

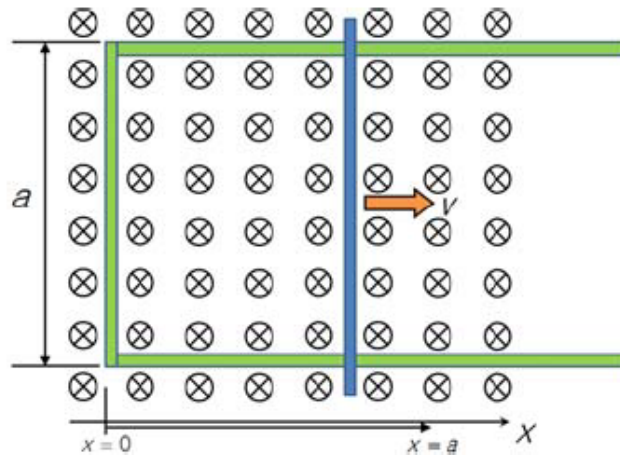
# Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8<sup>th</sup> and 9<sup>th</sup> Ed.
- The rest is made by me.

# 2012년 2차 시험

**Problem 1. (25 points)** As shown in Fig. 1, a straight segment of conducting wire is being pulled by an external force  $F_{ext}$  to the right and slides without friction at a constant velocity  $v$  in a vertical magnetic field  $\vec{B}$  that is uniform. The electrical resistivity  $\rho$  of the 'C-shaped' track (fixed in space) and that of the sliding segment are the same, and their cross-sectional areas  $A$  are also identical.

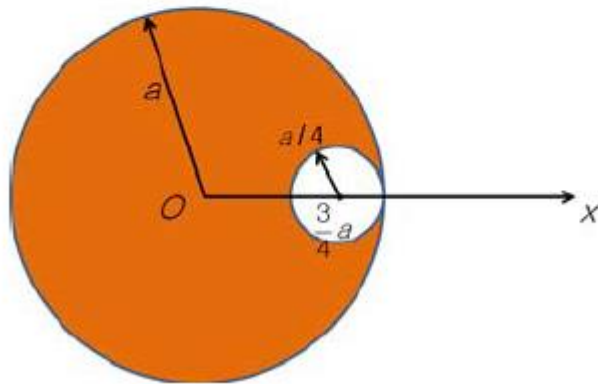
- (a) What will be the direction and magnitude of the induced current  $I$ ?
- (b) What should be the  $F_{ext}$  for maintaining  $v$ ?
- (c) How much energy is needed to bring the sliding segment from  $x = 0$  to  $a$ ?



<Fig. 1>

**Problem 2-A. (25 points)** Figure 2-A shows the cross-section of a long cylindrical conductor with radius  $a$ , containing a long cylindrical hole of radius  $a/4$ . The central axes of the cylinder and hole are parallel. The current density  $J$  is uniform over the shaded area.

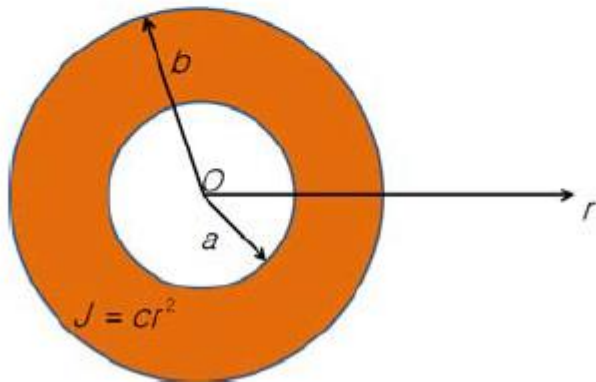
- (a) What is the magnetic field  $B(x)$  for  $0 < x < a/2$ ?
- (b)  $B(x)$  for  $a/2 < x < a$ ?
- (c)  $B(x)$  for  $x > a$ ?



<Fig. 2-A>

**Problem 2-B. (25 points)** Figure 2-B shows the cross-section of a long conducting cylinder with inner radius  $a$  and outer radius  $b$ . The cylinder carries a current out of the page, and the magnitude of the current density in the cross section is given by  $J = cr^2$  ( $c$ : constant,  $r$ : radial distance from the center  $O$ ).

- (a) What are the magnitude of  $\vec{B}$  and its direction for  $a < r < b$ ?
- (b) What are the magnitude of  $\vec{B}$  and its direction for  $r > b$ ?
- (c) Plot  $B(r)$  as a function of  $r$ .



<Fig. 2-B>

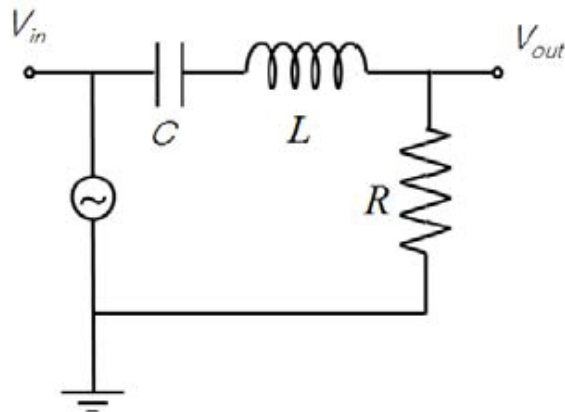
**Problem 3. (25 points)** The impedance  $Z$  of a series RLC circuit is

$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$  and the phase difference  $\phi$  between  $V_{in}(t)$  and  $I(t)$  is  $\phi = \tan^{-1}\left(\frac{\omega L - (\omega C)^{-1}}{R}\right)$ . Assume that  $V_{in} = V_0 \sin(\omega t)$  and  $I(t) = I_0 \sin(\omega t - \phi)$  is the time-varying current along the circuit.

(a) If we remove the capacitor (i.e.,  $C = \infty$ ), the remaining circuit can be viewed as a RL low-pass filter of the input  $V_{in}(t)$ , yielding  $V_{out}(t)$ . For what value of  $\omega$ ,  $V_{out}/V_{in} = 1/\sqrt{2}$ ? Draw  $V_{out}/V_{in}$  as a function of  $\omega$ , schematically.

(b) Then, consider the case in which  $C$ ,  $R$  and  $L$  all are finite. For what value of  $\omega$  the current  $I(t)$  will have the maximum amplitude  $I_{max}$ ? What will be the maximum  $I_{max}$  and the corresponding phase difference  $\phi_{max}$ ?

(c) When  $R$  becomes 0 (while  $C$  and  $L$  are kept finite), what will become  $\phi$ ?



<Fig. 3>

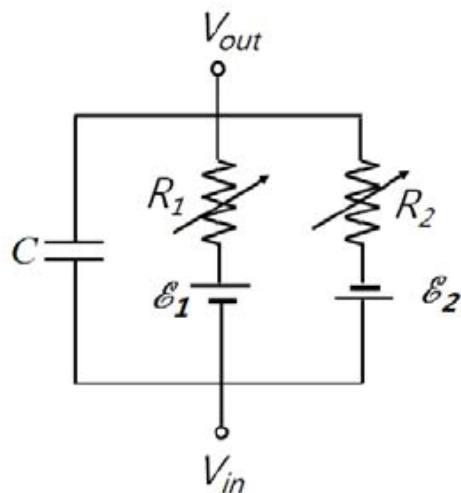
**Problem 4. (25 points)** The electrical activity across a cell membrane can be described by a simple circuit model depicted in Figure. 4, which includes two constant voltage sources,  $\mathcal{E}_1$  and  $\mathcal{E}_2$ , two variable resistors,  $R_1$  and  $R_2$ , and one capacitor  $C$ .

(a) Based on the Kirchhoff's circuit rules, write down the equation for the membrane potential  $V = V_{in} - V_{out}$ , and describe its steady state value  $V_\infty = V(t = \infty)$ , using the given set of parameter values.

(b) Suddenly, a biological event is triggered and  $R_2$  became very large compare to  $R_1$  so that one can regard  $R_2$  as  $\infty$ . What will become the new steady state membrane potential  $V_{new}$ ?

(c) How long will it take for  $V - V_{new}$  to reach the value of  $(V_\infty - V_{new})/e$ ?

[Note. The general solution of a first-order linear differential equation  $\frac{dx(t)}{dt} + \alpha x(t) + \beta = 0$ , is  $x(t) = x(0)e^{-\alpha t} - \frac{\beta}{\alpha}$ .]

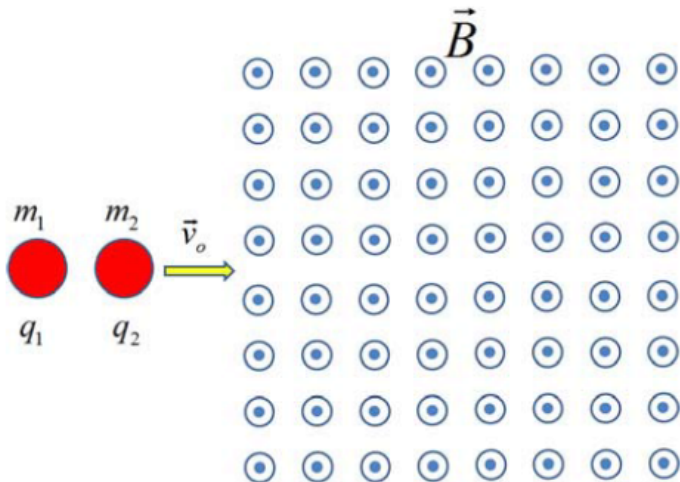


<Fig. 4>

# 2011년 2차 시험

**Problem 1. (25 points)** Two particles with masses  $m_1$  and  $m_2$  and charges  $q$  and  $2q$  enter a uniform magnetic field of strength  $B$  with an initial velocity  $\vec{v}_0$ , as shown in Fig. 1. In the magnetic field, they move in semi-circles with radii  $R$  and  $2R$ , respectively.

- (a) What is the ratio of their masses?
- (b) What are the time durations  $T_1$  and  $T_2$  of the two masses between the entrance to the field  $\vec{B}$  and its exit from the field?
- (c) If there is an electric field that would cause the particles to move in a straight line in the magnetic field, what should be its magnitude and direction?



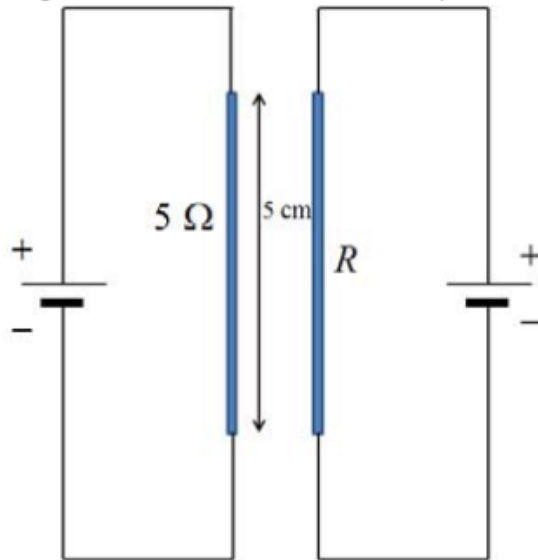
<Fig. 1>

**Problem 2. (25 points)** Two straight wires, each 5 cm long, are connected to two 9 V batteries as shown in Fig. 2. The resistance of the first wire is  $5 \Omega$ , and that of the other wire is unknown  $R$ .

(a) What is the magnetic field  $\vec{B}$  induced by the  $5 \Omega$  wire like? [Use Ampere's law and ignore the end effect.]

(b) If the separation between the wires is 4 mm, what value of  $R$  will produce a force of magnitude  $4 \times 10^{-5} \text{ N}$  between them?

(c) If the 9 V batteries are replaced by 18 V batteries, what will become the magnitude of the force for the separation of 4 mm?

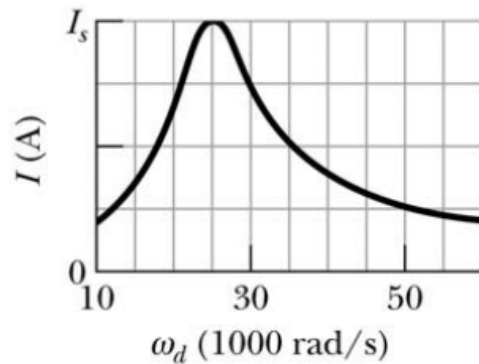


<Fig. 2>



**Problem 3. (25 points)** The current amplitude  $I$  versus driving angular frequency  $\omega_d$  for a driven  $RLC$  circuit (connected in series) is given in Fig. 3, where the vertical axis scale is set by  $I_s = 4.00$  A. The inductance  $L = 200 \mu\text{H}$ , and the emf amplitude  $V_m$  is 8.0 V.

- What is  $C$ ?
- What is  $R$ ?
- When  $L \neq 0$  but  $R = C = 0$ , what will be the phase difference  $\phi$  [when  $V_{emf}(t) = V_m \sin(\omega_d t)$  and  $I(t) = I_m \sin(\omega_d t - \phi)$ ].
- When  $L \neq 0$ ,  $C \neq 0$ , but  $R = 0$ , draw the phase difference  $\phi(\omega_d)$  as a function of  $\omega_d$ .

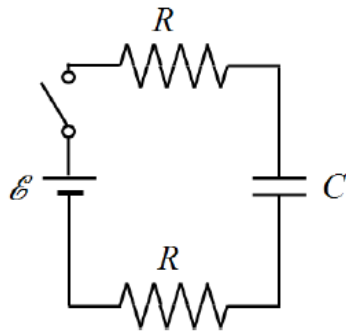


<Fig. 3>

# 2010년 2차 시험

**Problem 1.** (20 points) Two resistors (resistance  $R$ ) and a capacitor (capacitance  $C$ ) are serially connected to a constant electromotive force  $\mathcal{E}$ . When the switch was open, there was no charge in the capacitor. Once a switch is closed, the charge stored in the capacitor can be expressed as  $q(t) = \alpha[1 - \exp(-\gamma t)]$ , where  $\alpha$  and  $\gamma$  are constants and  $t$  is time.

- (a) Apply Kirchhoff's loop rule to find an equation including  $q(t)$ .
- (b) Substitute the  $q(t)$  given above to the equation and find the constants  $\alpha$  and  $\gamma$ .
- (c) It has passed long enough after the switch was closed. How much energy has been stored to the capacitor?



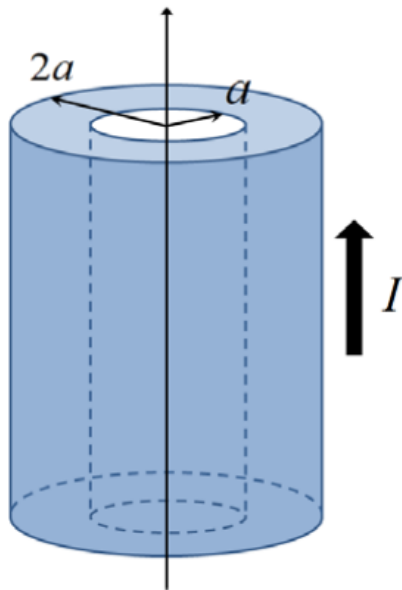
<Fig. 1>

**Problem 2.** (30 points) A straight conductor carries a current  $I$  that is uniformly distributed over the cross-sectional area bounded by two concentric circles of radii  $a$  and  $2a$ , as shown in Fig. 2.

(a) Find the direction of the magnetic field at hollow region, inside and outside the conductor made by the current.

(b) Find the magnitude of the magnetic field at hollow region, inside and outside the conductor.

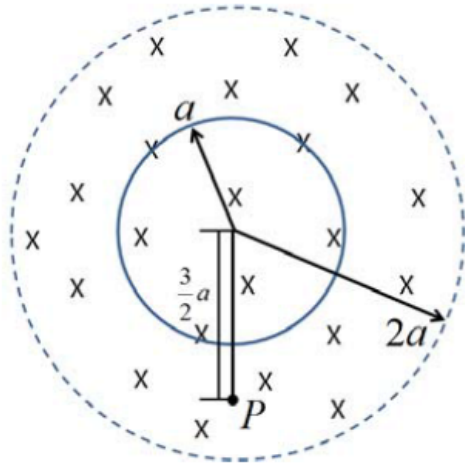
(c) Energy density in  $B$  field is  $u_B = \frac{B^2}{2\mu_0}$ . Find the energy stored in the region  $2a < r < 3a$ , where  $r$  is the distance from the axis of the conductor. The length of the region is  $l$ .



<Fig. 2>

**Problem 3.** (30 points) Magnetic field  $B = \beta t$ , increases with time at a circular region with radius  $2a$ . ( $\beta$  is a positive constant.) On the same plane, a closed conducting wire is placed as shown in Fig. 3. The net resistance of the wire is  $R$ .

- Find the direction and magnitude of an induced current.
- Find the power transferred from the source of the magnetic field.
- Find the electric field at point  $P$ . (The point  $P$  is located  $\frac{3}{2}a$  away from the center of the circle.)



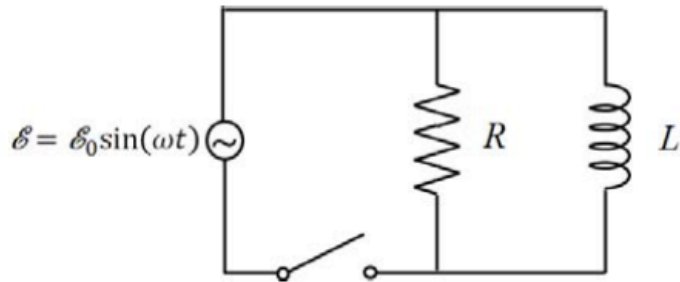
<Fig. 3>

**Problem 4.** (20 points) A resistor (resistance  $R$ ) and an inductor (inductance  $L$ ) are connected in parallel to an AC electromotive force  $\mathcal{E} = \mathcal{E}_0 \sin(\omega t)$ . The Kirchhoff's two rules can be applied to find a current.  $\omega = R/L$  is chosen. There was no current when the switch was open.

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha + \beta}{2}\right)\cos\left(\frac{\alpha - \beta}{2}\right).$$

(a) When the resistance  $R = \infty$ , the current can be expressed by  $I = I_0 \sin(\omega t - \phi)$ . Find  $I_0$  and  $\phi$ .

(b) When the resistance  $R \neq \infty$ , the current can be expressed by  $I = I_0 \sin(\omega t - \phi)$ . Find  $I_0$  and  $\phi$ .



<Fig. 4>