Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Wavelength and frequency

$$y(x,t) = y_m \sin(kx - \omega t)$$

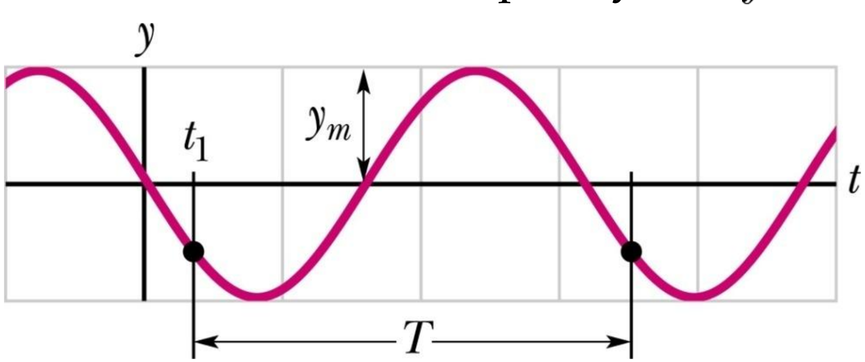
displacement — amplitude — angular wave number — ang. freq. — phase

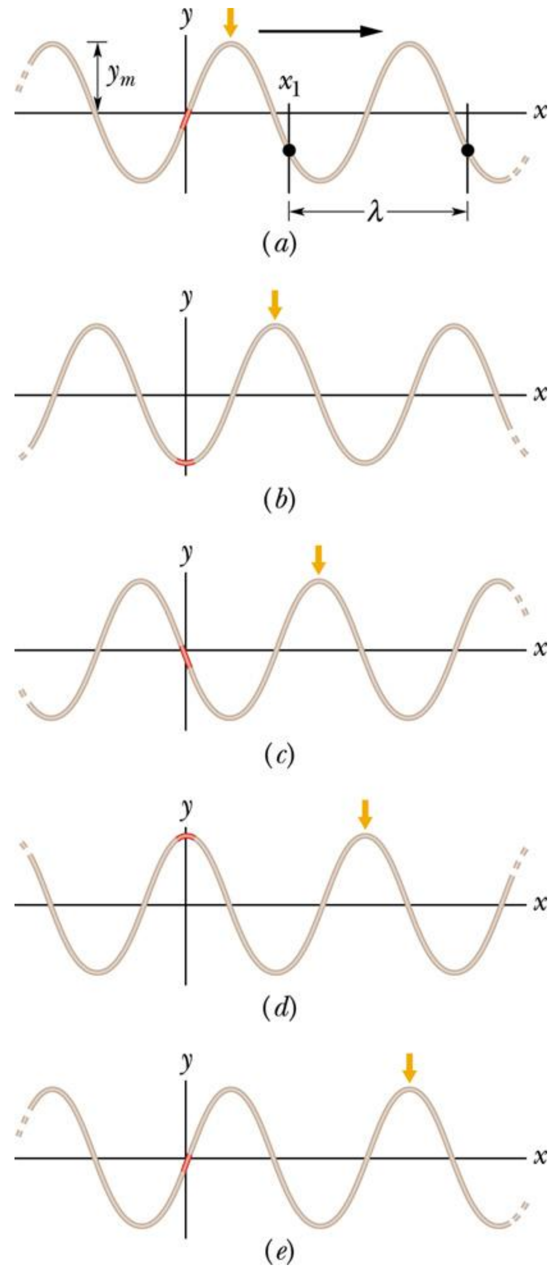
Angular wave number $k = \dfrac{2\pi}{\lambda}$

Angular frequency $\omega = \dfrac{2\pi}{T}$

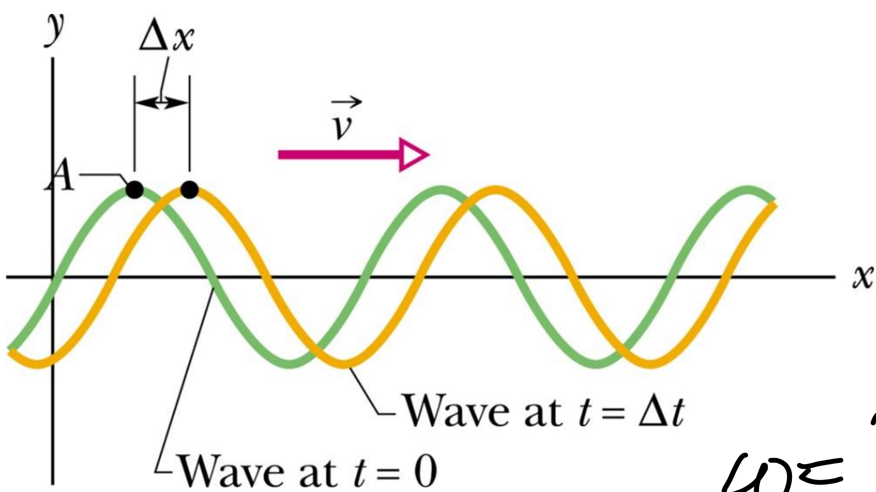
$$k\lambda = 2\pi = \omega T$$

frequency $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$





Speed of a travelling wave



$$kx - \omega t = \text{constant}$$

$$k\frac{dx}{dt} - \omega = 0$$

$$v = \frac{dx}{dt} = \frac{\omega}{k}$$

$\omega = \frac{2\pi}{T}$

phase velocity

$$y(x,t) = y_m \sin(kx - \omega t)$$

$k = \frac{2\pi}{\lambda}$

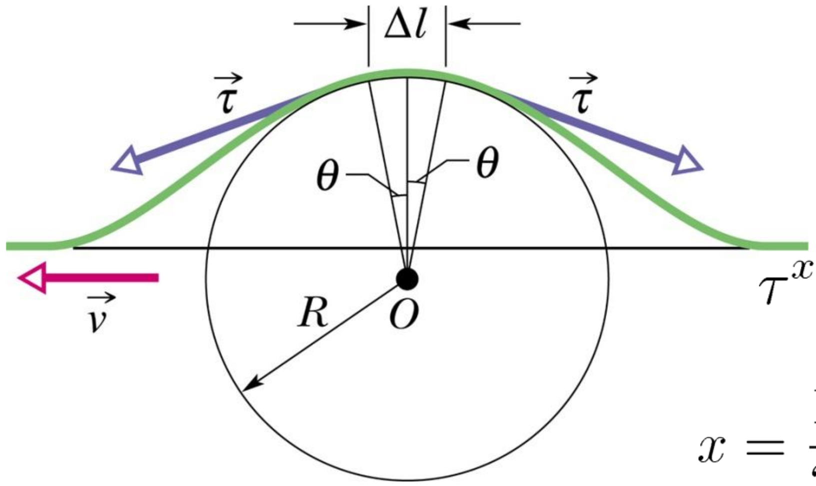
$$v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$$

왼쪽, 오른쪽으로 진행하는 파동의 일반적 표현

$$f(kx \pm \omega t)$$

$$\frac{\partial^2}{\partial t^2} f(x,t) - v^2 \frac{\partial^2}{\partial x^2} f(x,t) = 0$$

Wave velocity on a string



Dimensional analysis

$$[\tau] = MLT^{-2},\ [\mu] = ML^{-1}$$

$$\tau^x \mu^y = (MLT^{-2})^x (ML^{-1})^y = LT^{-1}$$

$$x = \frac{1}{2}, y = -\frac{1}{2} \qquad v = C\sqrt{\frac{\tau}{\mu}}$$

$$F = 2\tau \sin\theta \approx \tau(2\theta) = \tau\frac{\Delta l}{R}$$

$$\Delta m = \mu \Delta l$$

$$a = \frac{v^2}{R}$$

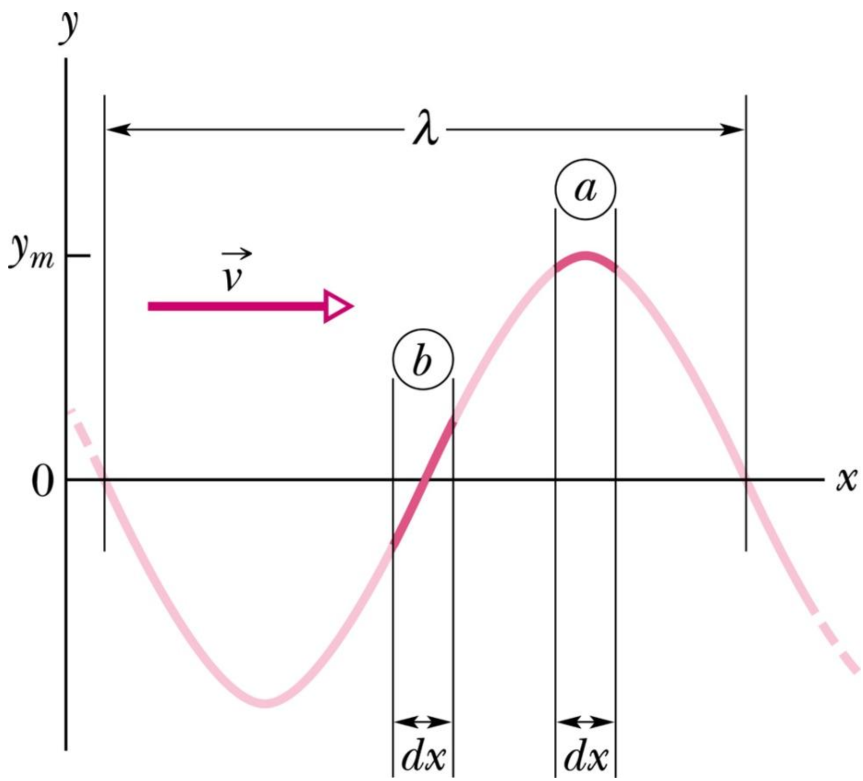
$$\tau\frac{\Delta l}{R} = \mu \Delta l \frac{v^2}{R}$$

$$v = \sqrt{\frac{\tau}{\mu}}$$

파동의 에너지와 일률

$\frac{1}{2}kx^2 = \frac{1}{2}\Delta m \omega^2 x^2$

$y = y_m \sin(kx - \omega t)$



운동에너지 $$dK = \frac{1}{2} dm u^2$$

$$u = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

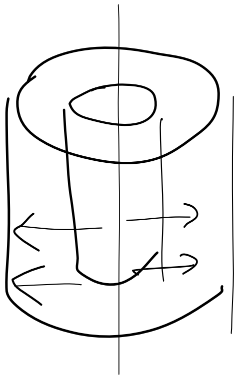
$$dK = \frac{1}{2}\mu dx (\omega y_m)^2 \cos^2(kx - \omega t)$$

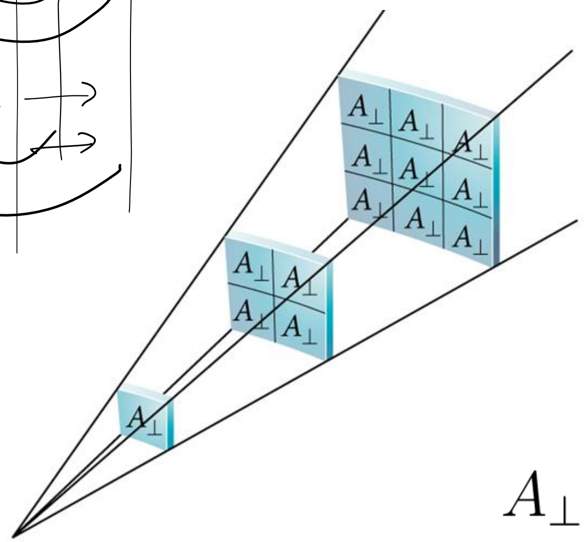
$$\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \cos^2(kx - \omega t)$$

$$\frac{dU}{dt} = \frac{1}{2}\mu v \omega^2 y_m^2 \sin^2(kx - \omega t)$$

$$P = \frac{dE}{dt} = \frac{d}{dt}(K + U) = \frac{1}{2}\mu v \omega^2 y_m^2$$

Energy, power, intensity

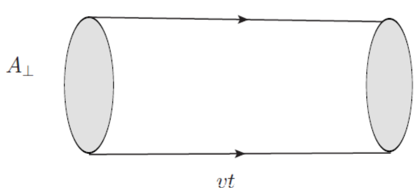




$$
\begin{aligned}
E &= \frac{1}{2} m\omega^2 y_m^2 = \frac{1}{2}\rho V \omega^2 y_m^2 \\
&= \frac{1}{2}\rho A_\perp v t \omega^2 y_m^2
\end{aligned}
$$

For spherical waves,

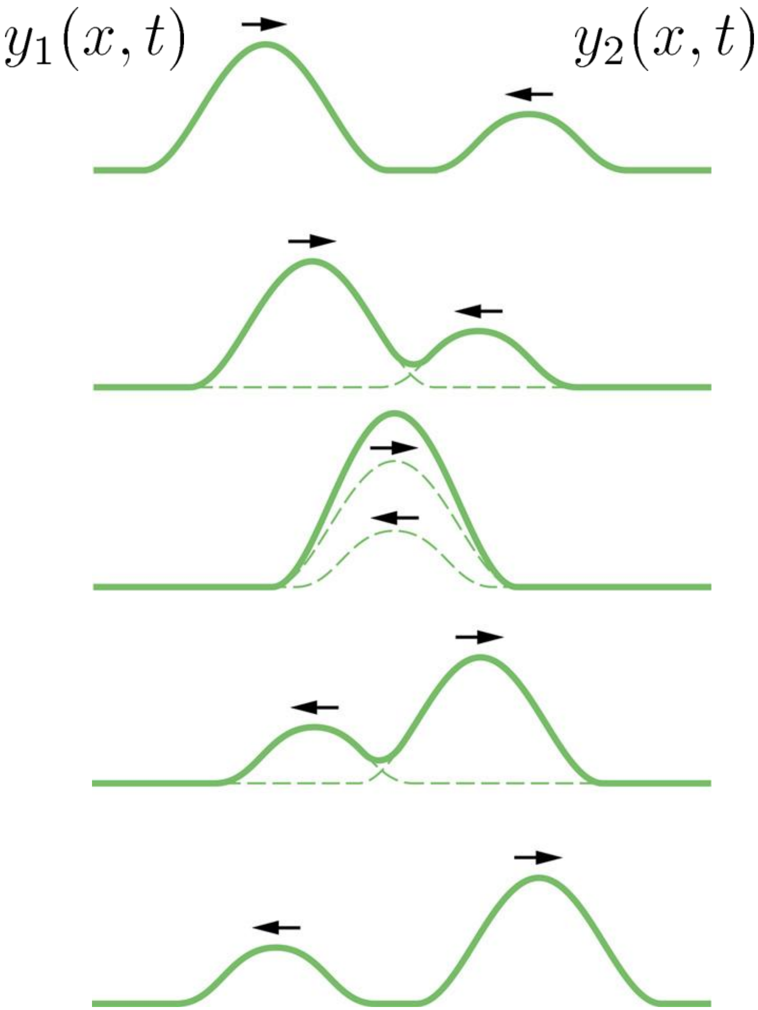
$$
A_\perp y_m^2 = \text{const.} \rightarrow y_m \propto \begin{cases} \dfrac{1}{r} & \text{spherical} \\ \dfrac{1}{\sqrt{r}} & \text{cylindrical} \end{cases}
$$



$$
\overline{P} = \frac{E}{t} = \frac{1}{2}\rho A_\perp v \omega^2 y_m^2
$$

intensity $$I = \frac{\overline{P}}{A_\perp} = \frac{1}{2}\rho v \omega^2 y_m^2$$

파동의 중첩원리



superposition principle

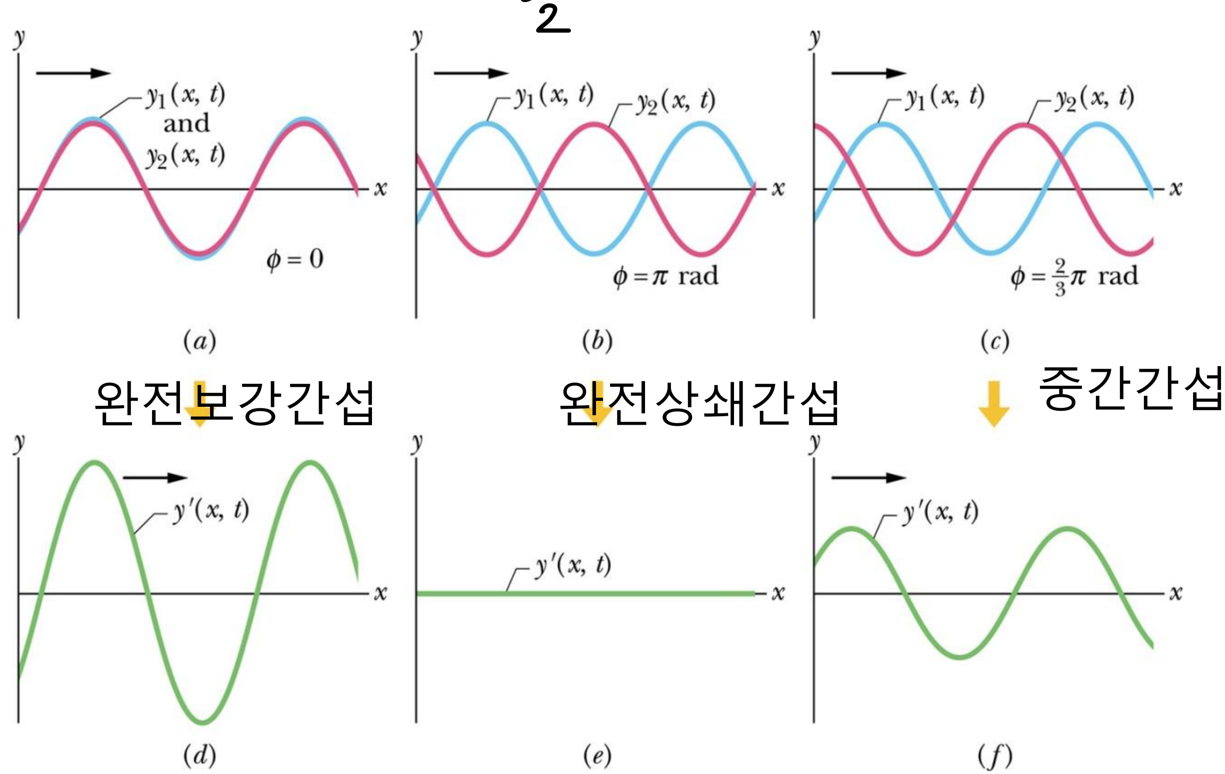
$$y(x,t) = y_1(x,t) + y_2(x,t)$$

$\sin\alpha + \sin\beta = 2\sin(\alpha+\beta)/2 \cdot \cos(\alpha-\beta)/2$

Interference of waves

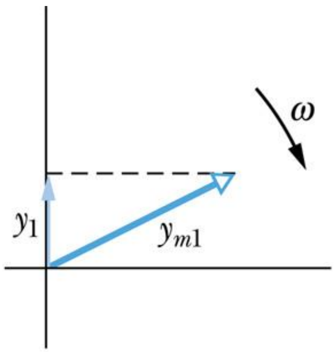
$$y_1(x,t) = y_m \sin(kx - \omega t),\; y_2(x,t) = y_m \sin(kx - \omega t + \phi)$$

$$\begin{aligned} y(x,t) &= y_1(x,t) + y_2(x,t) \\ &= y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \\ &= [2y_m \cos \tfrac{\phi}{2}] \sin(kx - \omega t + \phi/2) \end{aligned}$$



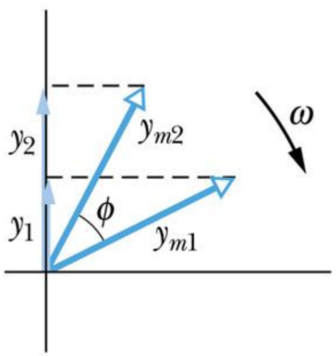
Phasor method

phase



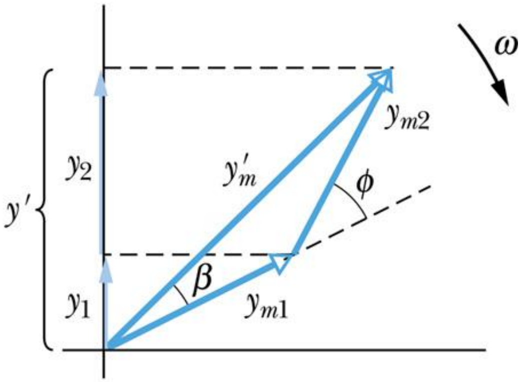
(a)

$$y_1(x,t) = y_{m1}\sin(kx - \omega t)$$



(b)

$$y_2(x,t) = y_{m2}\sin(kx - \omega t + \phi)$$



(c)

$$y'(x,t) = y'_m\sin(kx - \omega t + \beta)$$

Standing waves

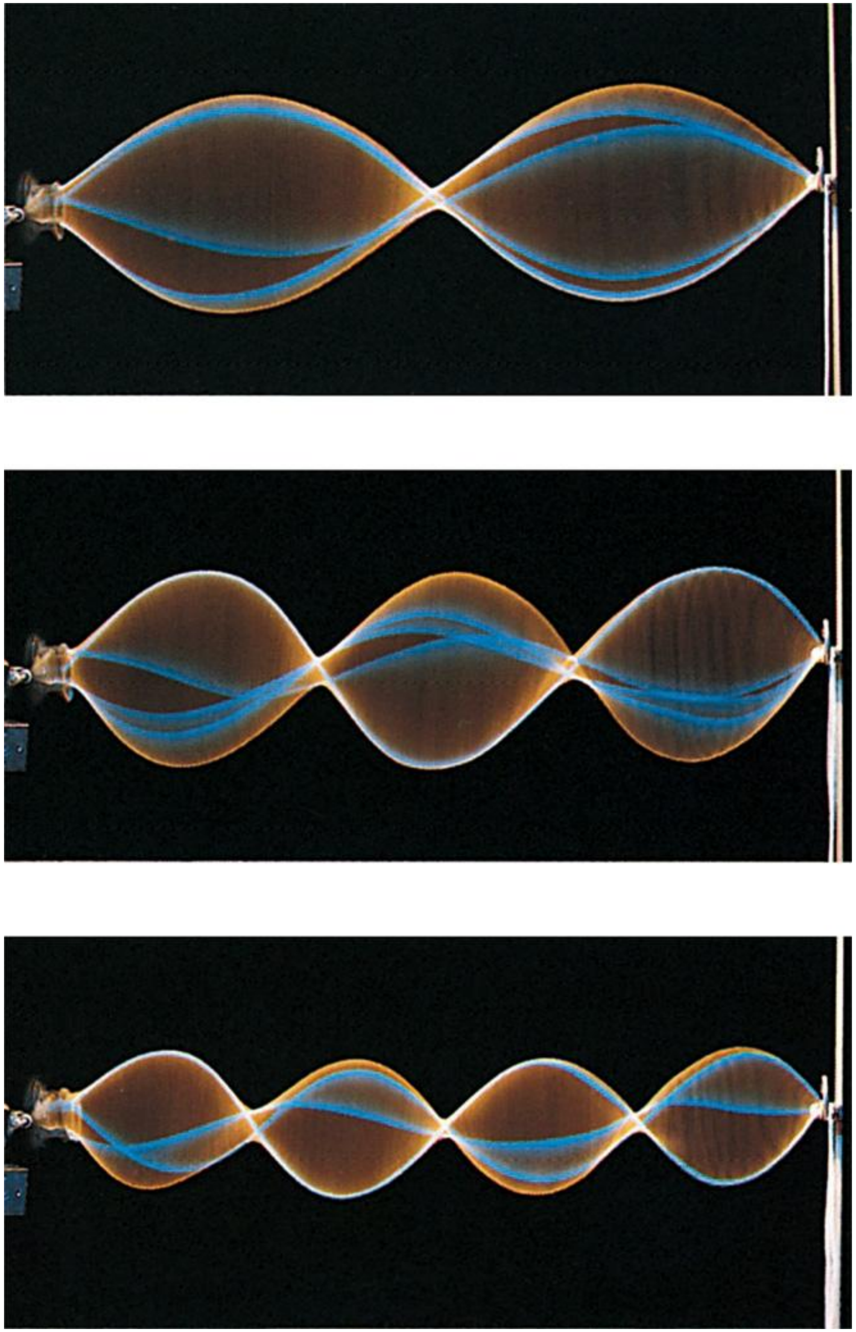
$$y_1(x,t) = y_m \sin(kx - \omega t),$$ 오른쪽으로 이동하는 파동

$$y_2(x,t) = y_m \sin(kx + \omega t)$$ 왼쪽으로 이동하는 파동

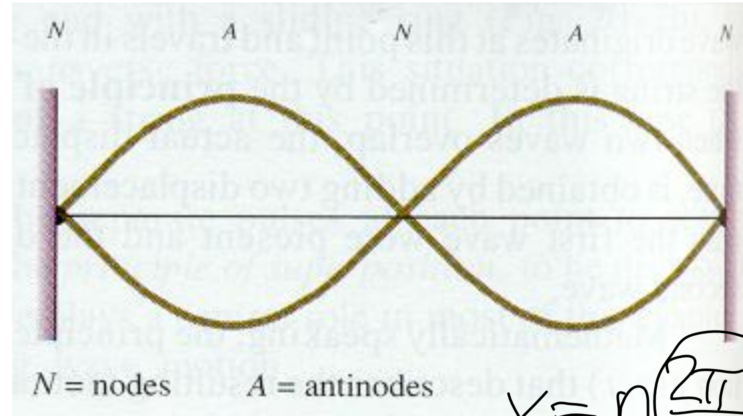
$$\begin{aligned} y'(x,t) &= y_1(x,t) + y_2(x,t) = y_m \sin(kx - \omega t) + y_m \sin(kx + \omega t) \\ &= [2y_m \sin kx] \cos \omega t \end{aligned}$$

진행파동이 아님

$$\overbrace{y'(x,t)}^{\text{Displacement}} = \underbrace{[2y_m \sin kx]}_{\substack{\text{Magnitude} \\ \text{gives} \\ \text{amplitude} \\ \text{at position } x}} \underbrace{\cos \omega t}_{\substack{\text{Oscillating} \\ \text{term}}}$$



$$y = [2y_m \sin kx]\cos\omega t$$



$$x = \frac{n}{2}\frac{2\pi}{k} = \lambda\frac{n}{2}$$

진폭이 0이 되는 곳 (node)

$$kx = n\pi \ (n = 0, 1, 2, \cdots)$$

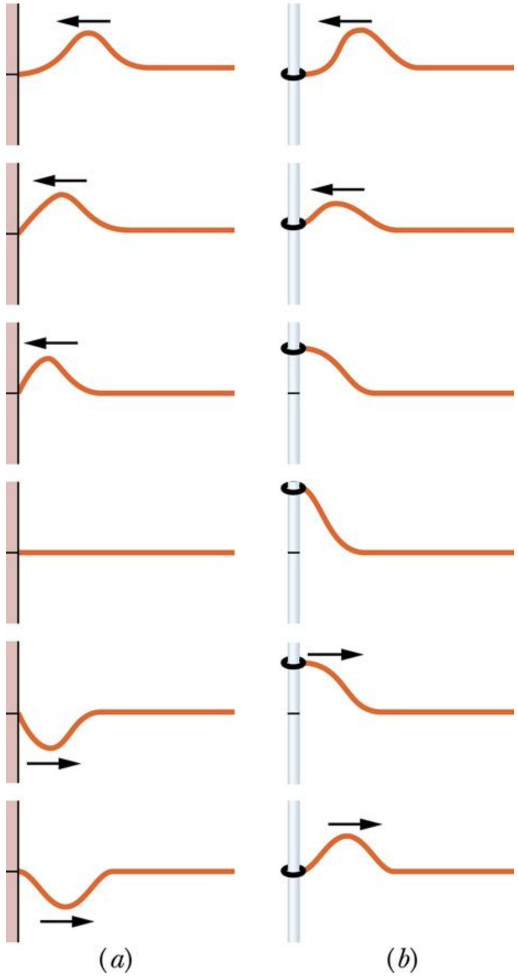
$$x = n\frac{\lambda}{2}, \ (n = 0, 1, 2, \cdots)$$

진폭이 최대가 되는 곳 (antinode)

$$kx = (n + \frac{1}{2})\pi$$

$$x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, \ (n = 0, 1, 2, \cdots)$$

경계면에서의 반사



Fixed end: 위상이 180도 차이

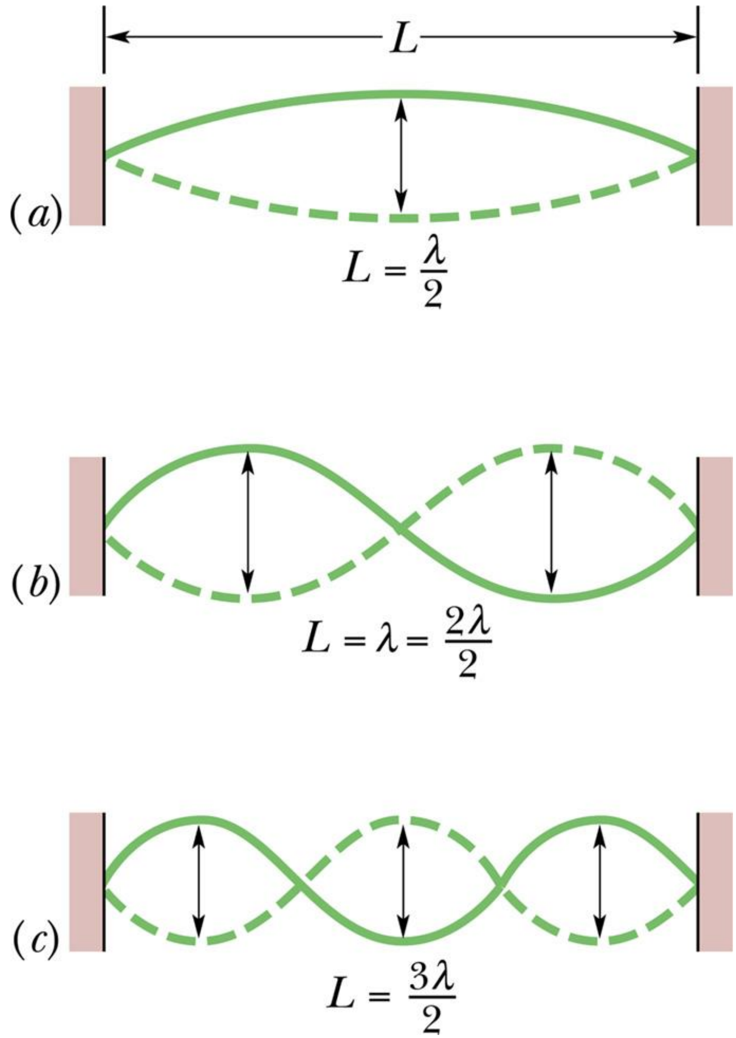
Free end: 위상의 변화 없음

quantize

Standing wave and resonance

$\lambda f = v$

$$f = \frac{v}{\lambda} = v\frac{n}{2L}$$

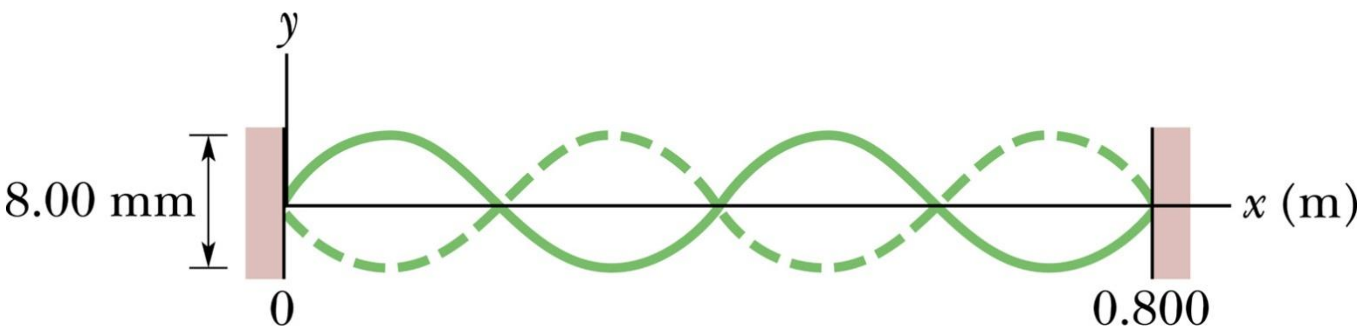


$$L = n\frac{\lambda}{2}$$

$$\lambda = \frac{2L}{n} \ (n = 1, 2, 3, \cdots)$$

$$f = \frac{v}{\lambda} = n\frac{v}{2L}, \ (n = 1, 2, 3, \cdots)$$

Example

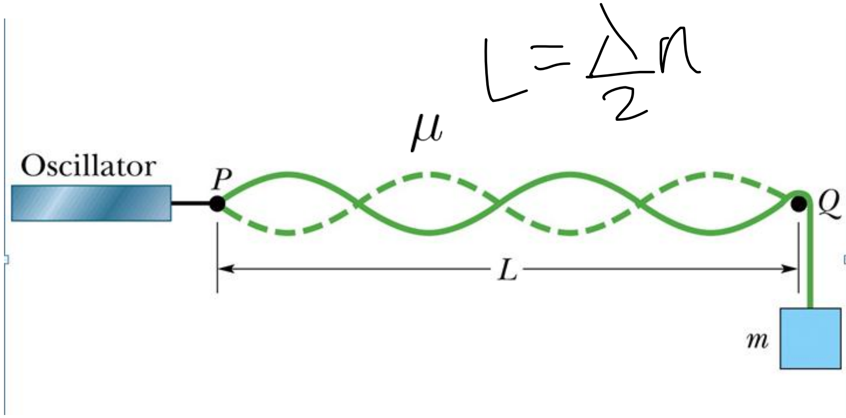


$$m = 2.500 \text{ g}, \ L = 0.800 \text{ m}, \ \tau = 325.0 \text{ N}$$

$$2\lambda = L \to \lambda = \frac{L}{2} \ (n = 4)$$

$$f = \frac{v}{\lambda} = \sqrt{\frac{\tau L}{m}}\frac{2}{L}$$

Problem 1 $\lambda = \frac{2L}{n}$



$$L = \frac{\lambda}{2} n$$

(a) If n=4, what is m?

(b) If m=1.00 kg, is standing wave possible?

$$L = 1.20\text{m},\ \ \mu = 1.6\text{g/m},\ f = 120\text{Hz}$$

$$\lambda = \frac{1}{f}\sqrt{\frac{mg}{\mu}} = \frac{2L}{n}$$

$$\lambda f = v = \sqrt{\frac{mg}{\mu}}$$

$$n = 2Lf\sqrt{\frac{\mu}{mg}}$$