

# LECTURE 15

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# Lecture 15

## 17. Nyquist-rate A/D converter

17.1 Integrating converters

**17.2 Successive-approximation converter**

17.3 Algorithmic (or cyclic) A/D converter

17.4 Pipelined A/D converter

17.5 Flash converters

17.6 Two-step A/D converters

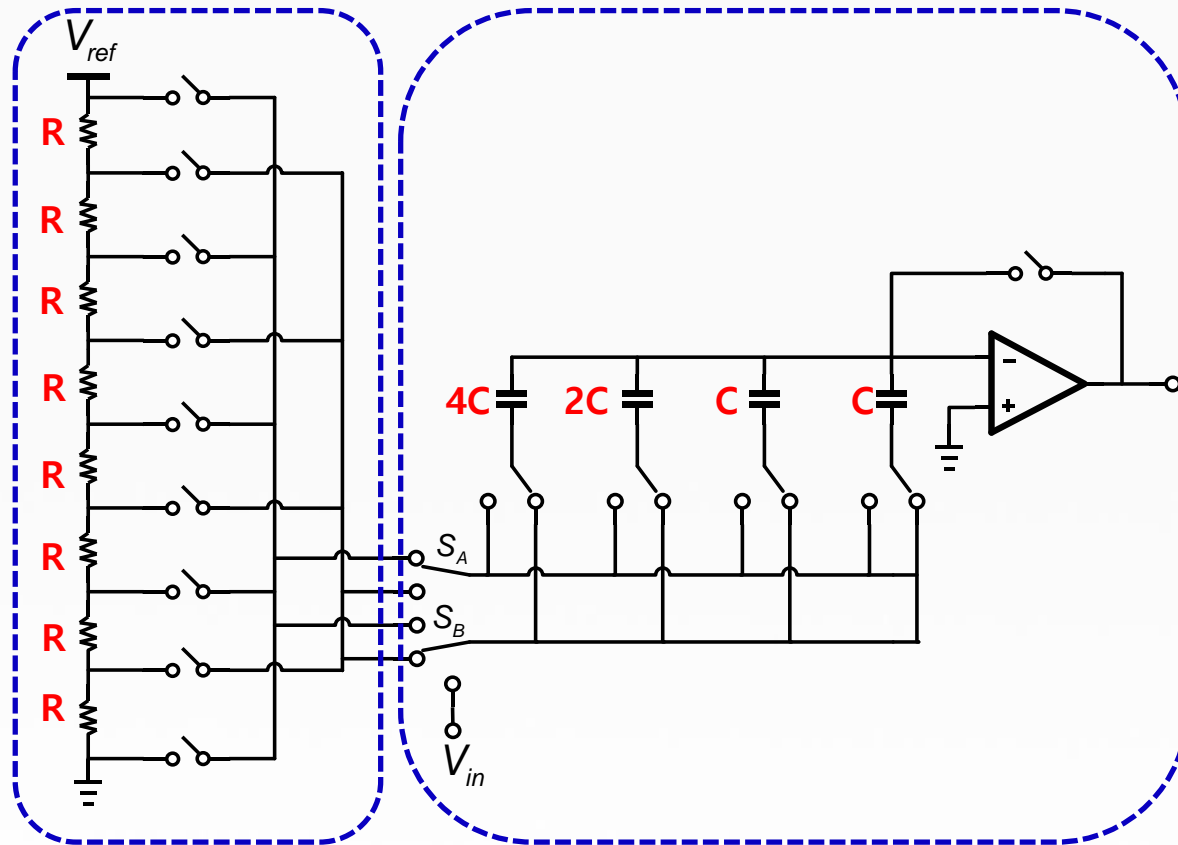
17.7 Interpolating A/D converters

17.8 Folding A/D converters

17.9 Time-interleaved A/D converters



# Resistor – Capacitor Hybrid(6-bit)



MSB(3-bit)

LSB(3-bit)



# Multi-Bit Successive-Approximation

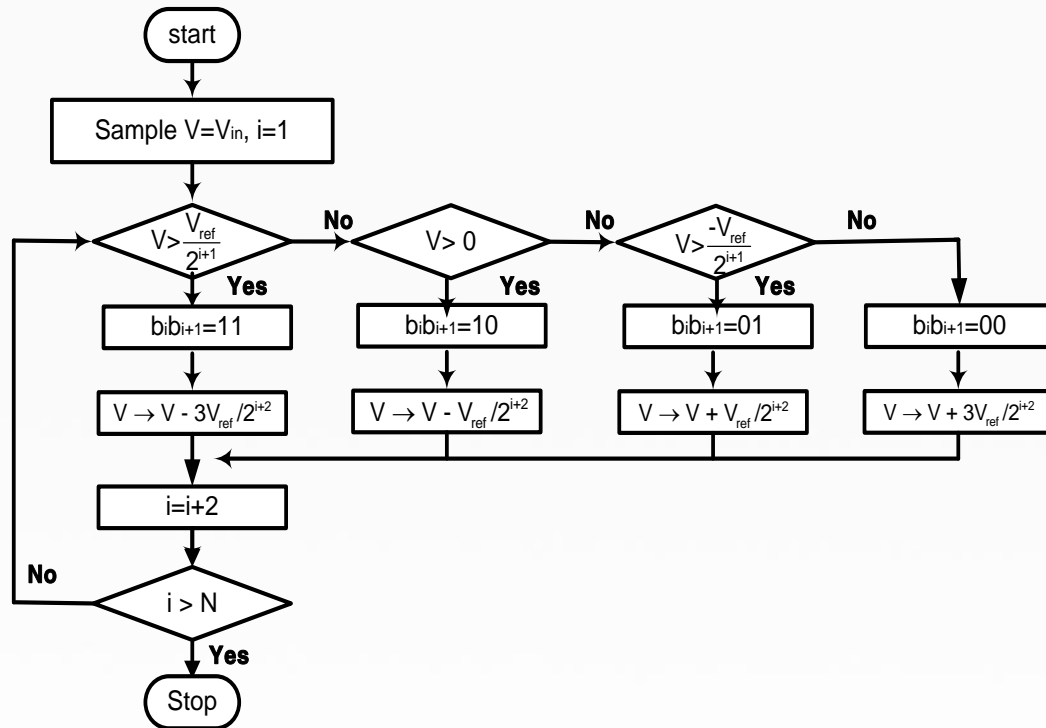
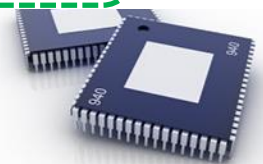
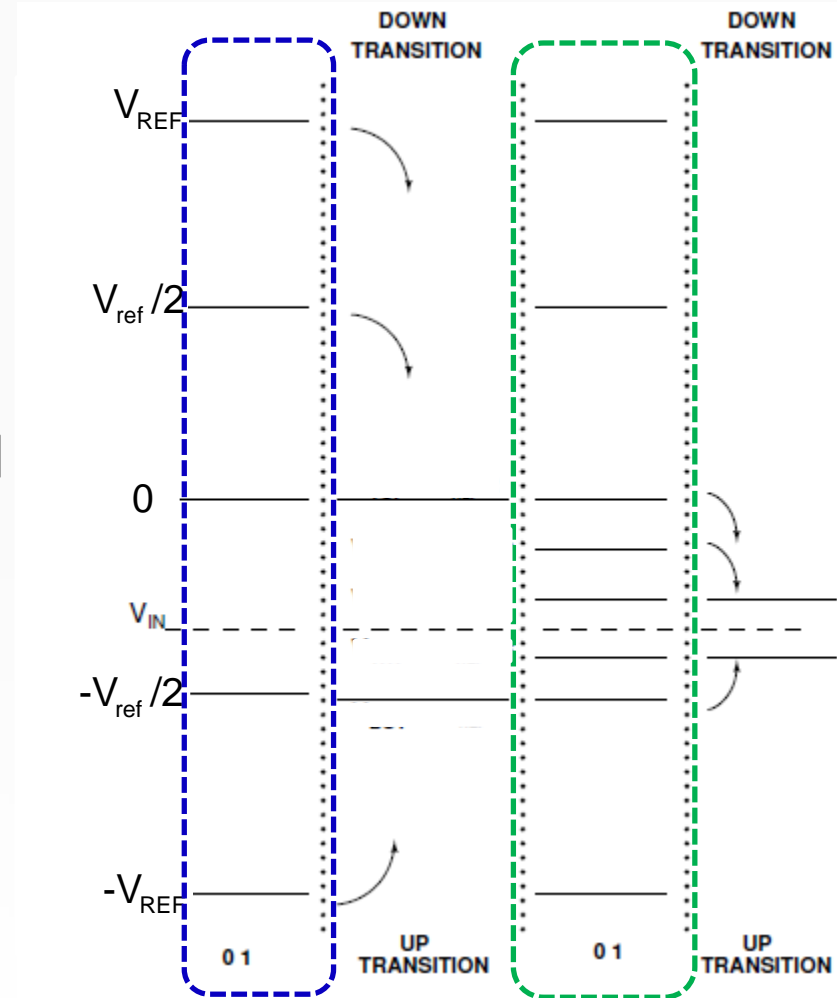


Fig. 17.13 Flow graph for 2-Bit Successive-Approximation



# 17. Nyquist-rate A/D converter

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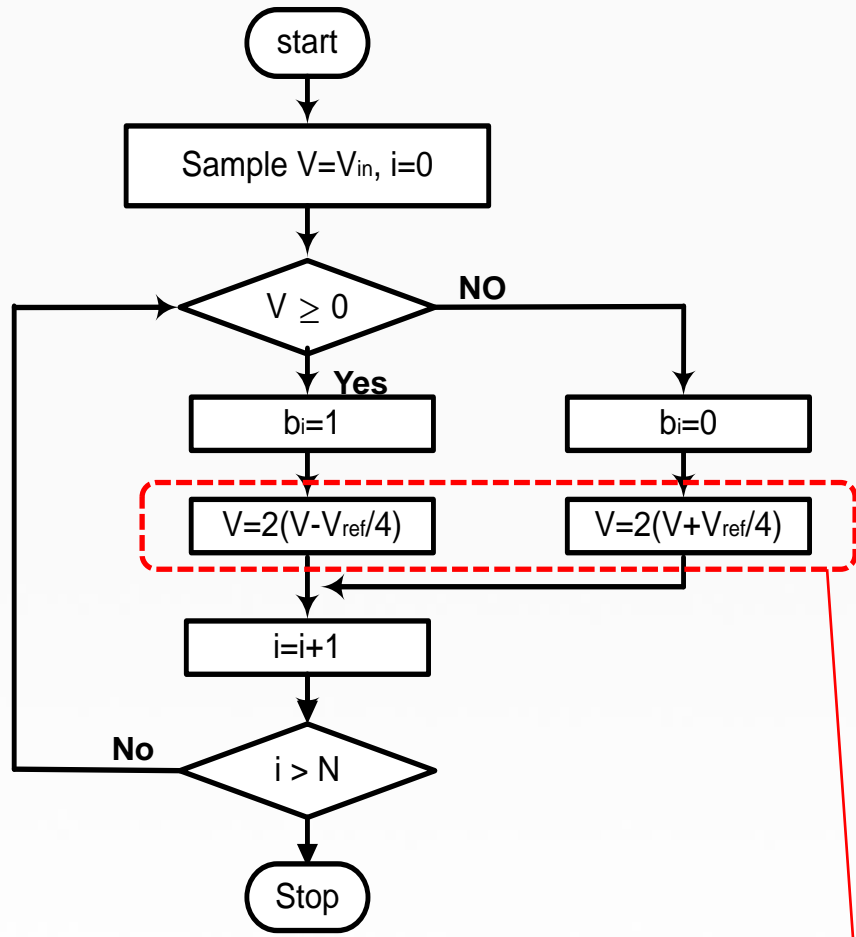
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# Basic Operation



Ex)  $N = 4\text{bit}$  ,  $V_{in} = \frac{10}{16}V_{ref}$

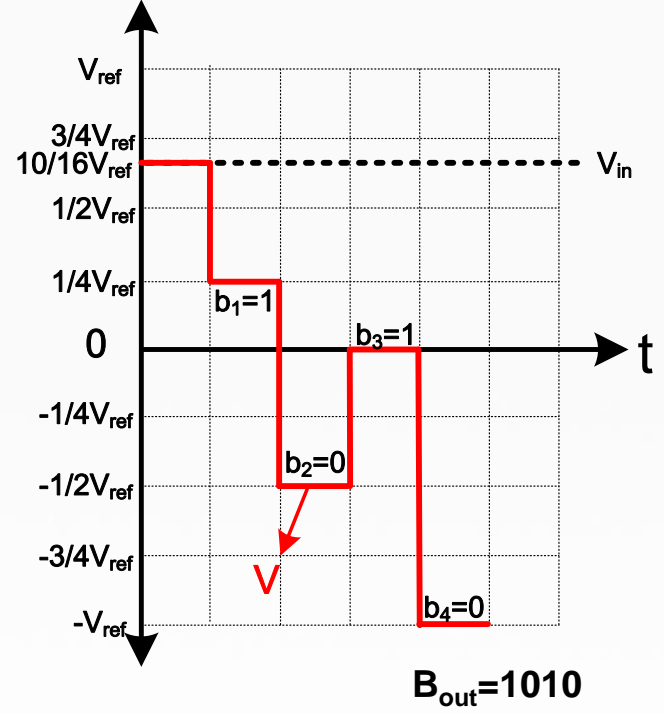
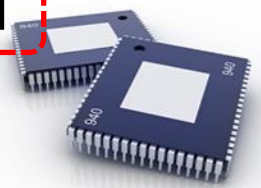


Fig. 17.14 Flow graph for the algorithmic and pipelined converter approach

$$V_{i+1} = 2[V_i + (b_i - 0.5)V_{ref} / 2]$$



# Algorithmic ADC block diagram

## Algorithmic converter

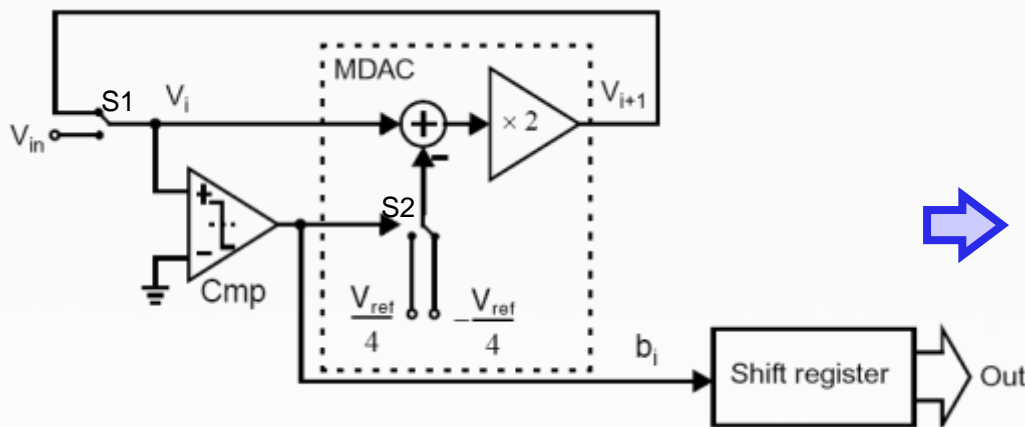
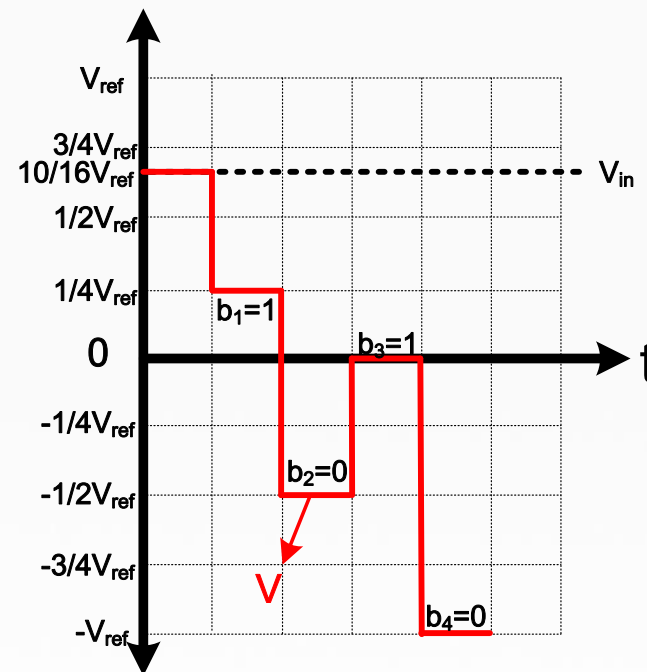


Fig. 17.15 Algorithmic converter

Ex)  $N = 4\text{bit}$ ,  $V_{in} = \frac{10}{16}FS$



$B_{out} = 1010$



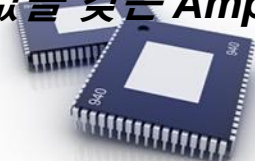
Small amount of analog circuitry



Requires  $N$  step for  $N$ -bit converter



SAR ADC와 달리, capacitor array & DAC 대신에 2배의 gain 값을 갖는 Amp 사용.



# Example 1

Find  $V_i$  and  $b_i(i=1,2,3,4,)$  during the operation of the 4-bit & 6bit algorithmic (or cyclic) converter. Assume that  $V_{in} = 1.23V$ ,  $V_{ref} = 10V$ .

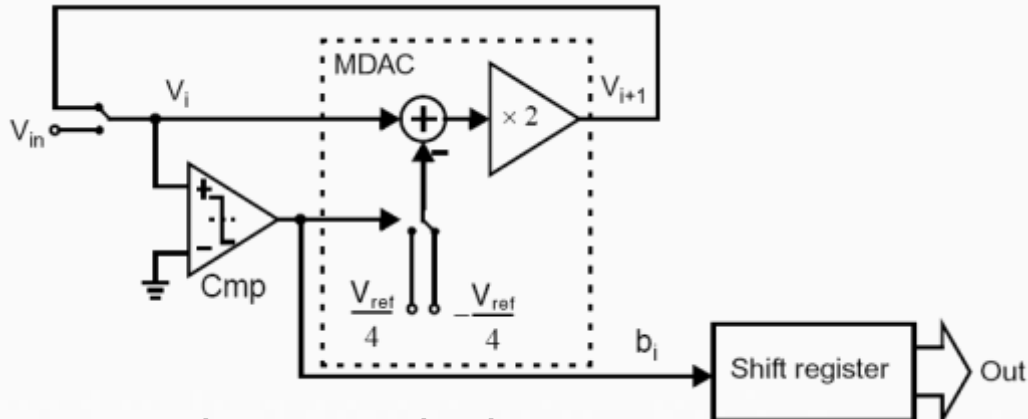
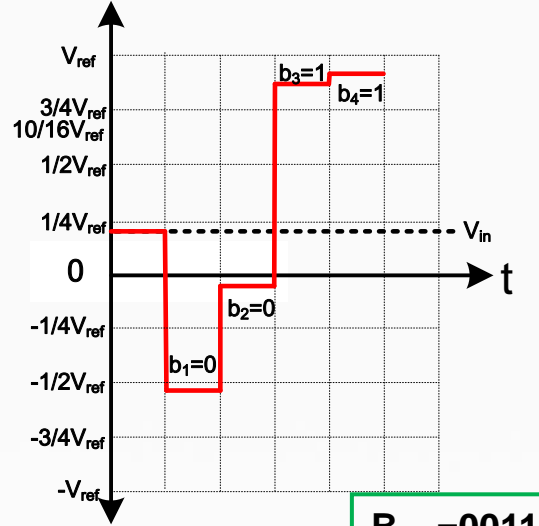


Fig. 17.15 Algorithmic converter



**B<sub>out</sub>=0011**

**- i = 1**

$$V_1 = V_{in} = 1.23V > 0$$

↓

$$V_1 \rightarrow 2 \left( V_1 - \frac{V_{ref}}{4} \right)$$

$$= -2.54 < 0$$

↓

$$b_1 = 0$$

**- i = 2**

$$V_2 = -2.54V < 0$$

↓

$$V_2 \rightarrow 2 \left( V_2 + \frac{V_{ref}}{4} \right)$$

$$= -0.08 < 0$$

↓

$$b_2 = 0$$

8

**- i = 3**

$$V_3 = -0.08V < 0$$

↓

$$V_3 \rightarrow 2 \left( V_3 + \frac{V_{ref}}{4} \right)$$

$$= 4.84 > 0$$

↓

$$b_3 = 1$$

**- i = 4**

$$V_4 = 4.84V > 0$$

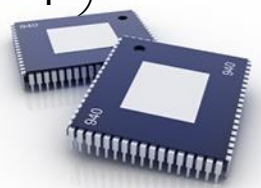
↓

$$V_4 \rightarrow 2 \left( V_4 - \frac{V_{ref}}{4} \right)$$

$$= 4.68 > 0$$

↓

$$b_4 = 1$$





# Example 1(Cont.)

Find  $V_i$  and  $b_i(i=1,2,3,4,)$  during the operation of the 4-bit & 6bit algorithmic (or cyclic) converter. Assume that  $V_{in} = 1.23V$ ,  $V_{ref} = 10V$ .

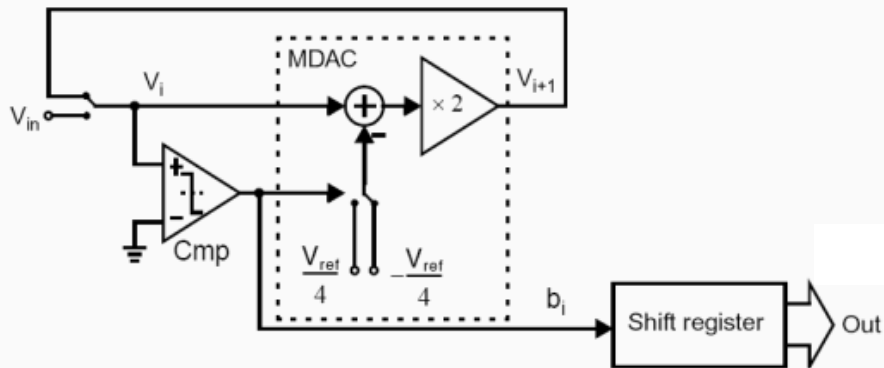


Fig. 17.15 Algorithmic converter

**- i = 5**

$$V_5 = 4.68V > 0$$

↓

$$V_5 \rightarrow 2\left(V_5 - \frac{V_{ref}}{4}\right)$$

$$= 4.36 > 0$$

↓

$$b_5 = 1$$

**- i = 6**

$$V_4 = 4.36V > 0$$

↓

$$V_6 \rightarrow 2\left(V_6 - \frac{V_{ref}}{4}\right)$$

$$= 3.72 > 0$$

↓

$$b_4 = 1$$

**- 4bit converter**

$$b_i = 0011$$



$$V_{DAC} = \frac{3}{16} \times 5 = 0.9375V$$

**- 6bit converter**

$$b_i = 001111$$



$$V_{DAC} = \frac{15}{64} \times 5 = 1.172V$$



0.2345V



# Example 2

What will happen if the **comparator** has an **offset of 30 mV** ?

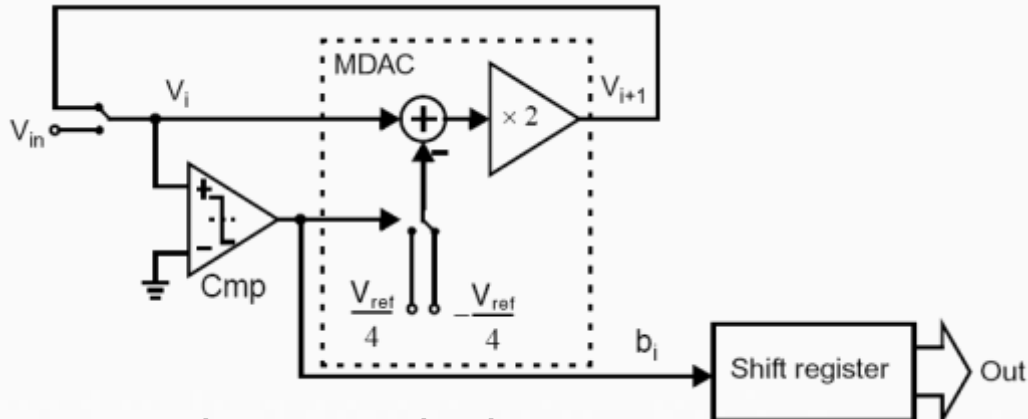
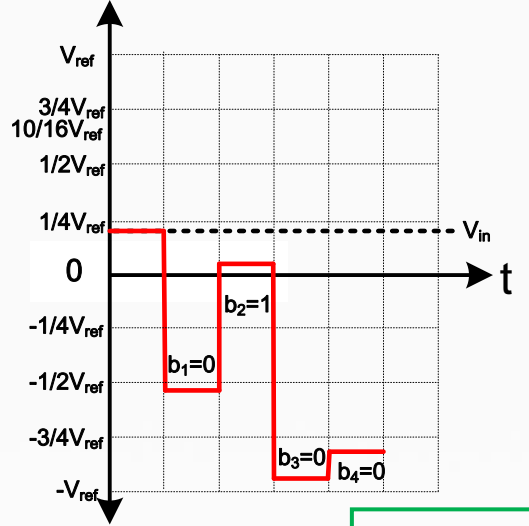


Fig. 17.15 Algorithmic converter



**B<sub>out</sub>=0100**

**- i = 1**

$$V_1 = V_{in} = 1.23V > 0$$

↓

$$V_1 \rightarrow 2 \left( V_1 - \frac{V_{ref}}{4} \right)$$

$$= -2.54 + 30m < 0$$

↓

**b<sub>1</sub> = 0**

**- i = 2**

$$V_2 = -2.51V < 0$$

↓

$$V_2 \rightarrow 2 \left( V_2 + \frac{V_{ref}}{4} \right)$$

$$= -0.02 + 30m > 0$$

↓

**b<sub>2</sub> = 1**

**- i = 3**

$$V_3 = 0.01V > 0$$

↓

$$V_3 \rightarrow 2 \left( V_3 - \frac{V_{ref}}{4} \right)$$

$$= -4.98 + 30m < 0$$

↓

**b<sub>3</sub> = 0**

**- i = 4**

$$V_4 = -4.95V < 0$$

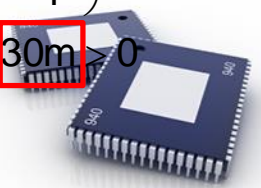
↓

$$V_4 \rightarrow 2 \left( V_4 + \frac{V_{ref}}{4} \right)$$

$$= -4.96 + 30m > 0$$

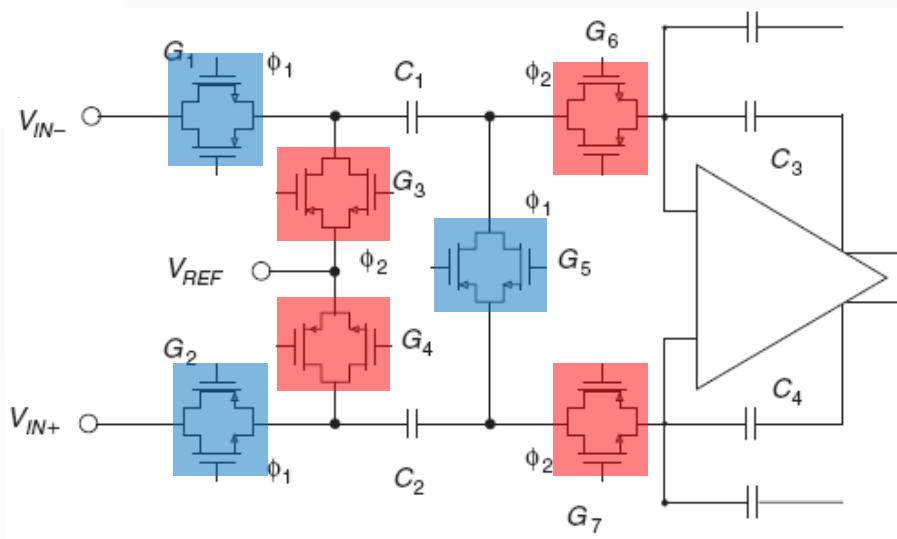
↓

**b<sub>4</sub> = 0**



# Algorithmic (or cyclic) A/D converters

## Multiply-by-two gain circuitry

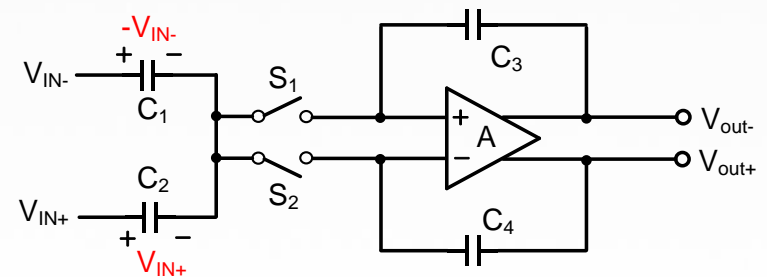
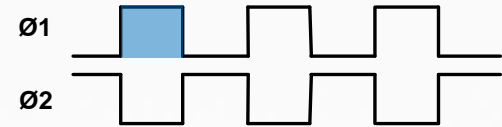


Multiply-by-two gain circuitry

- Operation
  - $\phi_1$  : G1, G2, G5
  - $\phi_2$  : G3, G4, G6, G7

①  $\phi_1$  (rising edge) : G1, G2, G5 turn on  $\Rightarrow V_{IN}$  sampling

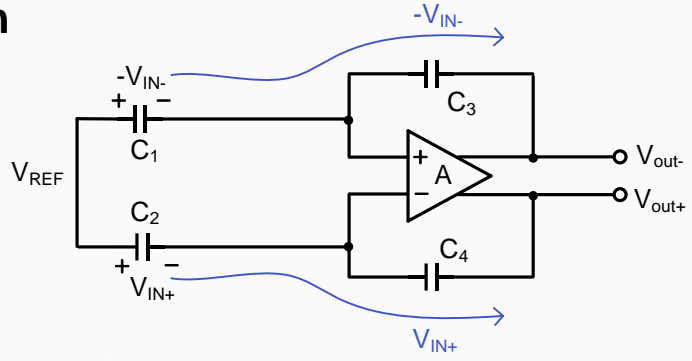
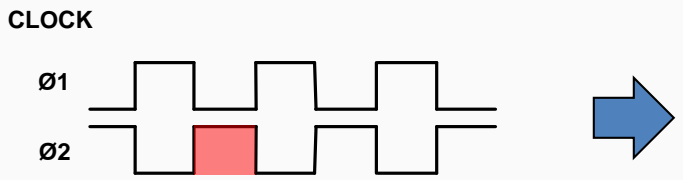
CLOCK



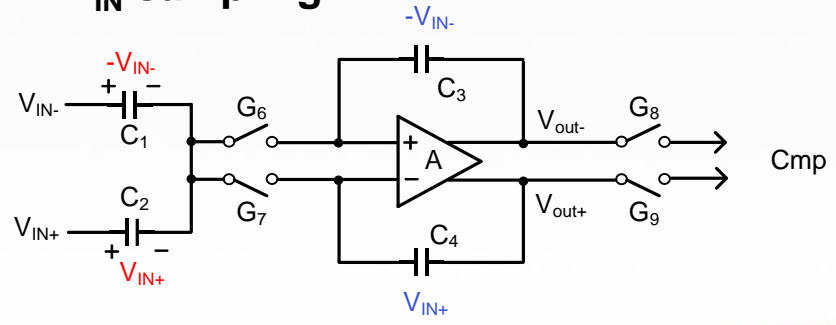
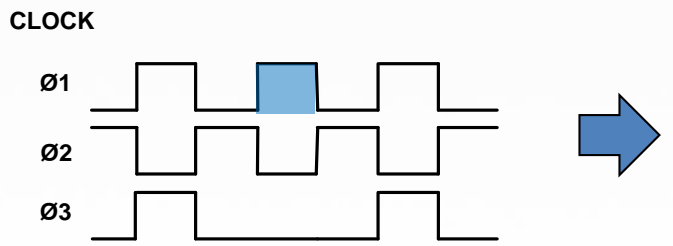
# Algorithmic (or cyclic) A/D converters

## Multiply-by-two gain circuitry

②  $\emptyset_2$  (falling edge) : G3, G4, G6, G7 turn on



③  $\emptyset_1$  (rising edge) : G1, G2, G5 turn on  $\Rightarrow V_{IN}$  sampling

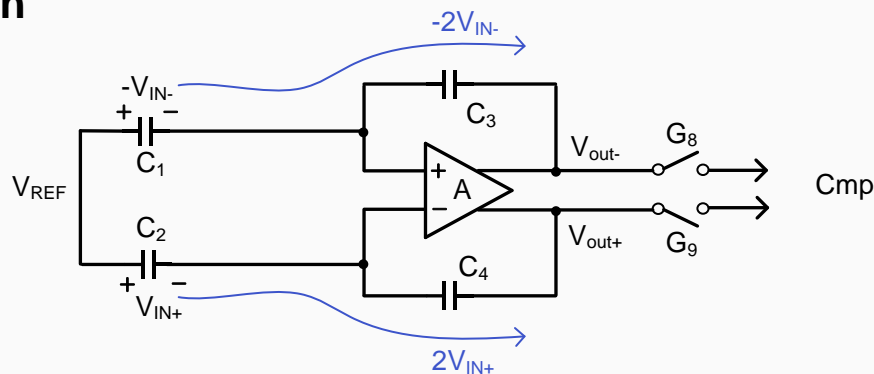
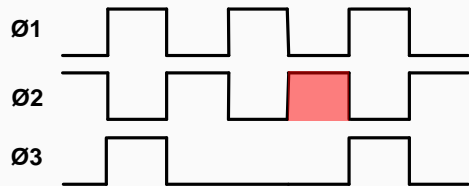


# Algorithmic (or cyclic) A/D converters

## Multiply-by-two gain circuitry

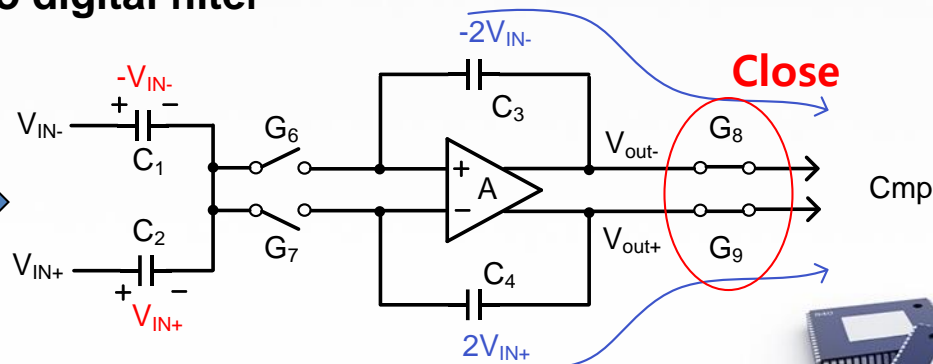
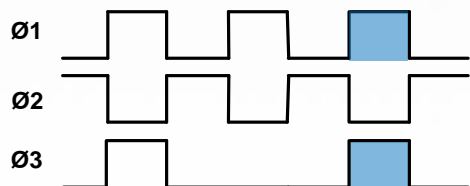
④  $\emptyset_2$  (falling edge) : G3, G4, G6, G7 turn on

CLOCK



⑤  $\emptyset_2$  (rising edge) : G1, G2, G5, G8, G9 turn on  
 $\Rightarrow V_{IN}$  sampling & send the signal to digital filter

CLOCK



# 17. Nyquist-rate A/D converter

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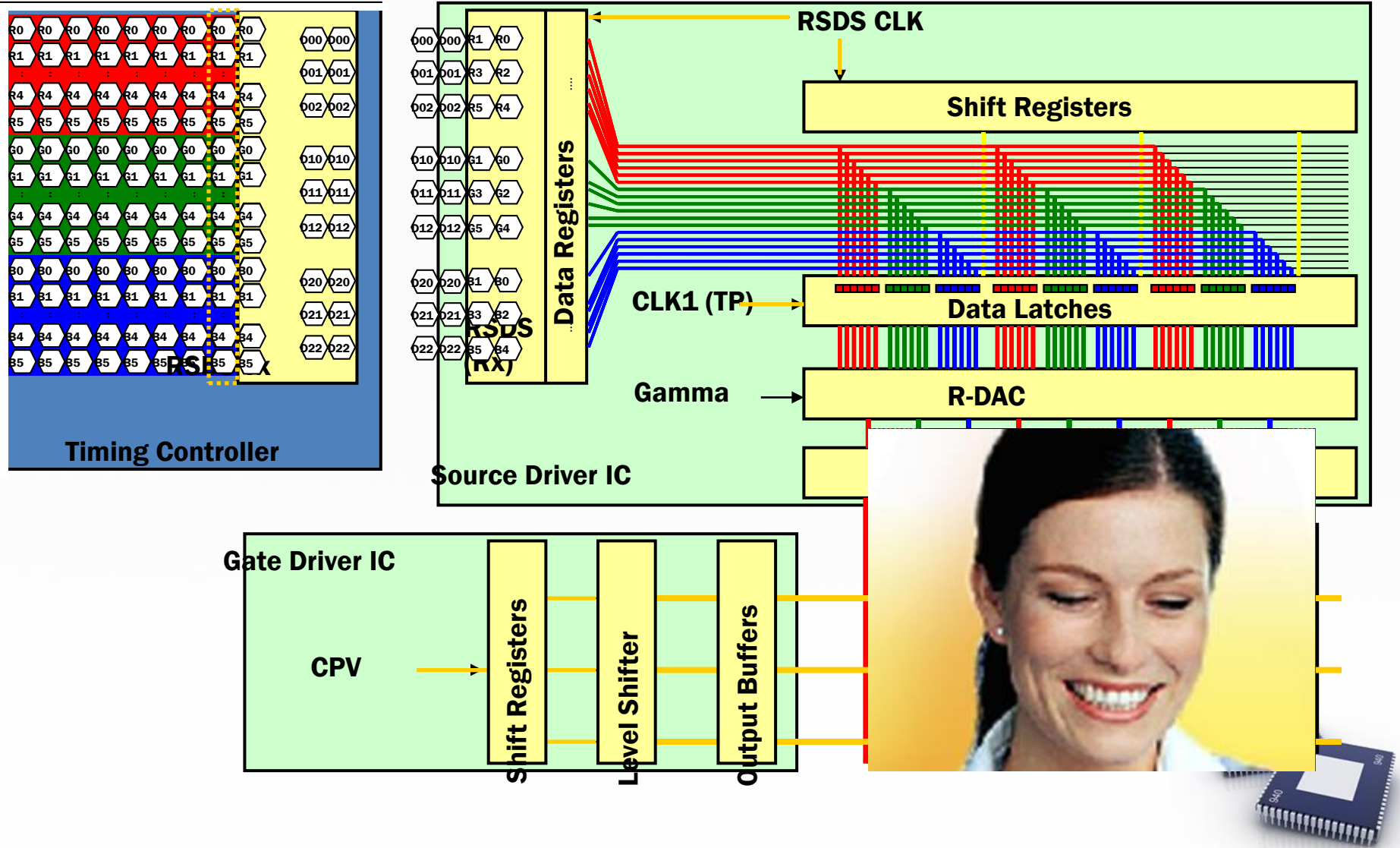
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# Operation of LCD



# Basic Operation

## SAR & Algorithmic converter

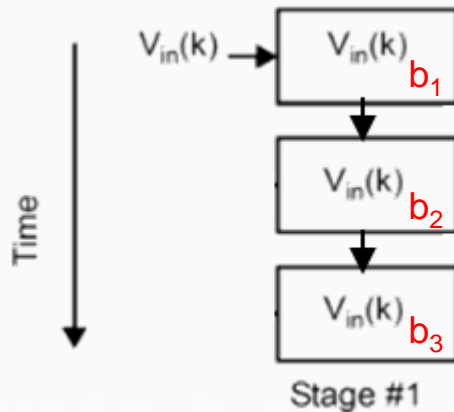


Fig. The signal flow in SAR & algorithmic converter.

- 😊 Area & power consumption ↓
- ☹️ Requires N step for N-bit converter

## Pipelined converter

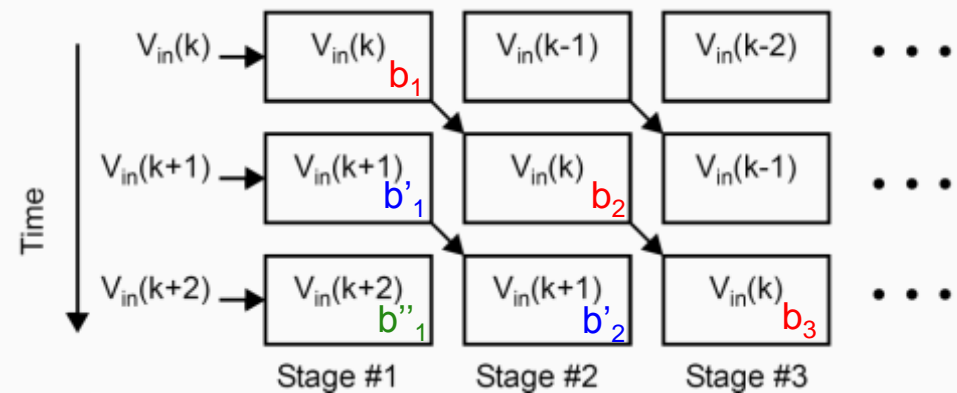


Fig. 17.17 The signal flow in a pipelined A/D converter.

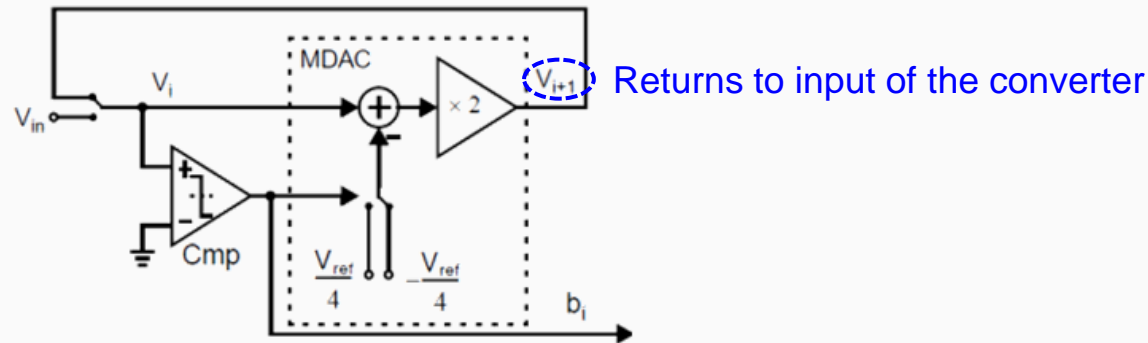
- 😊 Higher throughput
- ☹️ Need initial N step latency  
Area & power consumption ↑





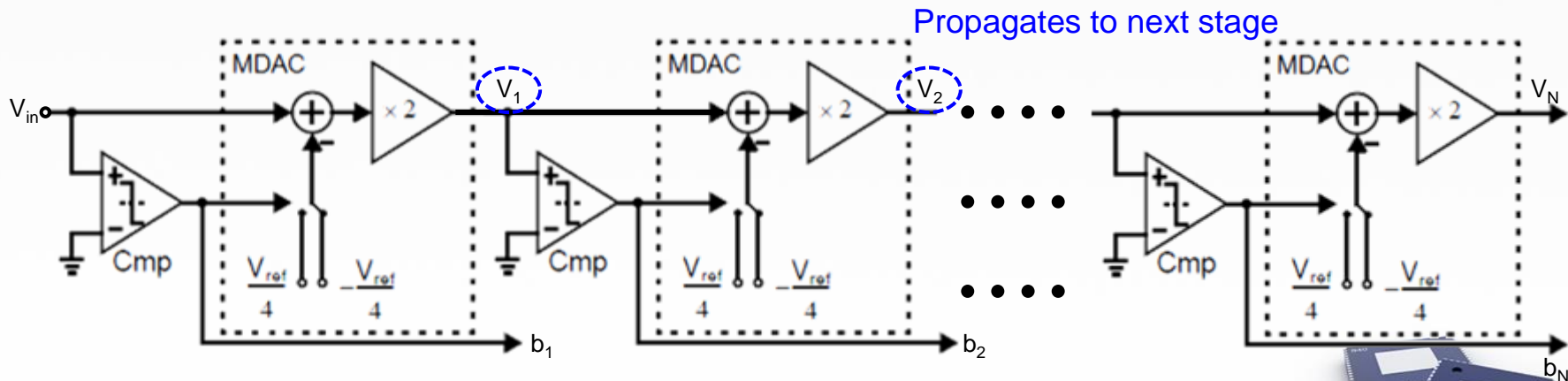
# Basic Operation

## Algorithmic converter

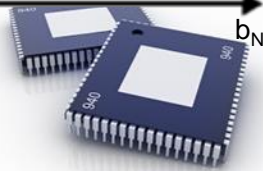


→ Iteration of **single** analog circuit

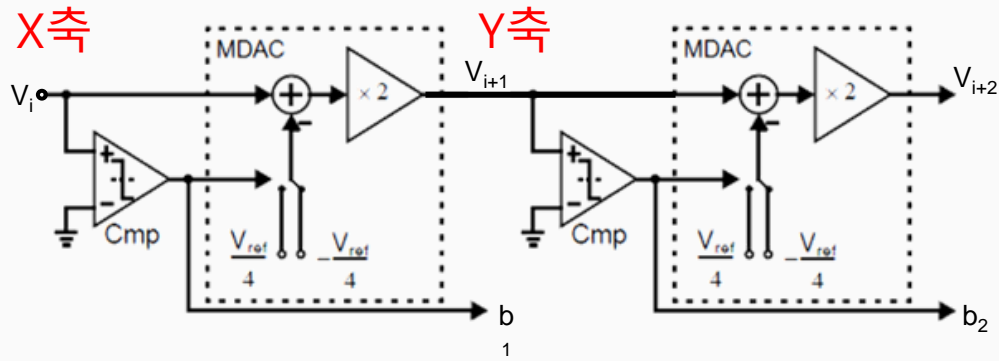
## Pipelined converter



→ Iteration of **N-stage** analog circuits



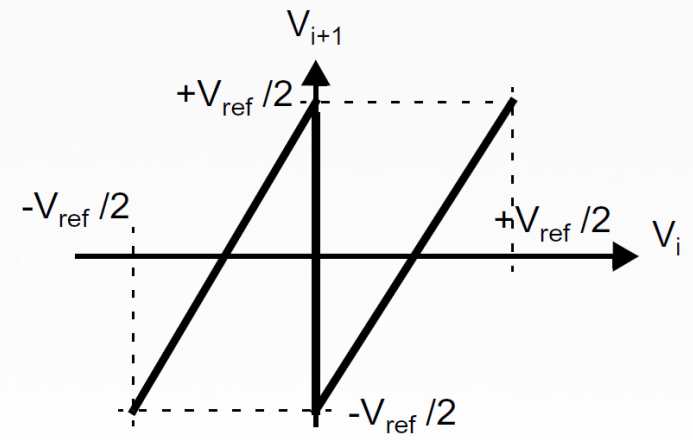
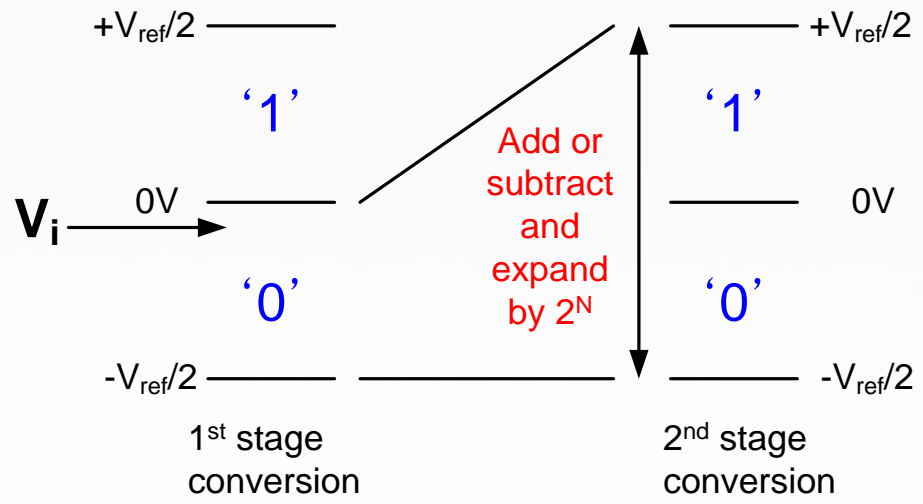
# One-Bit-Per-Stage Pipelined ADC



$$V_{i+1} = 2[V_i + (b_i - 0.5)V_{ref} / 2]$$

Residue signal (17.39)

$$V_{i+1} = 2V_i \pm V_{ref} / 2$$



😊 Each stages consists of same circuits in pipeline ADC



# One-Bit-Per-Stage Pipelined ADC

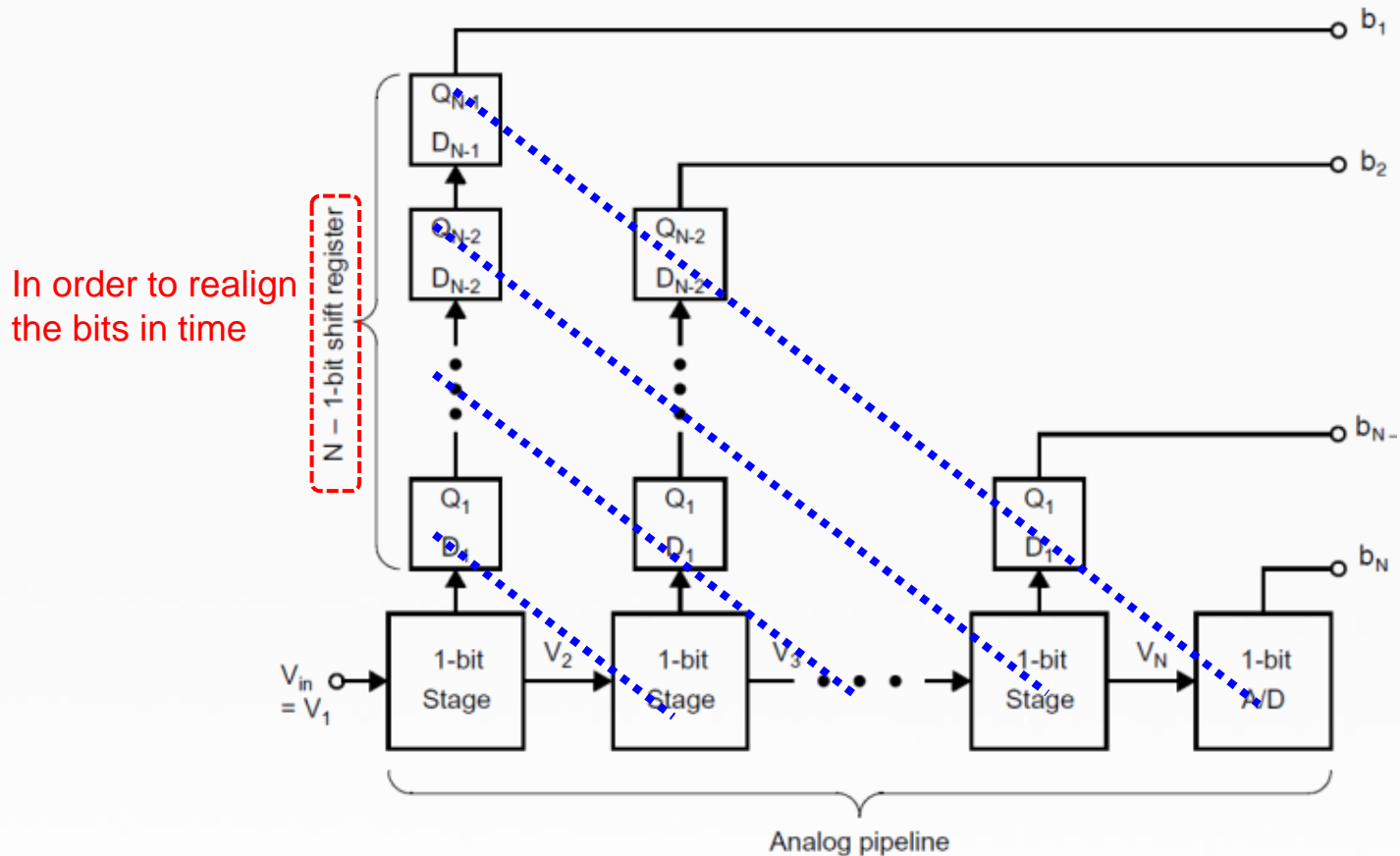
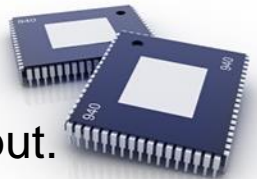


Fig. 17.19 A one-bit-per-stage pipelined A/D converter [Martin, 1981].

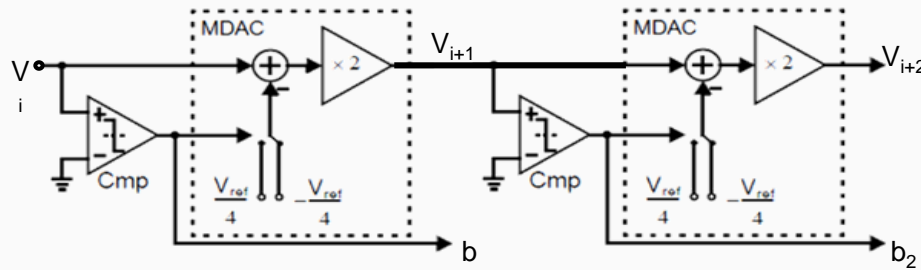
$$\hat{V}_{in} = V_{ref} \sum_{i=1}^N (b_i - 0.5) 2^{-i} \quad (17.40)$$

- Residue voltage : N-bit quantized estimate of the original input.



# Example 3

Consider the operation of a 3-bit, one bit per stage pipelined converter with  $V_{ref} = 1V(0V \pm 0.5V)$ . Find **final output digital code**, all the **residue voltages**,  $V_i$ , and the for a sampled input voltage of  $V_{in} = 240mV$ .



$$V_1 = V_{in} = 240mV \quad V_{i+1} = 2[V_i - (b_i - 0.5)V_{ref} / 2] \quad (17.39)$$

$$\textcircled{1} \quad b_1 = 1 \quad V_2 = 2(V_1 - V_{ref} / 4) = -20mV$$

$$\textcircled{2} \quad b_2 = 0 \quad V_3 = 2(V_2 + V_{ref} / 4) = 460mV$$

$$\textcircled{3} \quad b_3 = 1 \quad \blacktriangleright \text{Final output code } b_1 b_2 b_3 = 101$$

$\blacktriangleright$  All the residue voltage :

$$\hat{V}_{in} = V_{ref} \sum_{i=1}^N (b_i - 0.5)2^{-i} \quad (17.40)$$

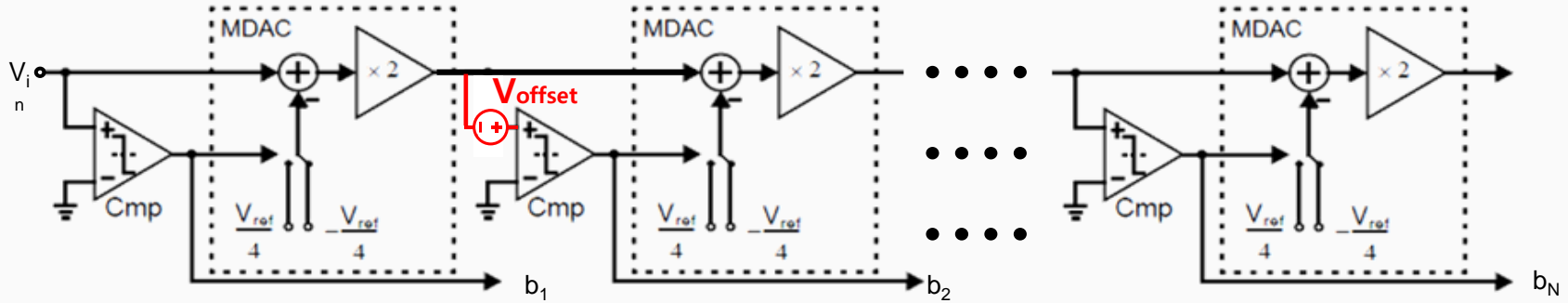
$$\hat{V}_{in} = 1 \times \left[ (1 - 0.5)2^{-1} \times (0 - 0.5)2^{-2} \times (1 - 0.5)2^{-3} \right]$$

$$\hat{V}_{in} = 187.5mV$$



# Example 4

What will happen if the **comparator** in the second stage has an **offset of 30 mV** ?



$$V_1 = V_{in} = 240mV \quad V_{i+1} = 2[V_i - (b_i - 0.5)V_{ref} / 2] \quad (17.39)$$

①  $b_1 = 1$        $V_2 = 2(V_1 - V_{ref} / 4) = -20mV$  (+ 30mV)

②  $b_2 = 1$        $V_3 = 2(V_2 - V_{ref} / 4) = -540mV$

③  $b_3 = 0$

➤ Final output code  $b_1 b_2 b_3 = 110$

$$\hat{V}_{in} = V_{ref} \sum_{i=1}^N (b_i - 0.5)2^{-i} \quad (17.40)$$

$$\hat{V}_{in} = 375mV$$



**오프셋**에 의한 입력전압과의 오차: **135 mV**,  
(오프셋이 없을 경우: **52.5 mV**)

