Communication Systems II

[KECE322_01]

<2012-2nd Semester>

Lecture #19

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Outline

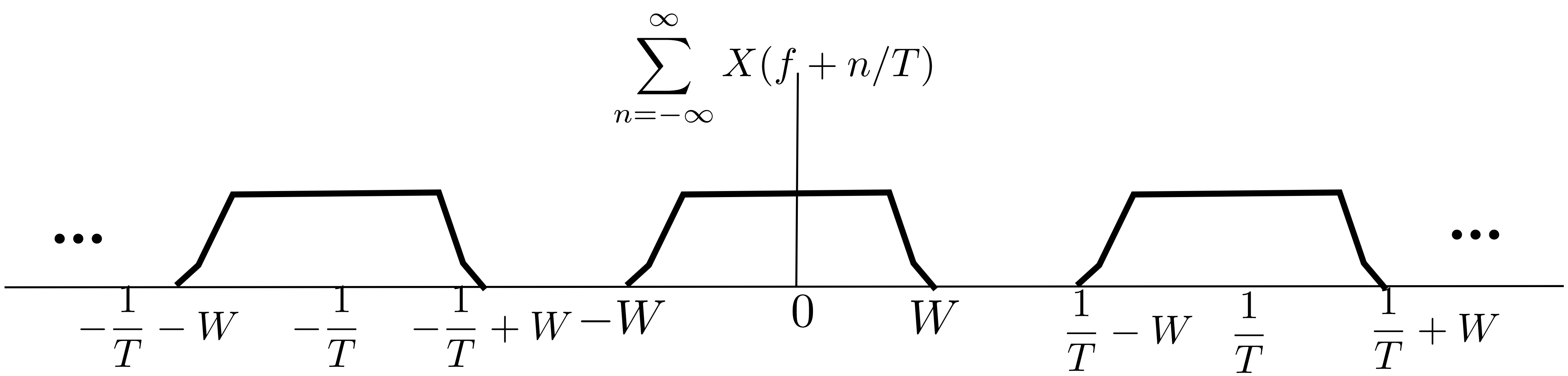
- Signal design for bandlimited channels (Chap.8.2.1)
- Transmission of digital information via carrier modulation (Chap. 9)

- Suppose that the channel has a bandwidth of W.
  - Then $C(f) = 0$ for $|f| > W$; consequently, $X(f) = 0$ for $|f| > W$.
  - Note that the Nyquist criterion for zero ISI is $\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T.$
- We distinguish three cases.

1. When $T < \frac{1}{2W}$, or equivalently, $\frac{1}{T} > 2W$
2. When $T = \frac{1}{2W}$
3. When $T > \frac{1}{2W}$

- When $T < \frac{1}{2W}$, or equivalently, $\frac{1}{T} > 2W$, $B(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$ consists of non-overlapping replicas of $X(f)$, separated by $1/T$.

  - There is no choice for $X(f)$ to ensure $B(f) \equiv T$ in this case and there is no way that we can design a system with no ISI.



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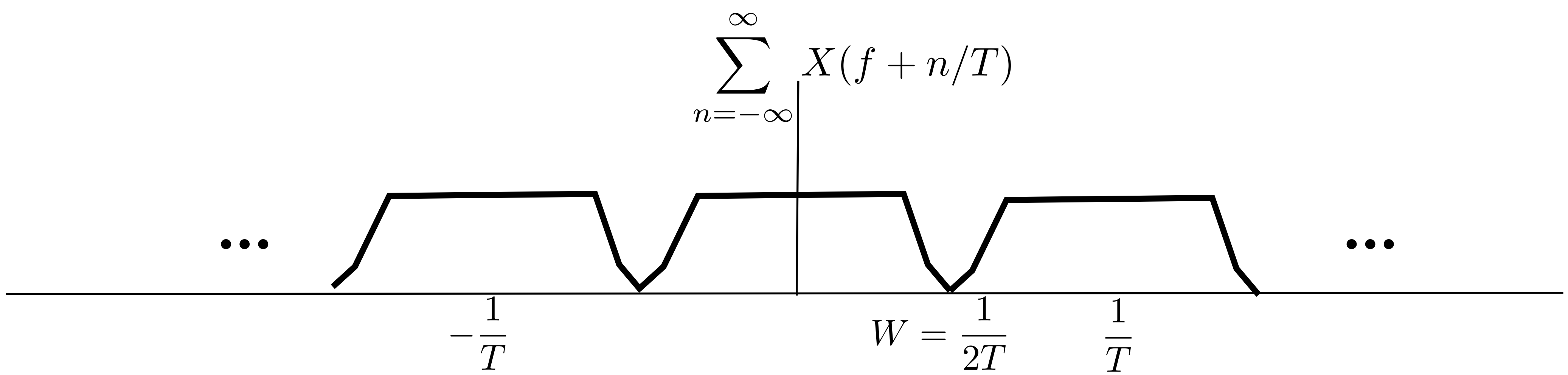
- When $T = 1/2W$, or equivalently, $1/T = 2W$, the replication of $X(f)$ separated by $1/T$ has the form as below



  - In this case there exists only one $X(f)$ that results $B(f) = T$, namely,

$$X(f) = \begin{cases} T, & (|f| < W) \\ 0, & (\text{otherwise}) \end{cases}$$

    - which corresponds to

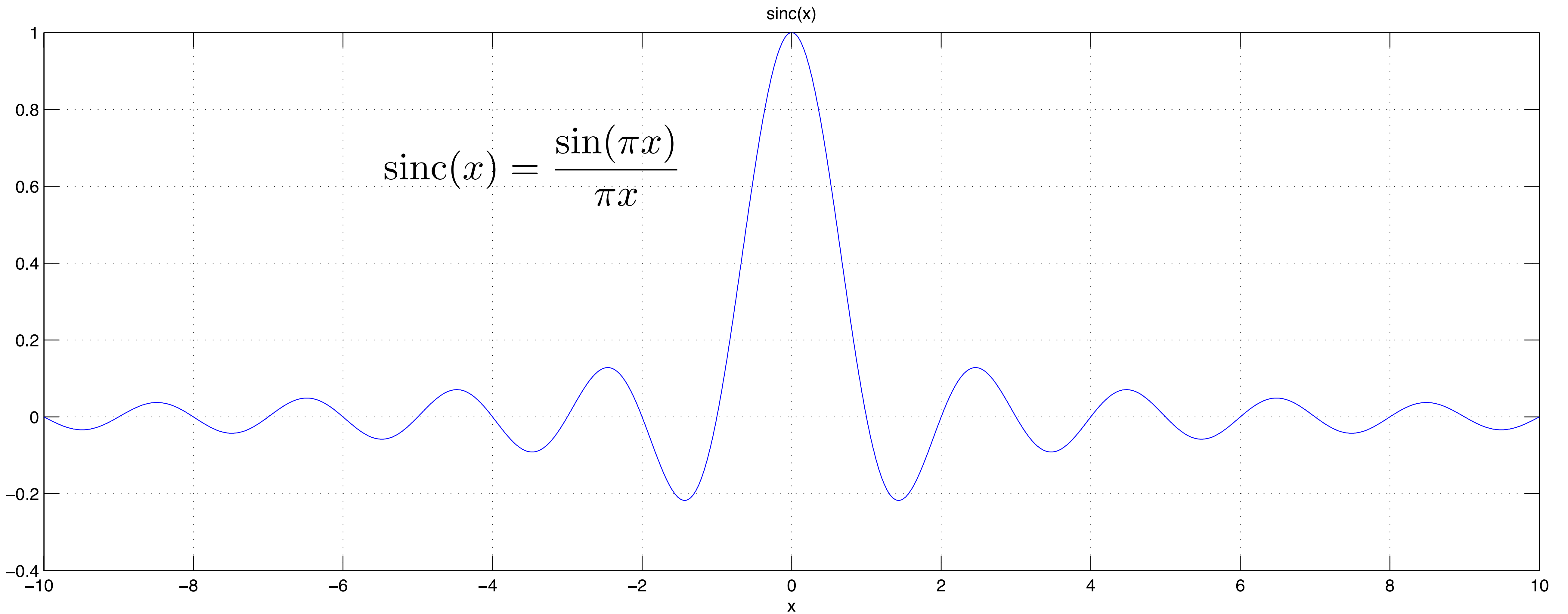
$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \equiv \text{sinc}\left(\frac{\pi t}{T}\right)$$

- This means that the smallest value of T for which transmission with zero ISI is possible is $T = 1/2W,$ and for this value, $x(t)$ has to be a sinc function.

- The **difficulty** with this choice of $x(t)$ is that it is non-causal and therefore non-realizable.

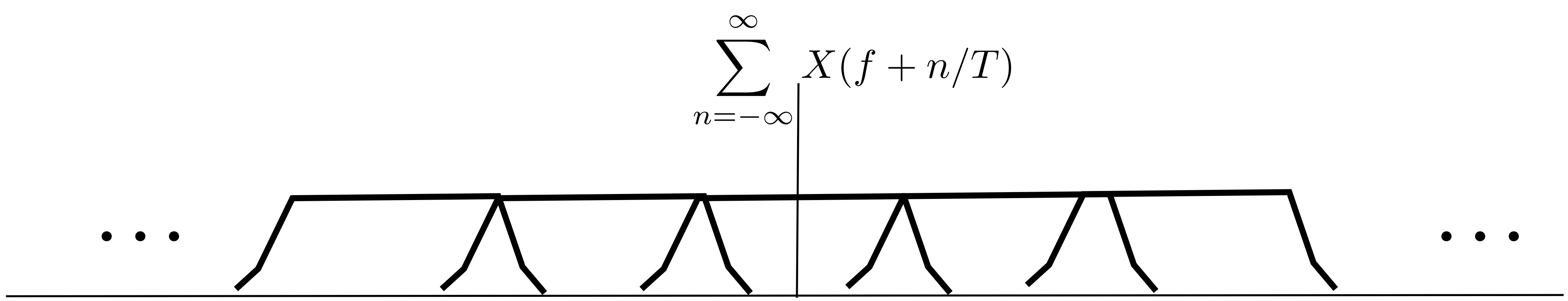
- To make it realizable, usually a delayed version of it, i.e., $\text{sinc}[\pi(t - t_0)/T]$ is used and t_0 is chosen such that for $t < 0$, we have $\text{sinc}[\pi(t - t_0)/T] \approx 0.$

- Of course, with this choice of $x(t)$, the sampling time must be shifted to $mT + t_0$.



- A second **difficulty** with this pulse shape is that its rate of convergence to zero is low. The tails of $x(t)$ decays as $1/t$, consequently, a small mistiming error in sampling the output of the matched filter at the demodulator results in an infinite series of ISI components.

- When $T > 1/2W$, $B(f)$ consists of overlapping replications of $X(f)$ separated by $1/T$.
  - In this case, there exists numerous choice for $X(f)$ such that $B(f) \equiv T$.



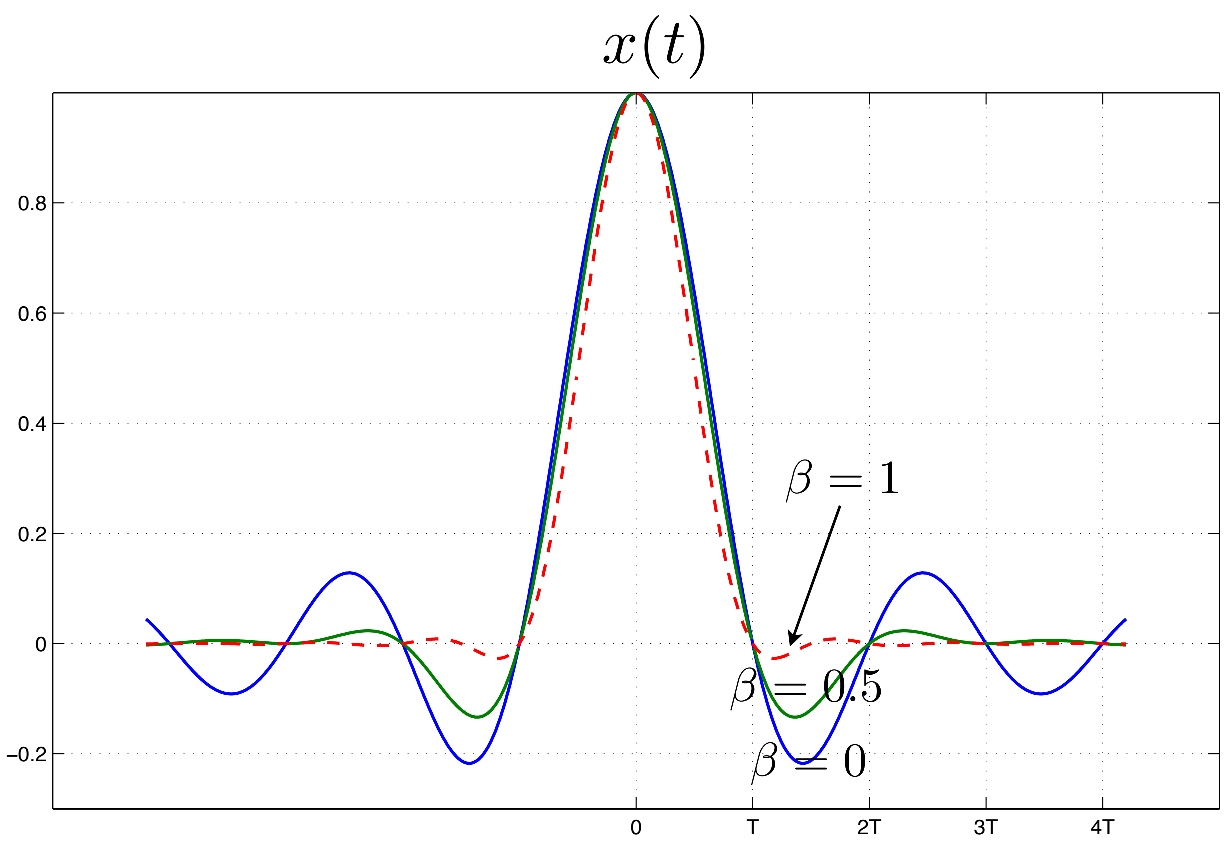
- A particular pulse spectrum for the $T > 1/2W$ case, that has desirable spectral properties and has been widely used in practice is the *raised cosine spectrum*.

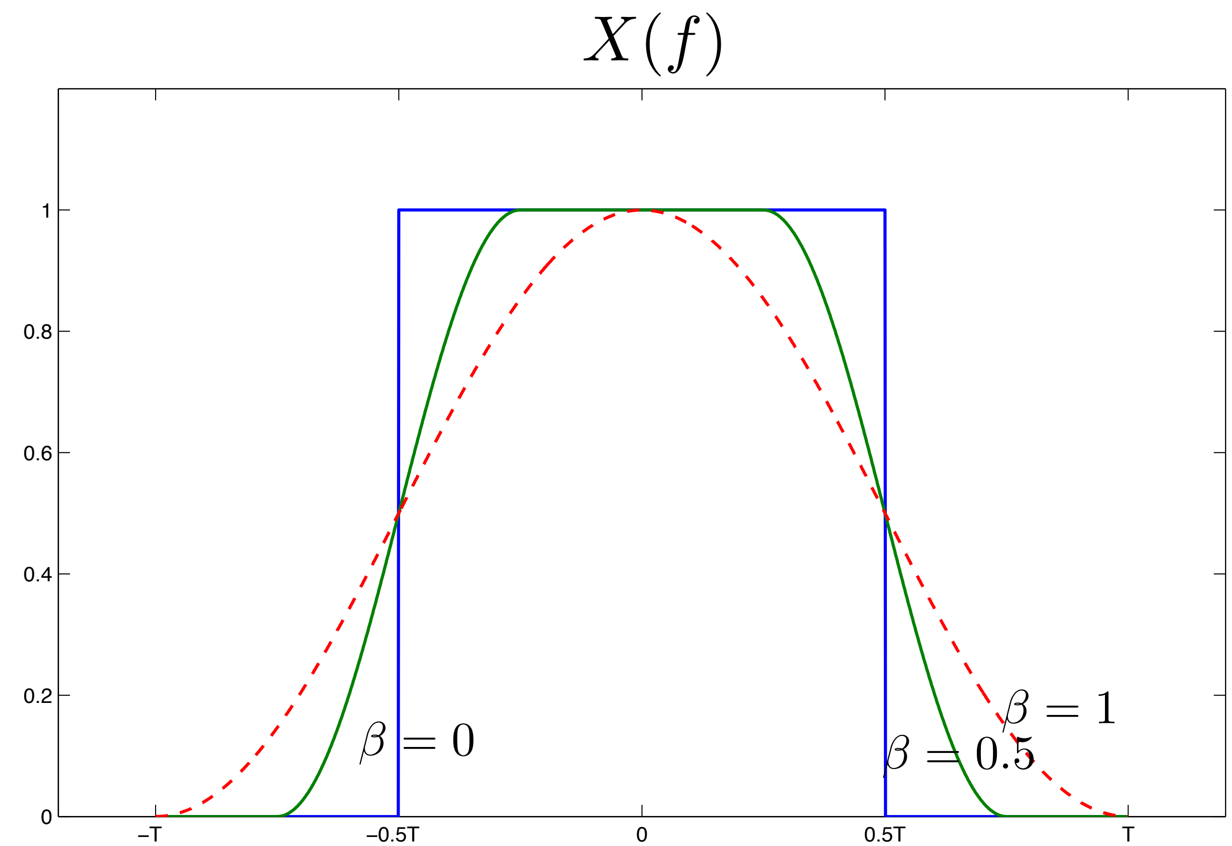
$$X_{rc}(f) = \begin{cases} T & \left(0 \le |f| \le \frac{1-\beta}{2T}\right) \\ \frac{T}{2}\left\{1 + \cos\left[\frac{\pi t}{\beta}\left(|f| - \frac{1-\beta}{2T}\right)\right]\right\} & \left(\frac{1-\beta}{2T} \le |f| \le \frac{1+\beta}{2T}\right) \\ 0 & \left(|f| > \frac{1+\beta}{2T}\right) \end{cases}$$

where $0 \le \beta \le 1$ is called roll-off factor.

Raised cosine pulse

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2} = \text{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi\beta t/T)}{1-4\beta^2t^2/T^2}$$





- Note that $x(t)$ is normalized so that $x(0) = 1$.

Raised cosine pulse

$$x(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\pi \beta t/T)}{1 - 4\beta^2 t^2/T^2} = \mathrm{sinc}\left(\frac{t}{T}\right) \frac{\cos(\pi \beta t/T)}{1 - 4\beta^2 t^2/T^2}$$

- For $\beta = 0$, the pulse reduces to $x(t) = \mathrm{sinc}\left(\frac{t}{T}\right)$ and the symbol rate is $\frac{1}{T} = 2W$.
- For $\beta = 1$, the symbol rate is $\frac{1}{T} = W$.
- In general, the tails of $x(t)$ decays as $1/t^3$ for $\beta > 0$. Consequently, a mistiming error in sampling leads to a series of ISI components that converges to a finite value.

- Special case when the channel is ideal with

$$C(f) = \prod\left(\frac{f}{2W}\right)$$

  - In this case, we have

$$X_{\mathrm{rc}}(f) = G_T(f)G_R(f) = |G_T(f)|^2$$

    - Ideally,

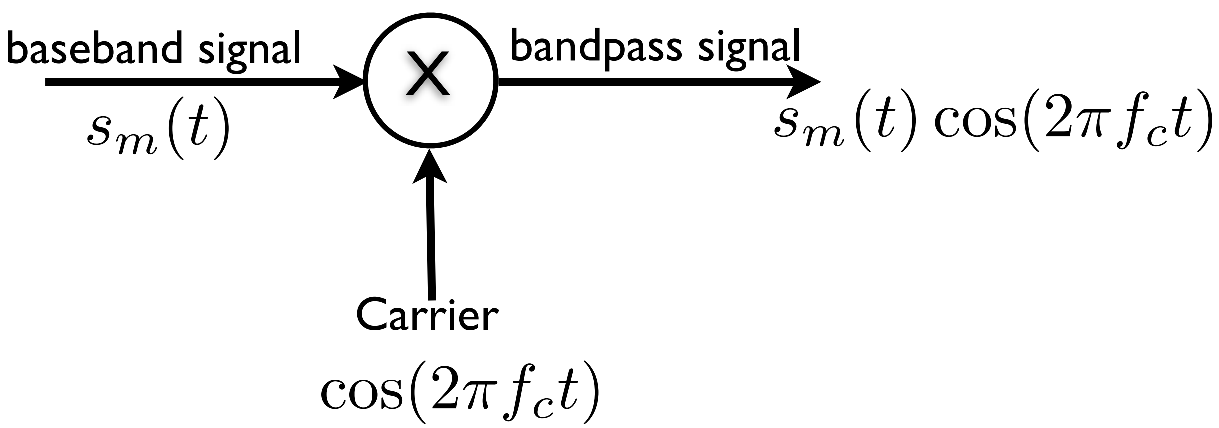
$$G_T(f) = \sqrt{|X_{\mathrm{rc}}(f)|}e^{-j2\pi f t_0}$$

      and

$$G_R(f) = G_T^*(f)$$

Transmission of Digital Information Via Carrier Modulation

- Signal transmission with carrier



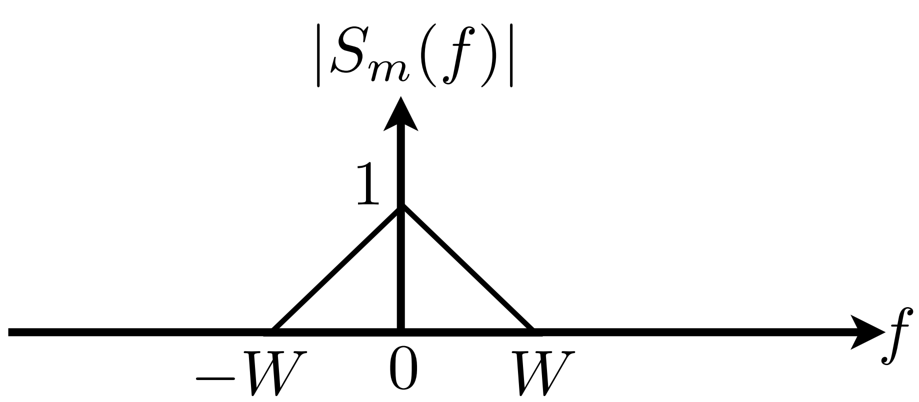
- We will learn
  - Amplitude-modulation (pulse amplitude modulation: PAM)
  - Phase-modulation (phase shift-keying: PSK)
  - Quadrature-amplitude modulation (QAM)
  - Frequency-shift keying (FSK)

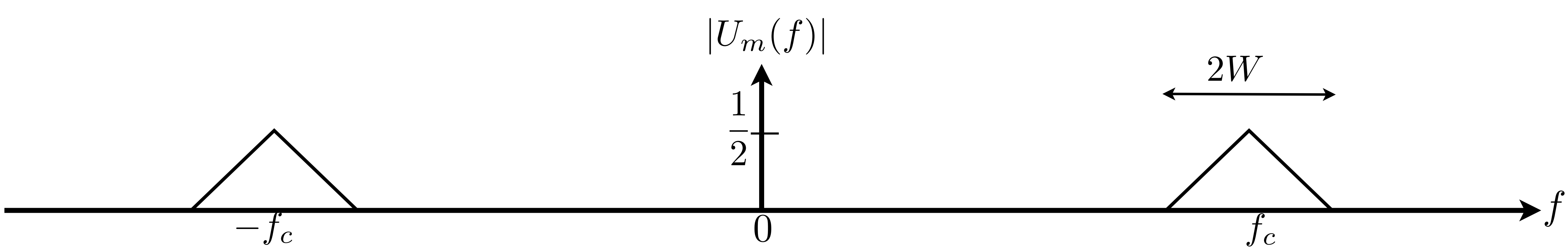
Amplitude-Modulated Digital Signals

- Transmitted signal waveform

$$u_m(t) = s_m(t)\cos(2\pi f_c t)$$

$$U_m(f) = \frac{1}{2}\left[S_m(f - f_c) + S_m(f + f_c)\right]$$





- Energy of the bandpass signal waveforms

$$\begin{aligned}\mathcal{E}_m &= \int_{-\infty}^{\infty} u_m^2(t)\,dt = \int_{-\infty}^{\infty} s_m^2(t)\cos^2 2\pi f_c t\,dt \\ &= \frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\,dt + \frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\cos(4\pi f_c t)\,dt = 0 \\ &= \frac{1}{2}\int_{-\infty}^{\infty} s_m^2(t)\,dt\end{aligned}$$

Geometric representation of bandpass signals

- Baseband signal

$$s_m(t) = s_m \psi_(t), \qquad m = 1, 2, \ldots, M$$

- Carrier modulated signals

$$u_m(t) = s_m(t)\cos(2\pi f_c t) = s_m \psi(t)\cos(2\pi f_c t), \qquad m = 1, 2, \ldots, M$$

- Let

$$\psi_c(t) = \sqrt{2}\psi(t)\cos(2\pi f_c t),$$

such that

$$\begin{aligned} \int_{-\infty}^{\infty} \psi_c^2(t)\,dt &= 2\int_{-\infty}^{\infty} \psi^2(t)\cos^2(2\pi f_c t)\,dt \\ &= \int_{-\infty}^{\infty} \psi^2(t)\,dt + \underbrace{\int_{-\infty}^{\infty} \psi^2(t)\cos(2\pi f_c t)\,dt}_{=0} \\ &= 1 \end{aligned}$$

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- Bandpass waveforms by the carrier-modulated basis function

$$u_m(t) = \frac{s_m}{\sqrt{2}}\psi_c(t) = s_{cm}\psi_c(t), \qquad m = 1, 2, \ldots, M$$

$$\psi_c(t) = \sqrt{2}\psi(t)\cos(2\pi f_c t)$$

and

$$s_{cm} = \frac{s_m}{\sqrt{2}} = (2m - 1 - M)d/\sqrt{2}$$

Demodulation and Detection of PAM

- Transmit signal

$$u_m(t) = s_m(t)\cos(2\pi f_c t) \qquad\qquad m = 1, 2, \ldots, M$$

- Received signal

$$r(t) = u_m(t) + n(t) \;= \frac{s_m}{\sqrt{2}}\psi_c(t) + n(t)$$

  - where

$$n(t) = n_c(t)\cos(2\pi f_c t) - n_s(t)\sin(2\pi f_c t)$$

- Cross-correlator output

$$\begin{aligned} y(T) &= \int_0^T r(t)\psi_c(t)\,dt \\ &= \sqrt{2}s_m \int_0^T \psi^2(t)\cos^2(2\pi f_c t)\,dt + \sqrt{2}\int_0^T n(t)\psi(t)\cos(2\pi f_c t)\,dt \\ &= \frac{s_m}{\sqrt{2}} + n = s_{cm} + n \end{aligned}$$

- Optimum detector

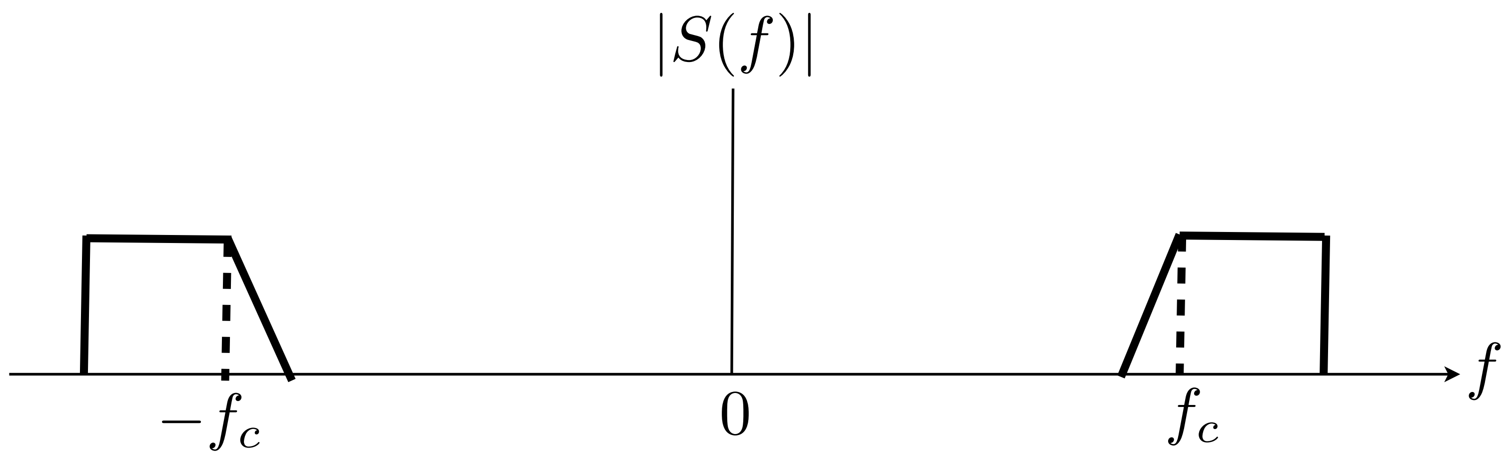
$$D(y, s_m) = (y - s_{cm})^2, \qquad m = 1, 2, \ldots, M$$

  - or equivalently

$$C(y, s_m) = 2ys_{cm} - s_{cm}^2$$

Representation of Band-Pass Signals

- Suppose a real-valued signal $s(t)$



- Define the analytic signal $s_+(t)$ such as its Fourier transform is given as $S_+(f) = 2u(f)S(f)$

- Then the analytic signal can be obtained by taking inverse Fourier transform as

$$s_+(t) = \int_{-\infty}^{\infty} S_+(f) e^{j2\pi f t} \, dt = \mathcal{F}^{-1}[2u(f)] * \mathcal{F}^{-1}[S(f)]$$

$$\begin{aligned} \mathcal{F}^{-1}[2u(f)] &= \delta(t) + \frac{j}{\pi t} \\ \mathcal{F}^{-1}[S(f)] &= s(t) \end{aligned}$$

- Then

$$\begin{aligned} s_+(t) &= \left[\delta(t) + \frac{j}{\pi t}\right] * s(t) \\ &= s(t) + \frac{j}{\pi t} * s(t) \end{aligned}$$

- We define $\hat{s}(t)$

$$\hat{s}(t) = \frac{1}{\pi t} * s(t) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\, d\tau$$

- Define $h(t)$ as

$$h(t) = \frac{1}{\pi t}, \qquad -\infty < t < \infty$$

- $h(t)$ is called a Hilbert transformer. The frequency response of this filter is

$$H(f) = \int_{-\infty}^{\infty} h(t) e^{-j2\pi f t}\, dt = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{t} e^{-j2\pi f t} = \begin{cases} -j, & (f > 0) \\ 0, & (f = 0) \\ j, & (f < 0) \end{cases}$$

- We observe that $|H(f)| = 1$ and the phase response $\Theta(f) = -\frac{1}{2}\pi$ for $f > 0$ and $\Theta(f) = \frac{1}{2}\pi$ for $f < 0$.
  - Therefor this filter is basically a 90° phase shifter for all frequencies in the input signal.

- Equivalent low-pass representation by performing a frequency translation of $S_+(f)$
  - Thus we define $S_l(f)$ as

$$S_l(f) = S_+(f + f_c)$$

  - Equivalent time-domain relation is

$$s_l(t) = s_+(t)e^{-j2\pi f_c t} = [s(t) + j\hat{s}(t)]e^{-j2\pi f_c t}$$

    - or equivalently

$$s(t) + j\hat{s}(t) = s_l(t)e^{j2\pi f_c t}$$

  - In general, the signal $s_l(t)$ is complex-valued and may be expressed as

$$s_l(t) = x(t) + jy(t)$$

- Then we can rewrite

$$s(t) + j\hat{s}(t) = s_l(t)e^{j2\pi f_c t} = (x(t) + jy(t))(\cos(2\pi f_c t) + j\sin(2\pi f_c t))$$

  - Hence we have

$$\begin{aligned} s(t) &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t) \\ \hat{s}(t) &= x(t)\sin(2\pi f_c t) + y(t)\cos(2\pi f_c t) \end{aligned}$$

- Another representation of the signal is

$$s(t) \quad = \quad \Re\{[x(t) + jy(t)]e^{j2\pi f_c t}]\} = \Re[s_l(t)e^{j2\pi f_c t}]$$

  - The low-pass signal $s_l(t)$ is usually called the *complex envelope* of the real signal $s(t)$ and is basically the *equivalent low-pass signal*.

- A third possible representation of a band-pass signal is obtained by expressing $s_l(t)$

$$s_l(t) = a(t)e^{j\theta(t)}$$

  - where

$$a(t) = \sqrt{x^2(t) + y^2(t)}, \qquad \theta(t) = \tan^{-1}\frac{y(t)}{x(t)}$$

- Then

$$\begin{aligned} s(t) &= \Re[s_l(t)e^{j2\pi f_c t}] \\ &= \Re[a(t)e^{j[2\pi f_c t + \theta(t)]}] \\ &= a(t)\cos(2\pi f_c t + \theta(t)) \end{aligned}$$

  - The signal $a(t)$ is called the *envelope* of $s(t)$, and $\theta(t)$ is called the *phase* of $s(t)$.

- The Fourier transform of $s(t)$

$$S(f) = \int_{-\infty}^{\infty} s(t)e^{-j2\pi f t}\,dt = \int_{-\infty}^{\infty} \{\Re[s_l(t)e^{j2\pi f_c t}]\}e^{-j2\pi f t}\,dt$$

- Use the identity

$$\Re(\zeta) = \frac{1}{2}(\zeta + \zeta^*)$$

- Then

$$\begin{aligned} S(f) &= \frac{1}{2}\int_{-\infty}^{\infty} [s_l(t)e^{j2\pi f_c t} + s_l^*(t)e^{-j2\pi f_c t}]e^{-j2\pi f t}\,dt \\ &= \frac{1}{2}[S_l(f - f_c) + S_l^*(-f - f_c)] \end{aligned}$$

- The energy in the signal $s(t)$

$$E = \int_{-\infty}^{\infty} s^2(t)\, dt = \int_{-\infty}^{\infty} \{\Re[s_l(t) e^{j2\pi f_c t}]\}^2\, dt$$

  - Using the $s_l(t) = a(t)e^{j\theta(t)}$

$$\begin{aligned} E &= \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2\, dt + \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2 \cos[4\pi f_c t + 2\theta(t)]\, dt \\ &\approx \frac{1}{2}\int_{-\infty}^{\infty} |s_l(t)|^2\, dt \end{aligned}$$

    - where $|s_l(t)|$ is just the envelope $a(t)$ of $s(t)$

Representation of Linear Band-Pass Systems

- A linear filter or system may be described either by its impulse response $h(t)$ or by its frequency response $H(f)$, which is the Fourier transform of $h(t)$.
  - Since $h(t)$ is real,
$$H^*(f) = H(f)$$
  - Let us define $H_l(f-f_c)$ as
$$H_l(f-f_c) = \begin{cases} H(f), & (f>0) \\ 0, & (f<0) \end{cases}$$
  - Then
$$H_l^*(f-f_c) = \begin{cases} 0, & (f>0) \\ H^*(-f), & (f<0) \end{cases}$$
  - Using $H^*(f) = H(f)$, we have
$$H(f) = H_l(f-f_c) + H_l^*(-f-f_c)$$
  - Inverse Fourier transform gives
$$h(t) = h_l(t)e^{j2\pi f_c t} + h_l^*(t)e^{-j2\pi f_c t} = 2\Re[h_l(t)e^{j2\pi f_c t}]$$

Response of a Band-Pass System to a Band-Pass Signal

- The output of the band-pass system is also a band-pass signal, and, therefore, it can be expressed in the form

$$r(t) = \Re[r_l(t)e^{j2\pi f_c t}]$$

  - where $r(t)$ is related to the input signal $s(t)$ and the impulse response $h(t)$ by the convolution integral

$$r(t) = \int_{-\infty}^{\infty} s(\tau)h(t-\tau)\, d\tau$$

  - Equivalently the output of the system, expressed in the frequency domain, is

$$R(f) = S(f)H(f)$$

    - which can be rewritten as

$$R(f) = \frac{1}{2}[S_l(f-f_c) + S_l^*(-f-f_c)][H(f-f_c) + H_l^*(-f-f_c)]$$

    - When $s(t)$ is a narrowband signal and $h(t)$ is the impulse response of a narrowband system, which follows

$$S_l(f-f_c)H_l^*(-f-f_c) = 0, \qquad S_l^*(-f-f_c)H_l(f-f_c) = 0$$

- Therefore, it simplifies to

$$\begin{aligned} R(f) &= \frac{1}{2}[S_l(f-f_c)H_l(f-f_c) + S_l^*(-f-f_c)H_l^*(-f-f_c)] \\ &= \frac{1}{2}[R_l(f-f_c) + R_l^*(-f-f_c)] \end{aligned}$$

  - where

$$R_l(f) = S_l(f)H_l(f)$$

  - The output signal in the time domain

$$r_l(t) = s_l(t) * h_l(t) = \int_{-\infty}^{\infty} s_l(\tau)h_l(t-\tau)\,d\tau$$

- PAM signal

$$s(t) = \Re[A_m g(t) e^{j2\pi f_c t}]$$

  - That is,

$$s_l(t) = A_m g(t)$$