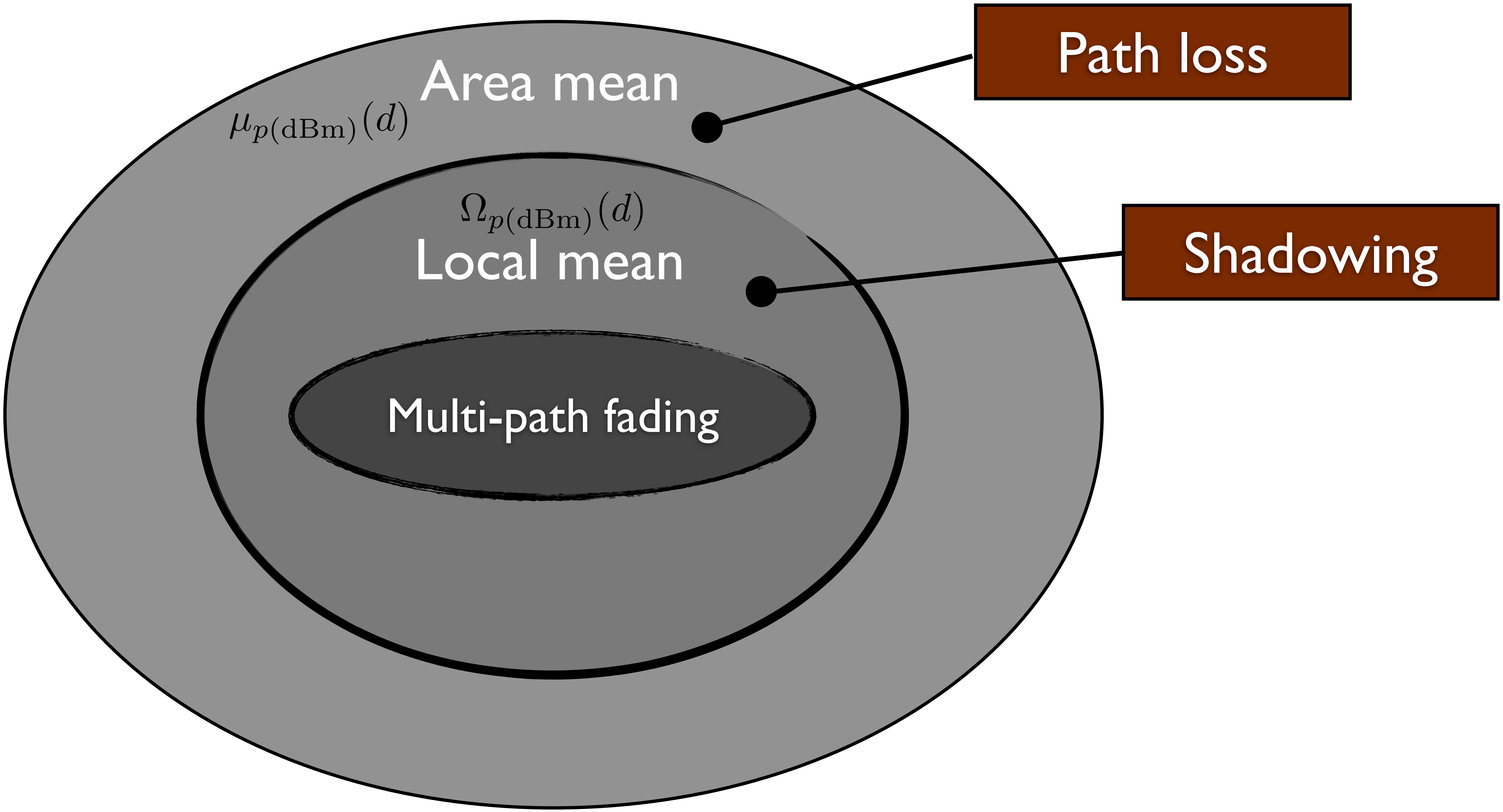


Co-channel interference on the uplink channel at a desired BS.
There are six interfering MSs.



PROPAGATION MODELING

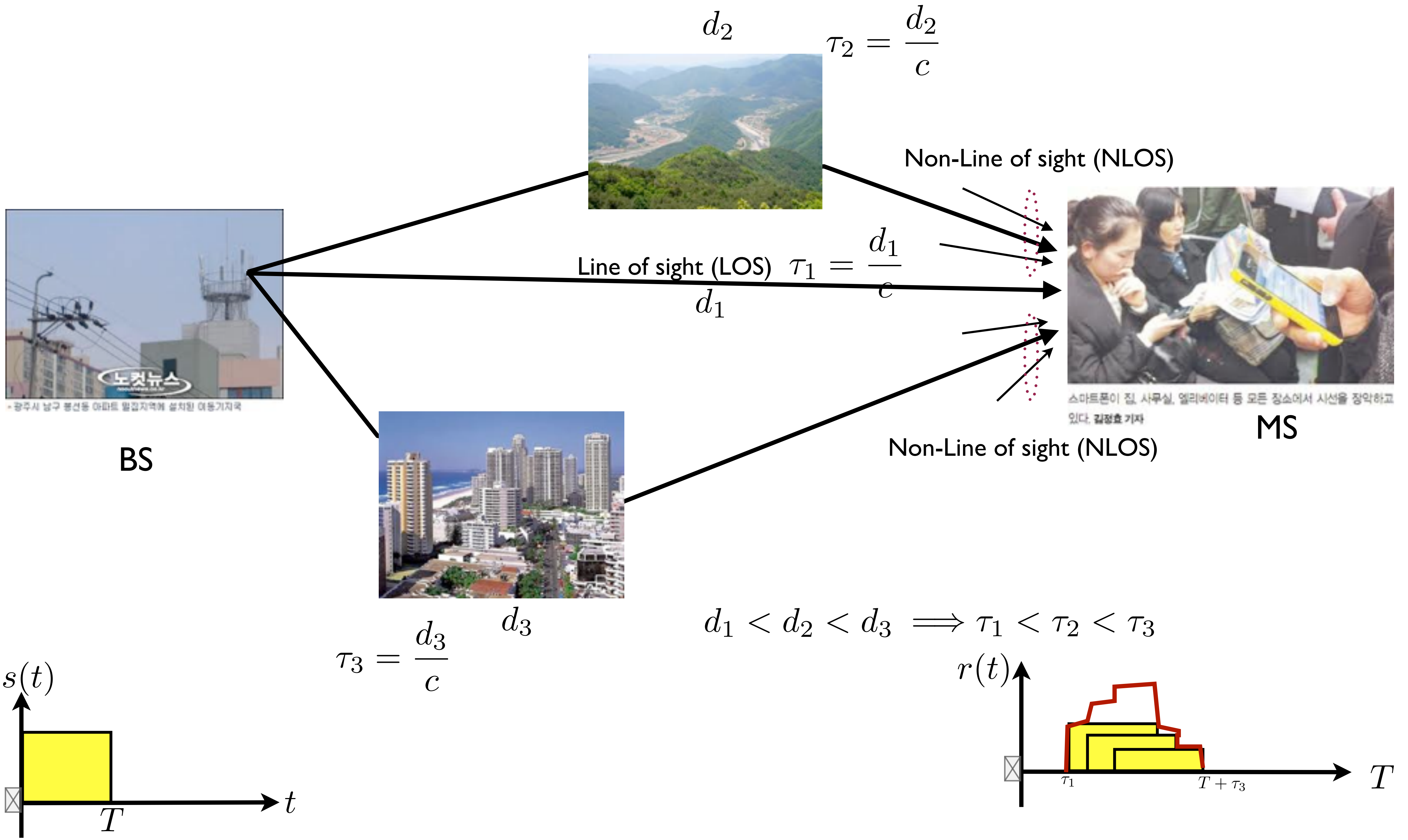

TX
d
d
d
d
d
$P_{r(\mathrm{dBm})}$
$\Omega_{p(\mathrm{dBm})}(d)$
t


Area mean
$\mu_{p(\mathrm{dBm})}(d)$
$\Omega_{p(\mathrm{dBm})}(d)$
Local mean
Multi-path fading
Path loss
Shadowing

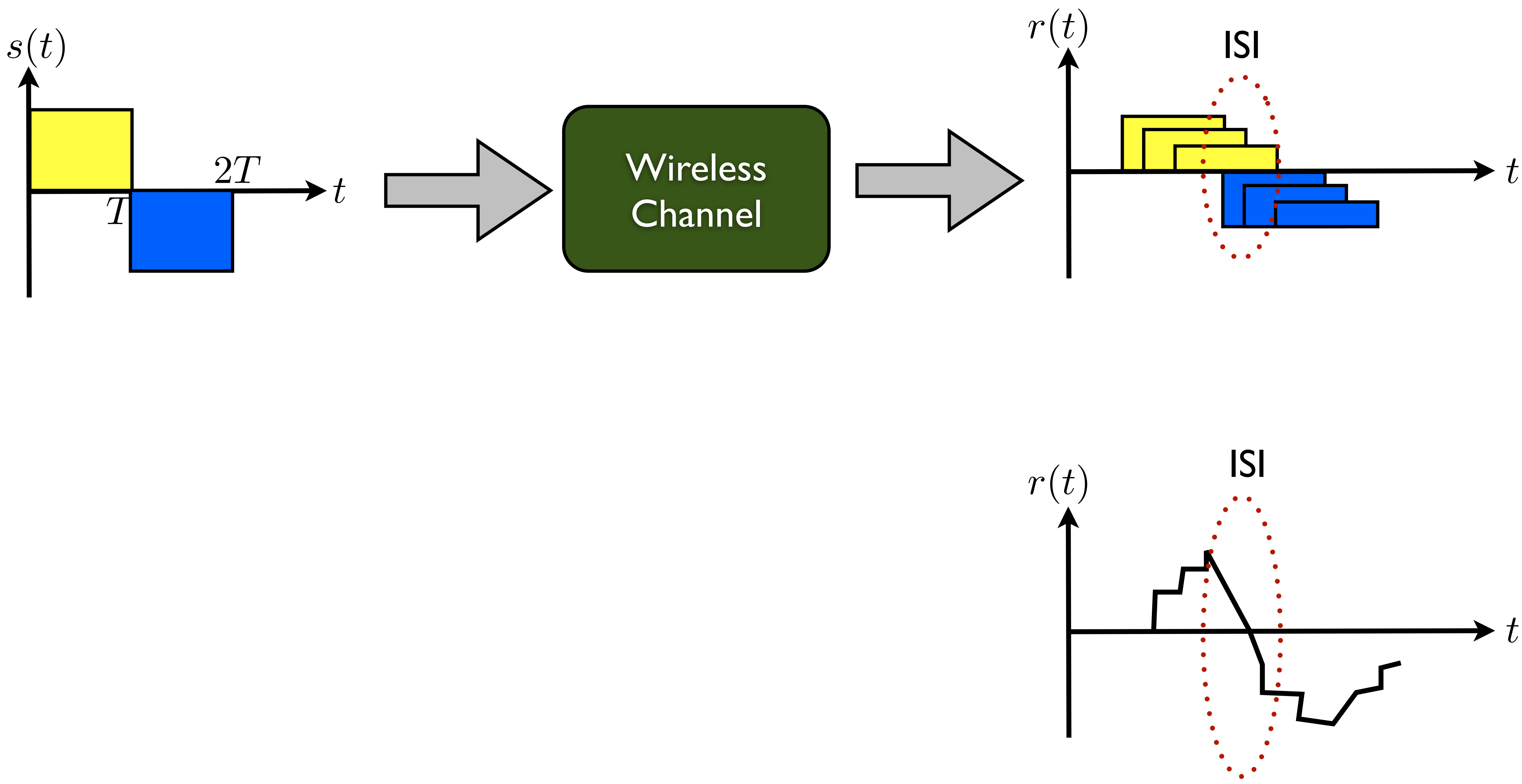
Multi-path Fading

- Categorization of multi-path fading
  - Depending on the vehicle speed
    - Fast fading vs. slow fading
  - Depending on the channel bandwidth and the signal bandwidth
    - Frequency flat fading vs. frequency selective fading

Multi-Path Phenomenon

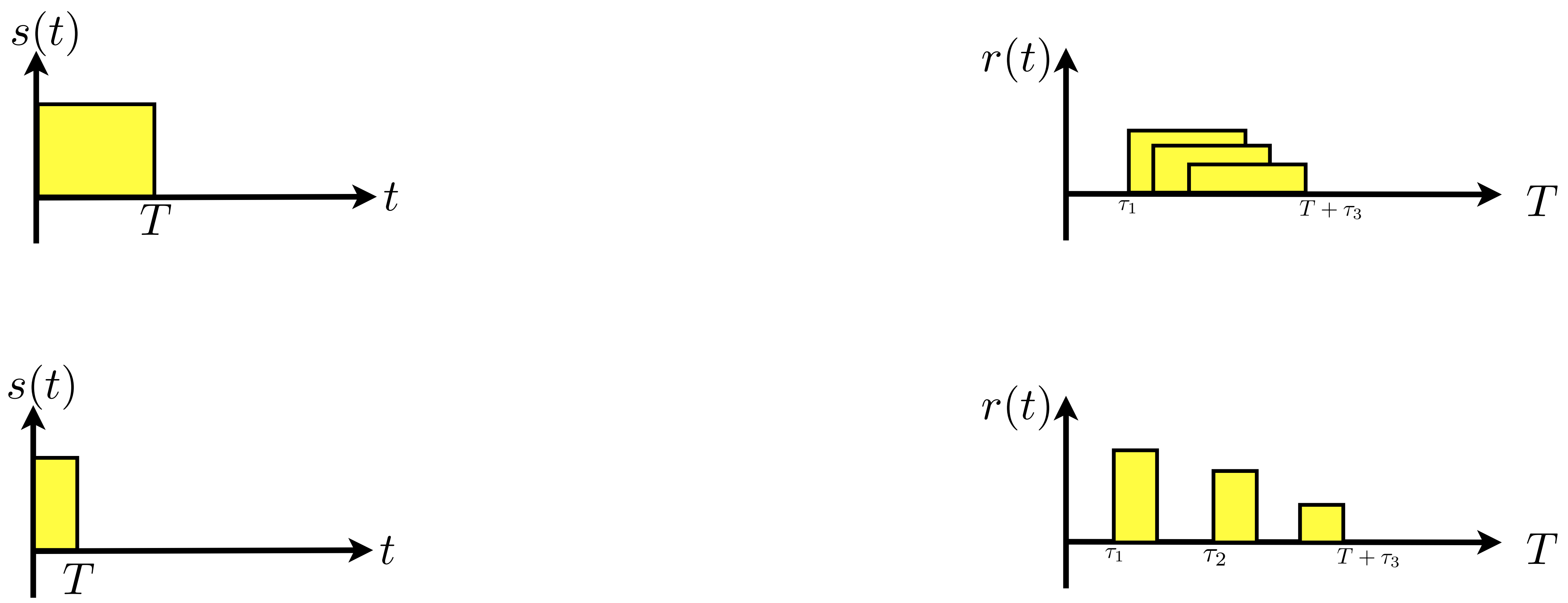


Inter-Symbol Interference due to Multi-Path Fading



Effect of Data Rate (or Bandwidth)

- Smaller time duration of the transmitted signal => higher data rate



ISI is getting severer when the data rate gets higher.

Wireless Channels

Wireless Channel

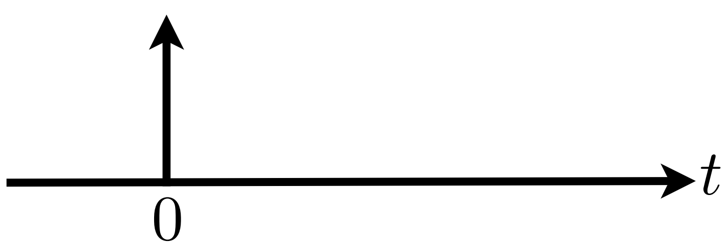
Received signal power

- *Time-varying random signal*
- *multi-path signal*

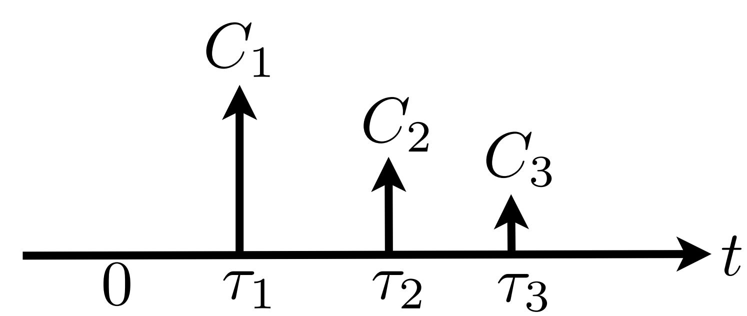
Impulse Response of the Wireless Channels

Equivalent low-pass signal and system representation

$$s(t) = \Re[\tilde{s}(t)e^{j2\pi f_c t}] \longrightarrow \boxed{\text{Wireless Channels} \quad h(t) = 2\Re[\tilde{h}(t)e^{j2\pi f_c t}]} \longrightarrow r(t) = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$$



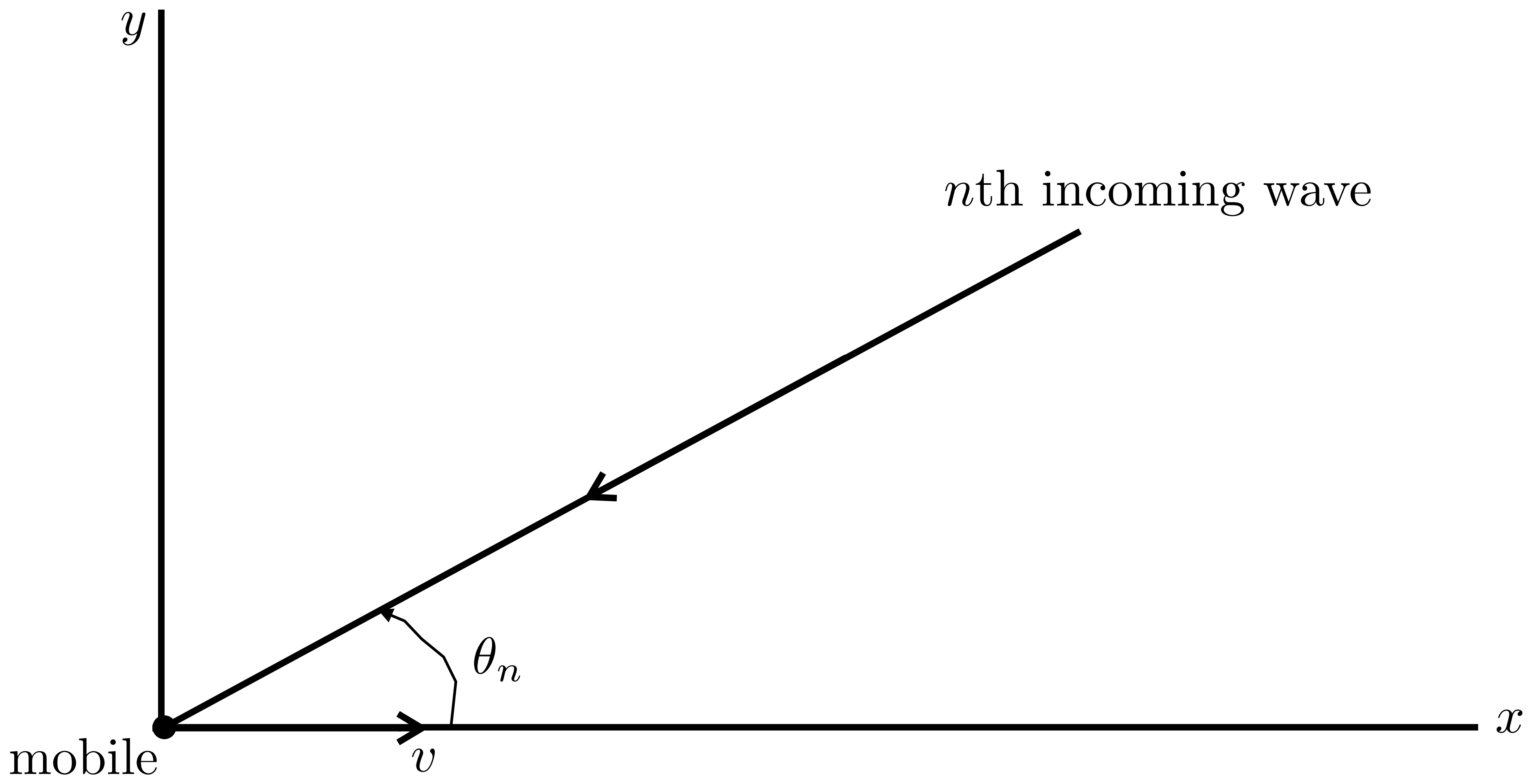
$$\tilde{r}(t) = \tilde{s}(t) * \tilde{h}(t)$$



In practice, there are many multiple paths arriving at the receiver with random amplitude and phase.

$$\tilde{h}(t,\tau) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \delta(\tau - \tau_n)$$

Doppler Effect



Doppler frequency

$$f_{D,n} = f_m \cos\theta_n \ \text{Hz}$$

where $f_m = \dfrac{v}{\lambda_c}$ is maximum Doppler frequency.

Frequency Shift Due to Doppler Effect

Example: $f_c = 2\,\mathrm{GHz}$ $\lambda_c = 15\,\mathrm{cm}$

Vehicle speed (km/hr)	Maximum Doppler freq. (Hz)	Frequency shift
3	5.56	2GHz + 5.56 Hz
30	55.56	2GHz + 55.56 Hz
60	111.11	2GHz + 111.11 Hz
120	222.22	2GHz + 222.22 Hz
300	555.56	2GHz + 555.56 Hz

Received Signal with Doppler Frequency

- Transmitted signals

$$s(t) = \Re[\tilde{s}(t)e^{j2\pi f_c t}]$$

- Received signals

$$r(t) = \Re\left[\sum_{n=1}^{N} C_n e^{j2\pi[(f_c+f_{D,n})(t-\tau_n)]}\tilde{s}(t-\tau_n)\right] = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$$

where the received complex envelope is

$$\tilde{r}(t) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)}\tilde{s}(t-\tau_n)$$

and

$$\phi_n(t) = 2\pi\left\{(f_c+f_{D,n})\tau_n - f_{D,n}t\right\}$$

Channel Response

- Channel impulse response

$$h(t,\tau) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \delta(\tau - \tau_n)$$

where $\phi_n(t) = 2\pi \left\{ (f_c + f_{D,n})\tau_n - f_{D,n} t \right\}$

- Flat fading

$\tau_i - \tau_j \approx 0$ or equivalently $|\tau_N - \tau_1| << T$ so that $\tau_i \approx \hat{\tau}, \quad \text{for all } n$

  - In this case, the effect of ISI is negligible.

$$h(t,\tau) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \delta(\tau - \hat{\tau}) = h(t)\delta(\tau - \hat{\tau})$$

where $h(t) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)}$

Channel Response of Flat Fading

- Channel impulse response

$$h(t,\tau) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \delta(\tau - \hat{\tau}) = h(t)\delta(\tau - \hat{\tau})$$

  - Now we can just omit $\delta(\tau - \hat{\tau})$ without loss of generality such as

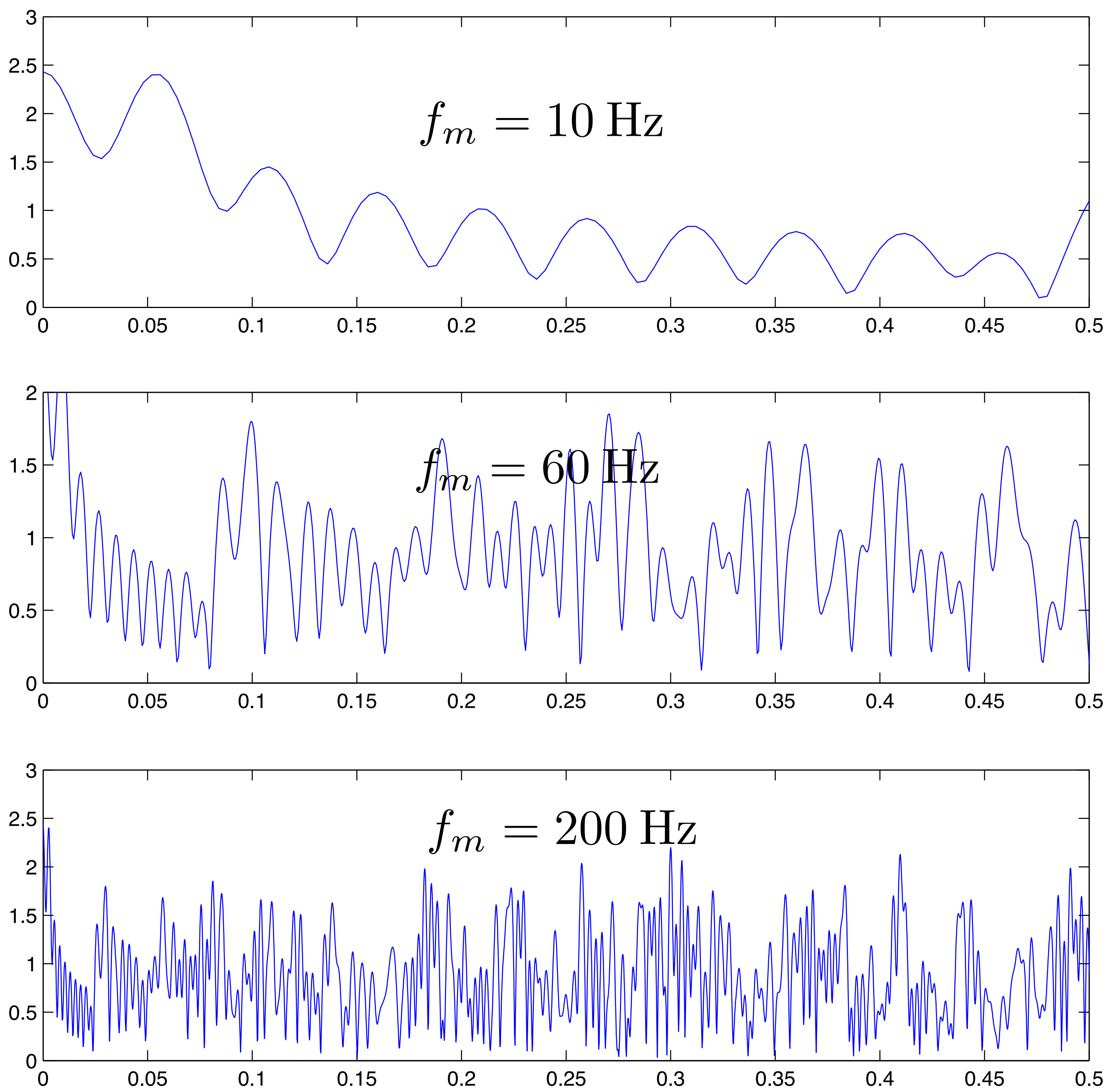
$$h(t,\tau) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} = \sum_{n=1}^{N} C_n \cos\phi_n(t) - j\sum_{n=1}^{N} C_n \sin\phi_n(t)$$

$$= h_I(t) + j h_Q(t)$$

where

$$\begin{aligned} h_I(t) &= \sum_{n=1}^{N} C_n \cos\phi_n(t) \\ h_Q(t) &= -\sum_{n=1}^{N} C_n \sin\phi_n(t) \end{aligned}$$

Fast vs. Slow Fading



Plot of Envelope

$$|h(t)| = \sqrt{h_I^2(t) + h_Q^2(t)}$$

Received Signal over Flat Fading Channel

Equivalent low-pass signal and system representation

$$\tilde{r}(t) = \tilde{s}(t) * \tilde{h}(t)$$

$$s(t) = \Re[\tilde{s}(t)e^{j2\pi f_c t}]$$

Wireless Channels

$$h_p(t) = 2\Re[h(t)e^{j2\pi f_c t}]$$

$$r(t) = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$$

$$\tilde{r}(t) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \tilde{s}(t - \tau_n) = \sum_{n=1}^{N} C_n e^{-j\phi_n(t)} \tilde{s}(t - \hat{\tau})$$

Let $\tilde{s}(t) = x(t) + jy(t)$.

Then $r(t) = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$

$$h(t) = h_I(t) + jh_Q(t)$$

$$r(t) = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$$

$$h(t) = h_I(t) + jh_Q(t)$$

$$\tilde{r}(t)$$

$$= \Re\left\{[h_I(t) + jh_Q(t)][x(t) + jy(t)]e^{j2\pi f_c t}\right\}$$

$$= [h_I(t)x(t) - h_Q(t)y(t)]\cos 2\pi f_c t - [h_I(t)y(t) + h_Q(t)x(t)]\sin 2\pi f_c t$$

Received Signal Correlation

- Let us consider the transmitted signal

$$\tilde{s}(t) = 1$$

  - Then the received signal over flat fading channel is

$$r(t) = h_I(t)\cos 2\pi f_c t - h_Q(t)\sin 2\pi f_c t$$

    - where

$$\begin{aligned} h_I(t) &= \sum_{n=1}^{N} C_n \cos\phi_n(t) \\ h_Q(t) &= -\sum_{n=1}^{N} C_n \sin\phi_n(t) \end{aligned}$$

Autocorrelation

$$\phi_{rr}(\tau) = E[r(t)r(t+\tau)]$$

$$= E[(h_I(t)\cos 2\pi f_c t - h_Q(t)\sin 2\pi f_c t)(h_I(t+\tau)\cos 2\pi f_c(t+\tau) - h_Q(t)\sin 2\pi f_c(t+\tau))]$$

$$= E[h_I(t)h_I(t+\tau)\cos 2\pi f_c t\cos 2\pi f_c(t+\tau)] - E[h_I(t)h_Q(t+\tau)\cos 2\pi f_c t\sin 2\pi f_c(t+\tau)]$$

$$-E[h_Q(t)h_I(t+\tau)\sin 2\pi f_c t\cos 2\pi f_c(t+\tau)] + E[h_Q(t)h_Q(t+\tau)\sin 2\pi f_c t\sin 2\pi f_c(t+\tau)]$$

Trigonometric identities:

$$\cos A\cos B = \frac{\cos(A-B)+\cos(A+B)}{2}$$

$$\sin A\sin B = \frac{\cos(A-B)-\cos(A+B)}{2}$$

$$\sin A\cos B = \frac{\sin(A+B)+\sin(A-B)}{2}$$

$$\cos A\sin B = \frac{\sin(A+B)-\sin(A-B)}{2}$$

$$\begin{aligned}
\phi_{rr}(\tau) &= \frac{1}{2}\{\underbrace{E[h_I(t)h_I(t+\tau)]}_{=\ \phi_{h_I h_I}(\tau)}\cos 2\pi f_c\tau + \underbrace{E[h_I(t)h_I(t+\tau)]}_{=\ \phi_{h_I h_I}(\tau)}\cos 2\pi(2f_c t+\tau)\} \\
&+ \frac{1}{2}\{\underbrace{E[h_I(t)h_Q(t+\tau)]}_{=\ \phi_{h_I h_Q}(\tau)}\sin 2\pi f_c\tau - \underbrace{E[h_I(t)h_Q(t+\tau)]}_{=\ \phi_{h_I h_Q}(\tau)}\sin 2\pi(2f_c t+\tau)\} \\
&+ \frac{1}{2}\{\underbrace{E[h_Q(t)h_I(t+\tau)]}_{=\ \phi_{h_Q h_I}(\tau)}\sin 2\pi f_c\tau - \underbrace{E[h_Q(t)h_I(t+\tau)]}_{=\ \phi_{h_Q h_I}(\tau)}\sin 2\pi(2f_c t+\tau)\} \\
&+ \frac{1}{2}\{\underbrace{E[h_Q(t)h_Q(t+\tau)]}_{=\ \phi_{h_Q h_Q}(\tau)}\cos 2\pi f_c\tau - \underbrace{E[h_Q(t)h_Q(t+\tau)]}_{=\ \phi_{h_Q h_Q}(\tau)}\cos 2\pi(2f_c t+\tau)\}
\end{aligned}$$

Assuming $h_I(t)$ and $h_Q(t)$ are wide sense stationary process and thus $r(t)$ is also WSS, then we can show

$$\begin{aligned}
\phi_{h_I h_I}(\tau) &= \phi_{h_Q h_Q}(\tau), \\
\phi_{h_I h_Q}(\tau) &= \phi_{h_Q h_I}(-\tau).
\end{aligned}$$

Then, we can write

$$\phi_{rr}(\tau) \quad = \quad E[h_I(t)h_I(t+\tau)]\cos 2\pi f_c\tau - E[h_Q(t)h_I(t+\tau)]\sin 2\pi f_c\tau$$

where

$$\begin{aligned}\phi_{h_I h_I}(\tau) &= \phi_{h_Q h_Q}(\tau),\\ \phi_{h_I h_Q}(\tau) &= \phi_{h_Q h_I}(-\tau).\end{aligned}$$

Assumption

$\phi_n(t)$ and $\phi_m(t)$ are independent

$\phi_n(t)$ are uniformly distributed RV over $[-\pi, \pi]$

$$h_I(t) = \sum_{n=1}^{N} C_n \cos\phi_n(t)$$

$$h_Q(t) = -\sum_{n=1}^{N} C_n \sin\phi_n(t)$$

$$\phi_n(t) = 2\pi\left\{(f_c + f_{D,n})\tau_n - f_{D,n}t\right\} \qquad \rightarrow\ f_c\tau_n >> 1$$

$$f_{D,n} = f_m\cos\theta_n \ \text{Hz}$$

$$\begin{aligned}\phi_{h_I h_I}(\tau) &= E_{\tau,\theta}[h_I(t)h_I(t+\tau)] \\ &= E\left[\sum_{n=1}^{N}\sum_{m=1}^{N} C_n C_m \cos\phi_n(t)\cos\phi_m(t)\right] \\ &= \sum_{n=1}^{N} C_n^2 E[\cos\phi_n(t)\cos\phi_n(t+\tau)]\end{aligned}$$

where
$$\begin{aligned}\phi_n(t) &= 2\pi\{(f_c + f_{D,n})\tau_n - f_{D,n}t\} \\ \phi_n(t+\tau) &= 2\pi\{(f_c + f_{D,n})\tau_n - f_{D,n}(t+\tau)\}\end{aligned}$$

and using the independence,

we have
$$E[\cos\phi_n(t)\cos\phi_n(t+\tau)] = E_\theta[\cos(2\pi f_m \tau\cos\theta)]$$

Then the auto-correlation can be written as

$$\phi_{h_I h_I}(\tau) = \frac{\Omega_p}{2} E_\theta\left[\cos(2\pi f_m \tau\cos\theta)\right]$$

where $\Omega_p = \sum_{n=1}^{N} C_n^2$

In a similar way, we obtain

$$\begin{aligned}\phi_{h_I h_Q}(\tau) &= E[h_I(t)h_Q(t+\tau)] \\ &= \frac{\Omega_p}{2} E_\theta[\sin(2\pi f_m \tau \cos\theta)]\end{aligned}$$

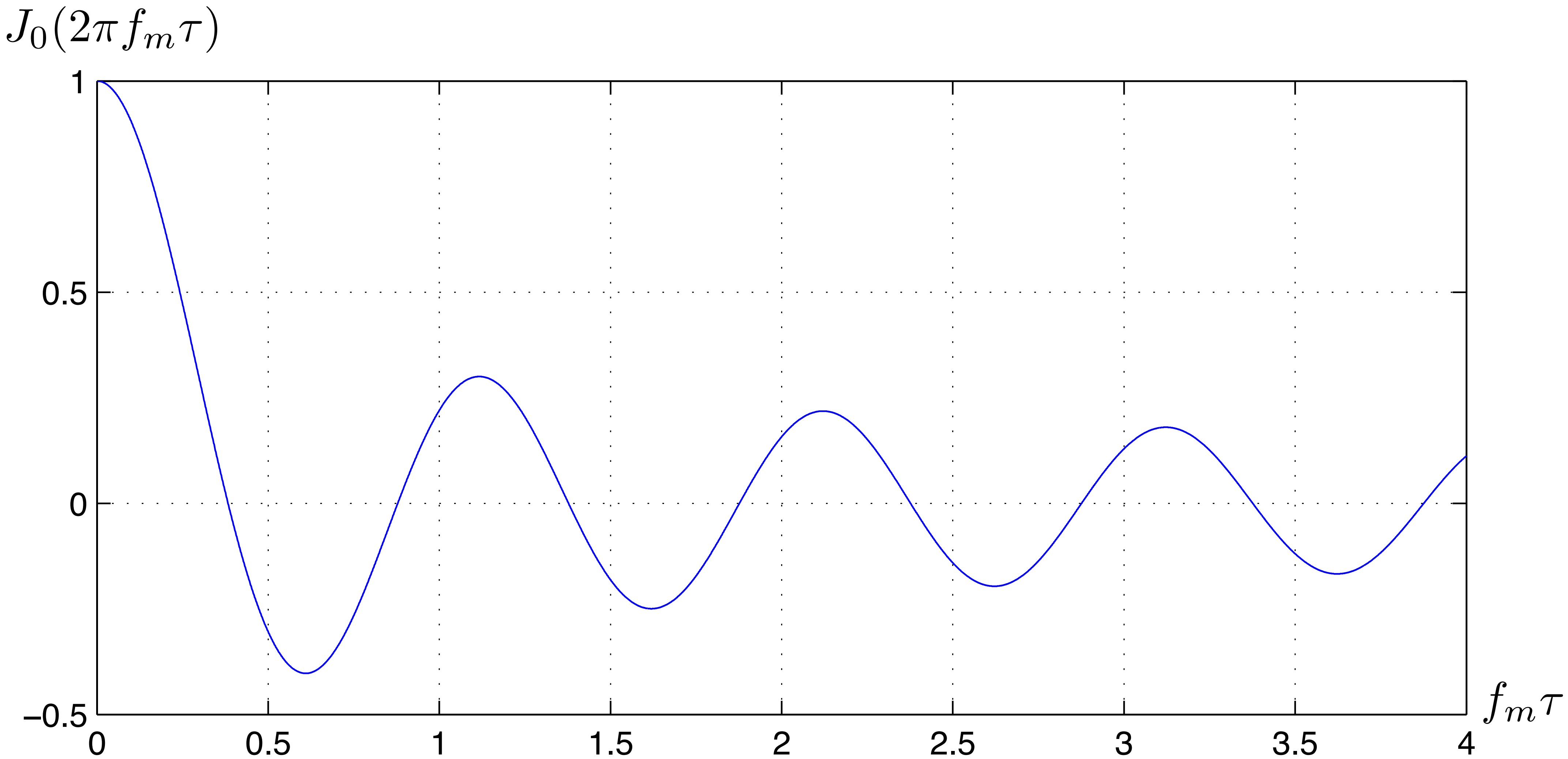
$$\begin{aligned}\phi_{h_I h_I}(\tau) &= \frac{\Omega_p}{2}\int_{-\pi}^{\pi} \cos(2\pi f_m \tau \cos\theta) p_\theta(\theta)\, d\theta \\ &= \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi} \cos(2\pi f_m \tau \cos\theta)\, d\theta \\ &= \frac{\Omega_p}{2}\frac{1}{\pi}\int_{0}^{\pi} \cos(2\pi f_m \tau \cos\theta)\, d\theta \quad = \quad \frac{\Omega_p}{2} J_0(2\pi f_m \tau)\end{aligned}$$

Zero-order Bessel function of the first kind

$$\phi_{h_I h_Q}(\tau) = \frac{\Omega_p}{2}\frac{1}{2\pi}\int_{-\pi}^{\pi} \sin(2\pi f_m \tau \cos\theta)\, d\theta = 0$$

Hence,

$$
\begin{aligned}
\phi_{rr}(\tau) &= \phi_{h_I h_I}(\tau)\cos 2\pi f_c\tau - \phi_{h_Q h_I}(\tau)\sin 2\pi f_c\tau \\
&= \frac{\Omega_p}{2} J_0(2\pi f_m\tau)\cos(2\pi f_c\tau)
\end{aligned}
$$

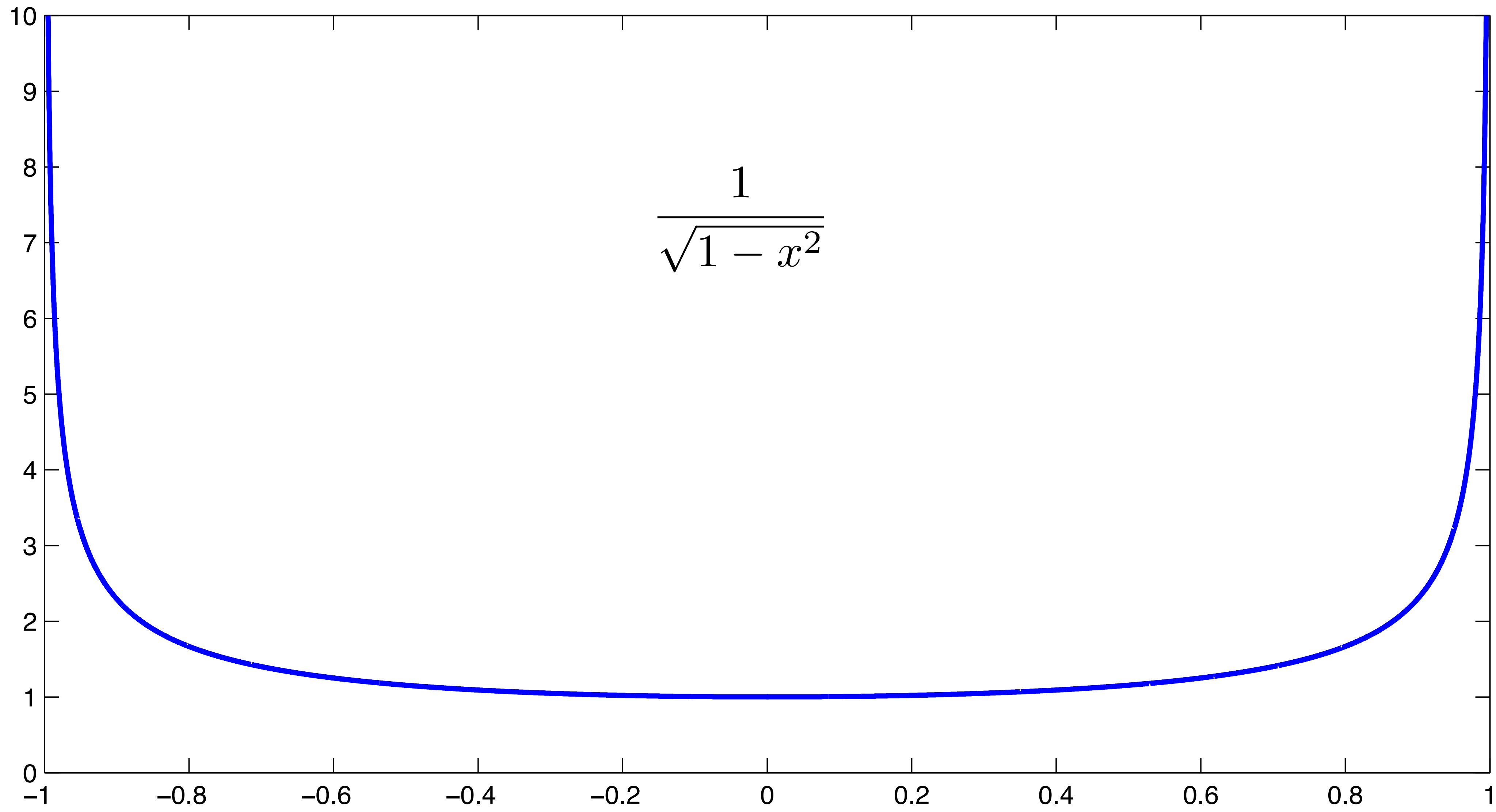


Power Spectral Density (PSD)

$$
\begin{aligned}
S_{h_I h_I}(f) &= \mathcal{F}[\phi_{h_I h_I}(\tau)] \\
&= \begin{cases} \frac{\Omega_p}{2\pi f_m} \frac{1}{1-(f/f_m)^2}, & |f| \le f_m \\ 0, & \text{otherwise.} \end{cases}
\end{aligned}
$$

$$
\phi_{rr}(\tau) = \Re\left[\phi_{h_I h_I}(\tau) e^{j2\pi f_c \tau}\right]
$$

$$
\begin{aligned}
S_{rr}(f) &= \frac{1}{2}\left[S_{hh}(f-f_c) + S_{hh}(-f-f_c)\right] \\
&= \frac{\Omega_p}{4\pi f_m} \frac{1}{\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}, \qquad |f-f_c| \le f_m.
\end{aligned}
$$


$\frac{1}{\sqrt{1-x^2}}$
10
9
8
7
6
5
4
3
2
1
0
–1
–0.8
–0.6
–0.4
–0.2
0
0.2
0.4
0.6
0.8
1

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HW #1
Wireless Communications System

Due by March 19, Tuesday.

Problem 1. By using geometric arguments, show that the co-channel refuse factor for cellular deployments based on hexagonal cells is given by

$$Q = D/R = \sqrt{3N}.$$

Problem 2. Show that the area noise outage probability is given by

$$O_N = Q(X) - \exp\{XY + Y^2/2\}Q(X+Y)$$

where $X = \frac{M_{\text{shad}}}{\sigma_\Omega}$ and $Y = \frac{2\sigma_\Omega}{\beta\zeta}$.

Problem 3. Consider the worst case forward channel co-channel interference situation shown in Fig. 1. The path loss is described by the following simple model:

$$\mu_{\Omega_p} = \frac{\Omega_t (h_b h_m)^2}{d^4}$$

where

$$\begin{aligned} \mu_{\Omega_p} &= \text{received power} \\ \Omega_t &= \text{transmitted power} \\ h_b &= \text{base station antenna height} \\ h_m &= \text{mobile station antenna height} \\ d &= \text{radio path length} \end{aligned}$$

(a) Assuming that $h_b = 30$ m, $h_m = 1.5$ m, and all BS transmit powers are the same what is the worst case Λ for a cluster size $N = 4$?

(b) Now suppose that the antenna height of the serving BS (in the center) is increased to 40m while the other BS antenna heights remain at 30 m. This has the effect of enlarging the center cell. Assuming that we wish to maintain the same worst case Λ value obtained in part a), what is the new radius of the center cell?

(c) Now suppose that the antenna height of one of the co-channel BSs is increased to 40m while the antenna heights of the other BSs antenna heights, including the serving BS, remain at 30m. This has the effect of shrinking the center cell and making it a non-regular hexagon. Assuming, again, that we wish to maintain the same worst case Λ value obtained in part a), what are the new dimensions of the center cell?

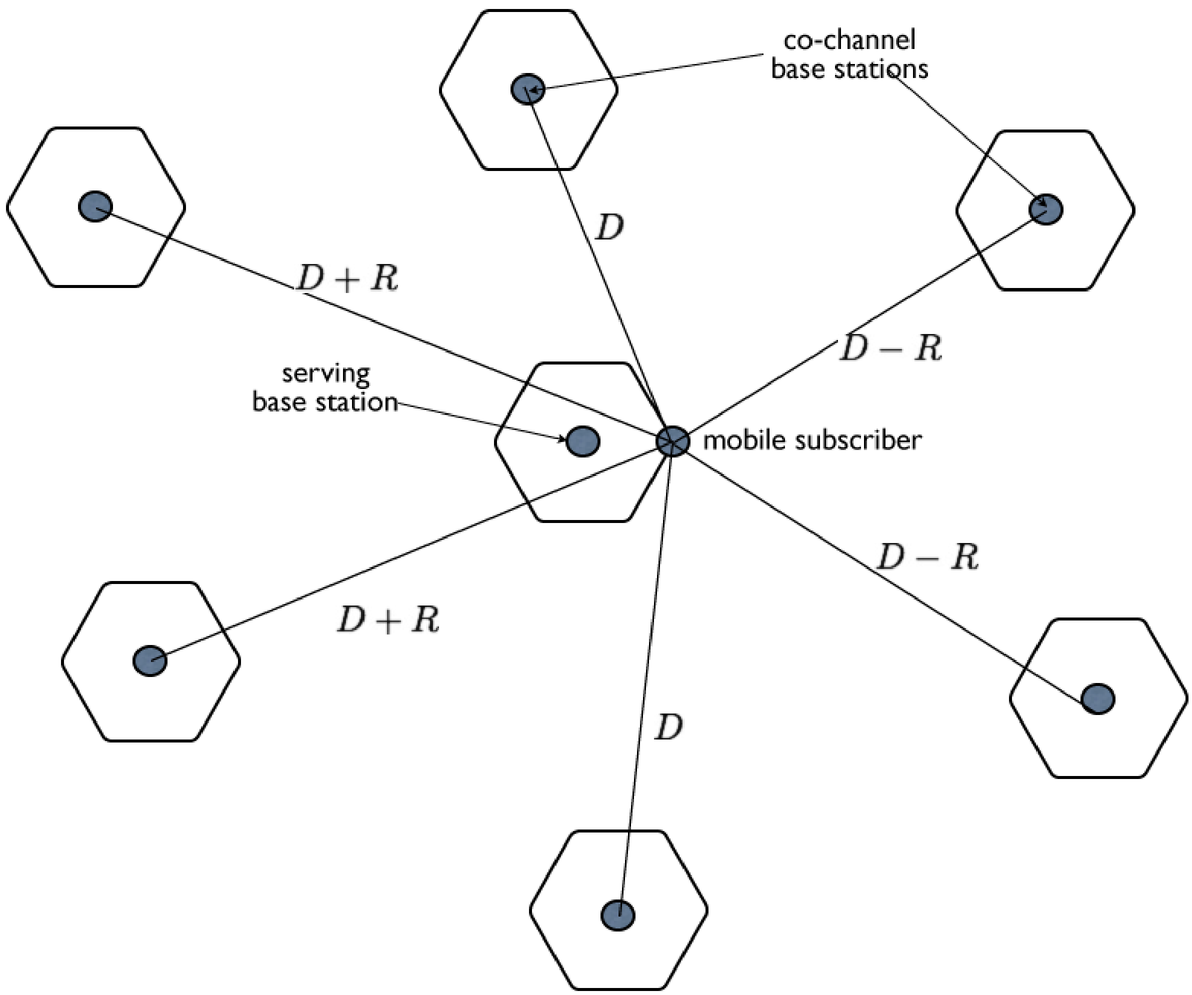

Figure 1: Worst case co-channel interference on the forward channel