Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

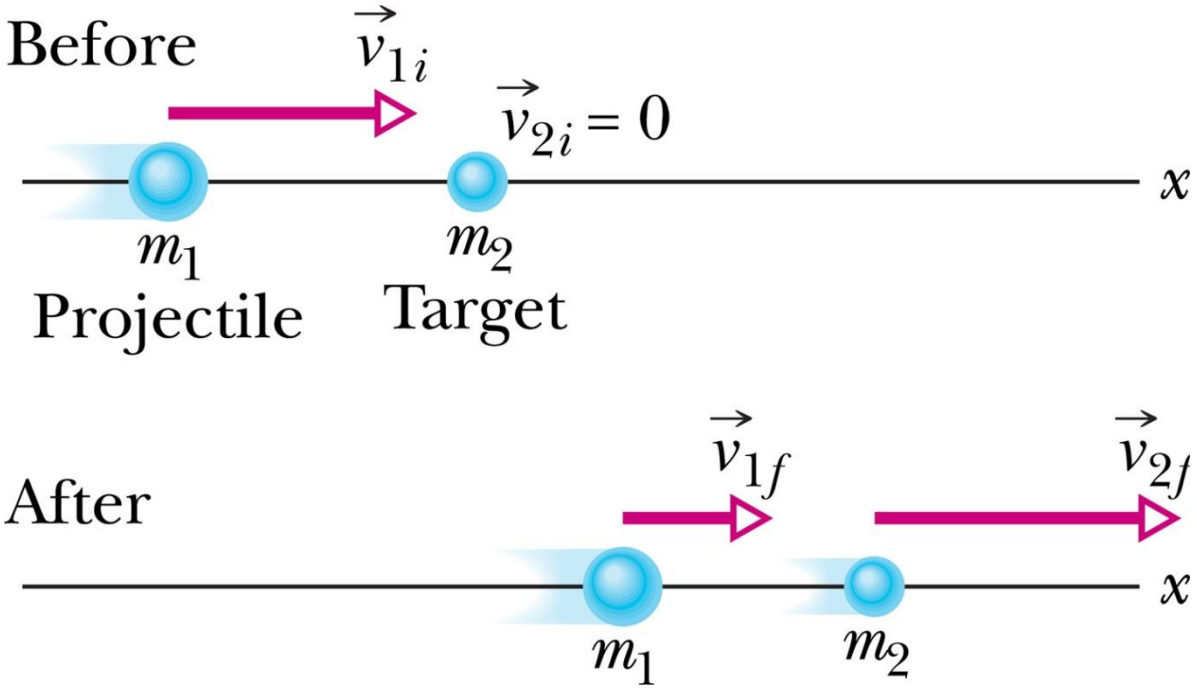
collision의 종류

(1) Elastic collision: 충돌 과정에서 운동에너지가 보존
(2) Inelastic collsion: 충돌과정에서 운동에너지가 보존되지 않음.

*완전 비탄성충돌: 두 물체가 붙어버리는 경우

충돌의 종류에 따라 운동에너지는 보존되거나 보존되지 않지만, 운동량은 항상 보존된다.

1D elastic collison



충돌 전 운동에너지 = 충돌 후 운동에너지

$m_1 v_{1i} = m_1 v_{1f} + m_2 (v_{1i} + v_{1f})$

(1) 운동량 보존

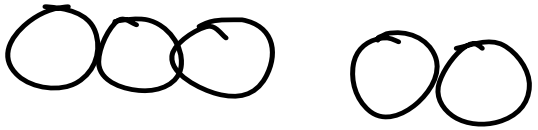
$(m_1 + m_2) v_{1f} = (m_1 - m_2) v_{1i}$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f} \longrightarrow m_1 (v_{1i} - v_{1f}) = m_2 v_{2f}$$

$v_{1i} + v_{1f} = v_{2f}$

(2) 운동에너지 보존

$v_{2f} =$



$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 \longrightarrow m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 v_{2f}^2$$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{i1}, \quad v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i}$$

1) 같은 질량 $(m_1 = m_2)$ $\quad v_{1f} = 0, \quad v_{2f} = v_{1i}$

2) 질량이 큰 표적 $(m_2 \gg m_1)$ $\quad v_{1f} \approx -v_{1i}, \quad v_{2f} \approx \frac{2m_1}{m_2} v_{1i}$

3) 질량이 큰 발사체 $(m_1 \gg m_2)$ $\quad v_{1f} \approx v_{1i}, \quad v_{2f} \approx 2v_{1i}$

$v_{2f} = v_{1i} - v_{2i} + \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$

일반적인 1차원 탄성충돌

$v_{1i} + v_{1f} = v_{2i} + v_{2f}$

$+ m_2 v_{1i} - m_2 v_{2i}$

$= m_1 v_{1f} + m_2 (v_{1i} + v_{1f} - v_{2i}) = v_{1f}(m_1 + m_2)$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \longrightarrow m_1 (v_{1i} - v_{1f}) = -m_2 (v_{2i} - v_{2f})$$

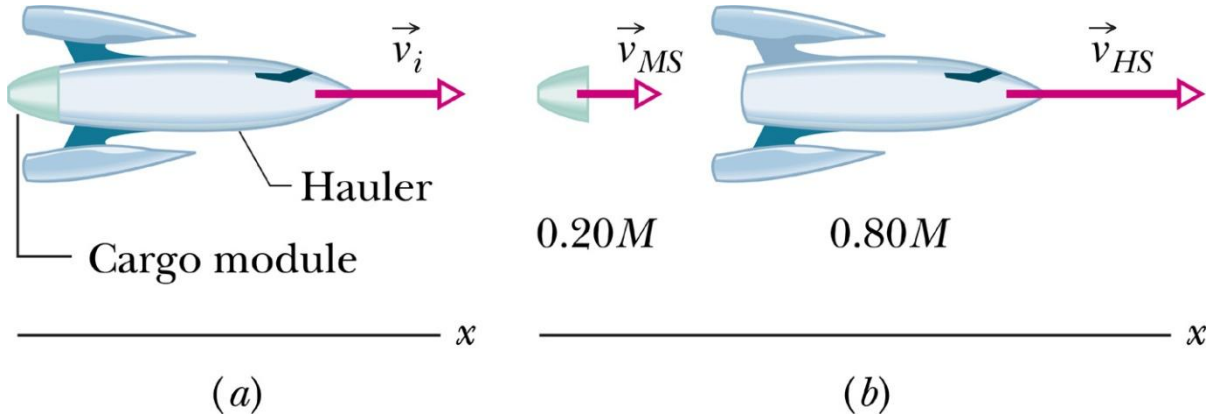
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\longrightarrow m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = -m_2 (v_{2i} - v_{2f})(v_{2i} + v_{2f})$$

$$\begin{aligned} v_{1f} &= \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}, \\ v_{2f} &= \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}. \end{aligned}$$

Sample problem



$$M; v_i = 2100\text{km/h},\ 0.80M; v_f = 2600\text{km/h},\ 0.20M; v_{\text{rel}} = 500\text{km/h}$$

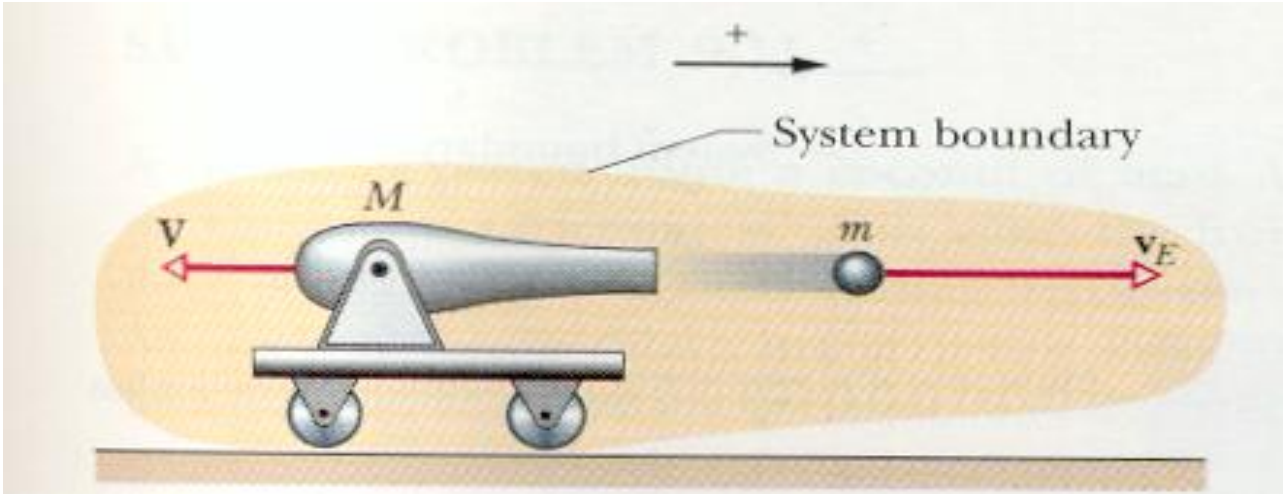
$$P_i = Mv_i$$
$$P_f = (0.20M)v_{MS} + (0.80M)v_{HS}$$

$$v_{HS} = v_{\text{rel}} + v_{MS}$$

$$Mv_i = 0.20M(v_{HS} - v_{\text{rel}}) + 0.80Mv_{HS}$$

$$v_{HS} = v_i = 0.20v_{\text{rel}} = 2200\ \text{km/h}$$

Sample problem



$$M = 1300\text{kg}$$
$$m = 72\text{kg}$$
$$\mathbf{v} = 55\text{m/s}$$

$(M+m)V+mv=0$

$m(v+V)$

지구에 대한 대포의 상대속도 V

지구에 대한 총알의 상대속도 v_E

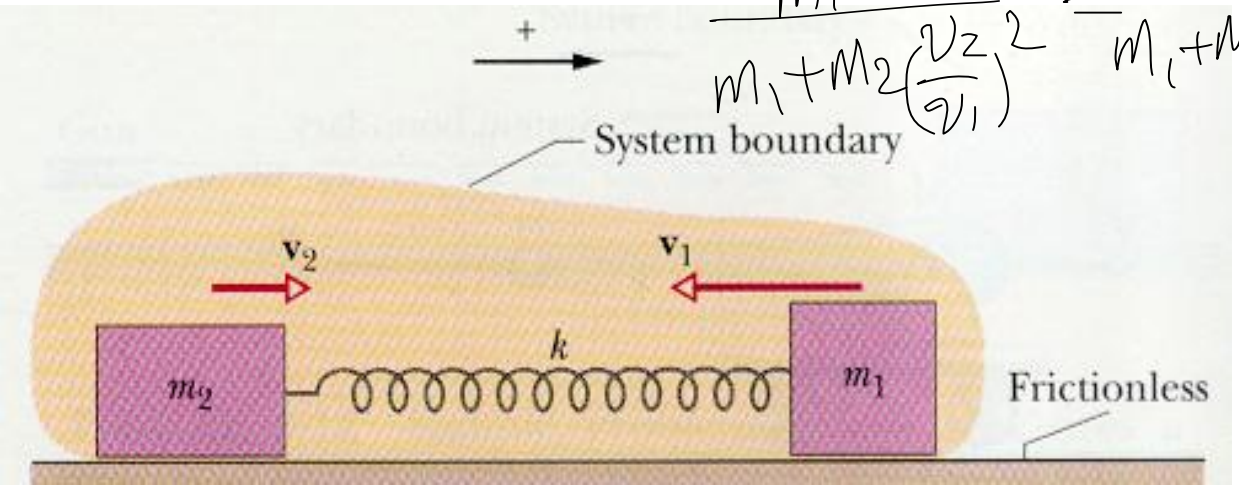
총알의 속도 $v = v_E - V$

$$P_i = 0$$
$$P_f = MV + mv_E$$

$$V = -\frac{mv}{M+m} = -2.9\text{m/s}$$

$$v_E = v + V = 52\text{m/s}$$

Sample problem



$$P_i = 0$$

$$P_f = m_1 v_1 + m_2 v_2$$

$$\frac{v_1}{v_2} = -\frac{m_2}{m_1}$$

운동에너지 부분율

$$\frac{K_1}{K_1 + K_2} = \frac{\frac{1}{2} m_1 v_1^2}{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2} = \frac{m_2}{m_1 + m_2}$$

$$\frac{K_2}{K_1 + K_2} = \frac{\frac{1}{2} m_2 v_1^2}{\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2} = \frac{m_1}{m_1 + m_2}$$

$$m_2 = 10 m_1$$

$$\frac{K_1}{K_1 + K_2} = \frac{10 m_1}{m_1 + 10 m_1} = 0.91$$

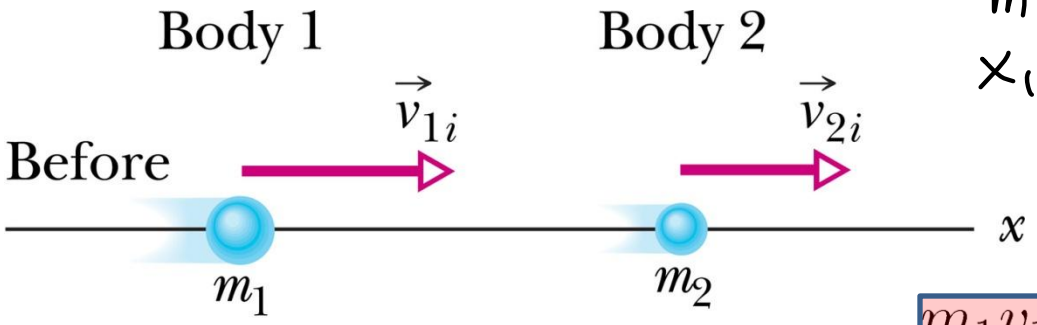
$$\frac{K_2}{K_1 + K_2} = \frac{m_1}{m_1 + 10 m_1} = 0.09$$

1D inelastic collision

m_1, m_2

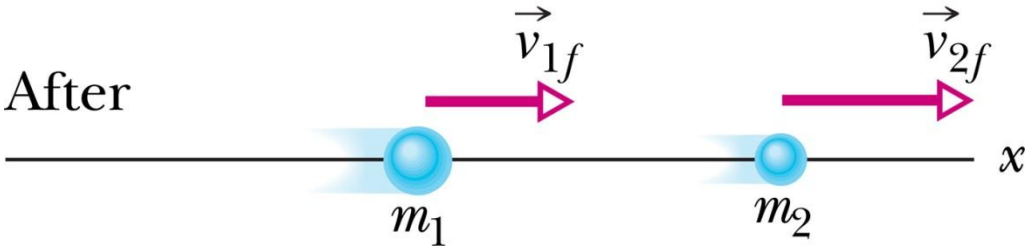
x_1 x_2

$$x_{com} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$



$$p_{1i} + p_{2i} = p_{1f} + p_{2f}$$

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$



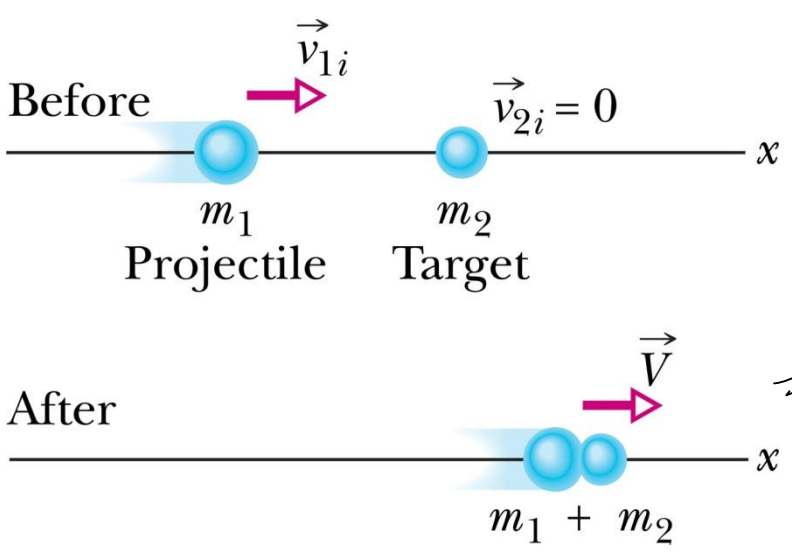
질량중심의 속도 $\mathbf{P} = M\mathbf{v}_{\text{com}} = (m_1 + m_2)\mathbf{v}_{\text{com}}$

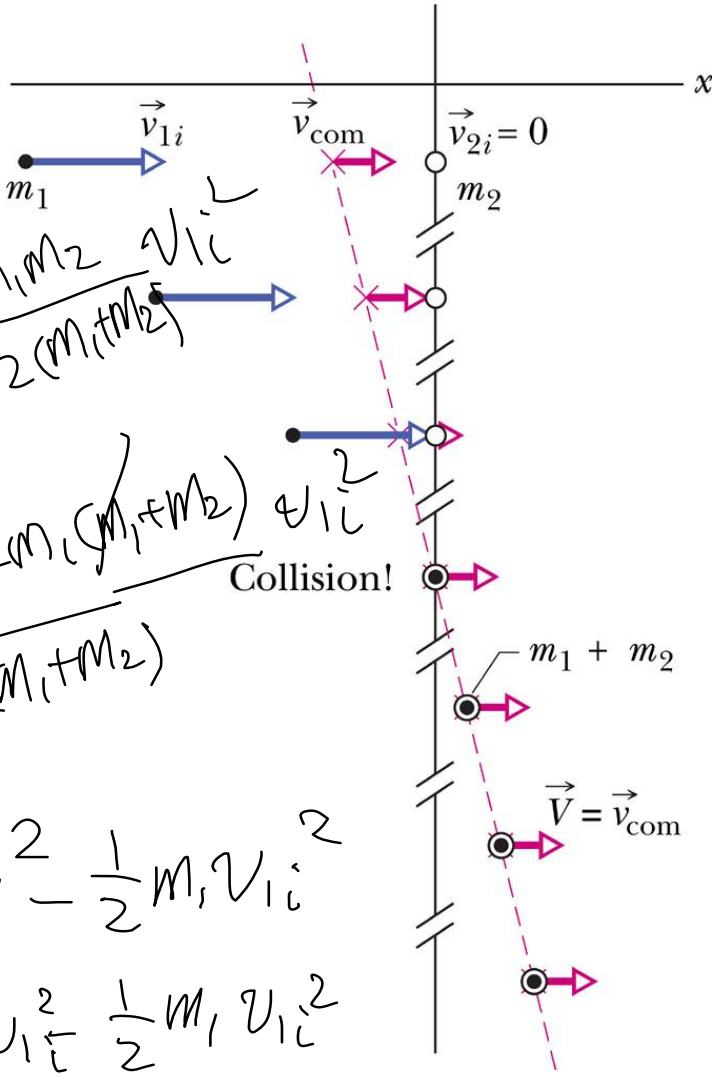
$$\mathbf{P} = \mathbf{p}_{1i} + \mathbf{p}_{i2} = \mathbf{p}_{1f} + \mathbf{p}_{2f}$$

$$\mathbf{v}_{\text{com}} = \frac{\mathbf{P}}{m_1 + m_2} = \frac{\mathbf{p}_{1i} + \mathbf{p}_{2i}}{m_1 + m_2} = \frac{\mathbf{p}_{1f} + \mathbf{p}_{2f}}{m_1 + m_2}$$

항상 일정

1차원 완전 비탄성충돌





$$m_1 v_{1i} = (m_1 + m_2)V$$

$$V = \frac{m_1}{m_1 + m_2} v_{1i} < v_{1i}$$

질량중심의 속도

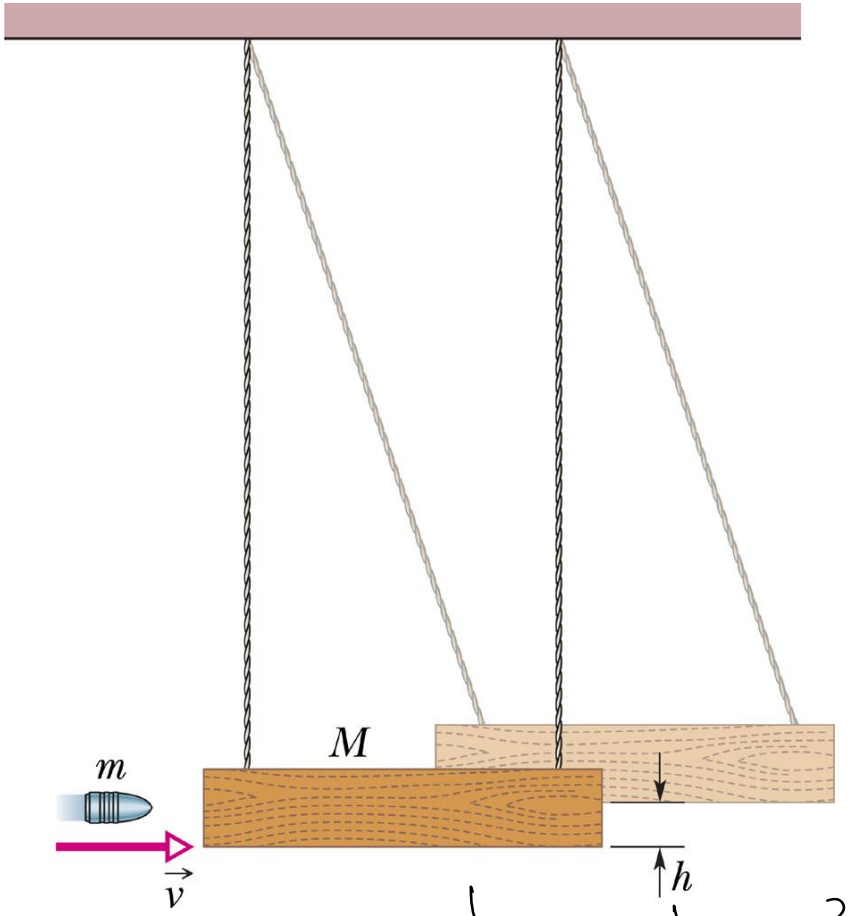
$$= -\frac{m_1 m_2}{2(m_1+m_2)} v_{1i}^2$$

$$= \frac{m_1^2 - m_1(m_1+m_2)}{2(m_1+m_2)} v_{1i}^2$$

$$\Delta E = \frac{1}{2}(m_1+m_2)V^2 - \frac{1}{2}m_1 v_{1i}^2$$

$$= \frac{m_1^2}{2(m_1+m_2)} v_{1i}^2 - \frac{1}{2}m_1 v_{1i}^2$$

ballistic pendulum



(1) 완전 비탄성충돌

$$V = \frac{m}{m+M}v$$

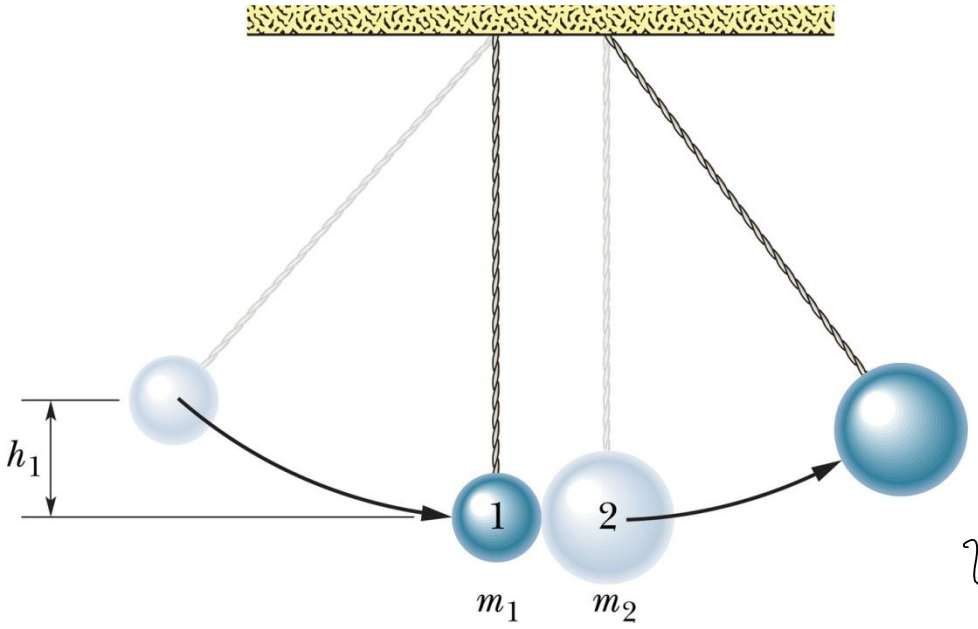
(2) 충돌후 역학적 에너지보존

$$\frac{1}{2}(m+M)V^2 = (m+M)gh$$

$$v = \frac{m+M}{m}\sqrt{2gh}$$

$\frac{1}{2}\frac{m^2}{(m+M)^2}v^2 = gh$, $v =$ $\frac{2gh(m+M)}{m}$

Sample problem



$m_1 = 30\text{g},\ h_1 = 8.0\text{cm}$

$m_2 = 75\text{g}$

$v_{if} = ?$

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i}$$

$$v_{1i} = \sqrt{2gh_1} = -0.537\text{m/s} \approx -0.54\text{m/s}$$

$v_{2i} = 0$

$$m_1 g h_1 = \frac{1}{2} m_1 v_{1i}^2$$

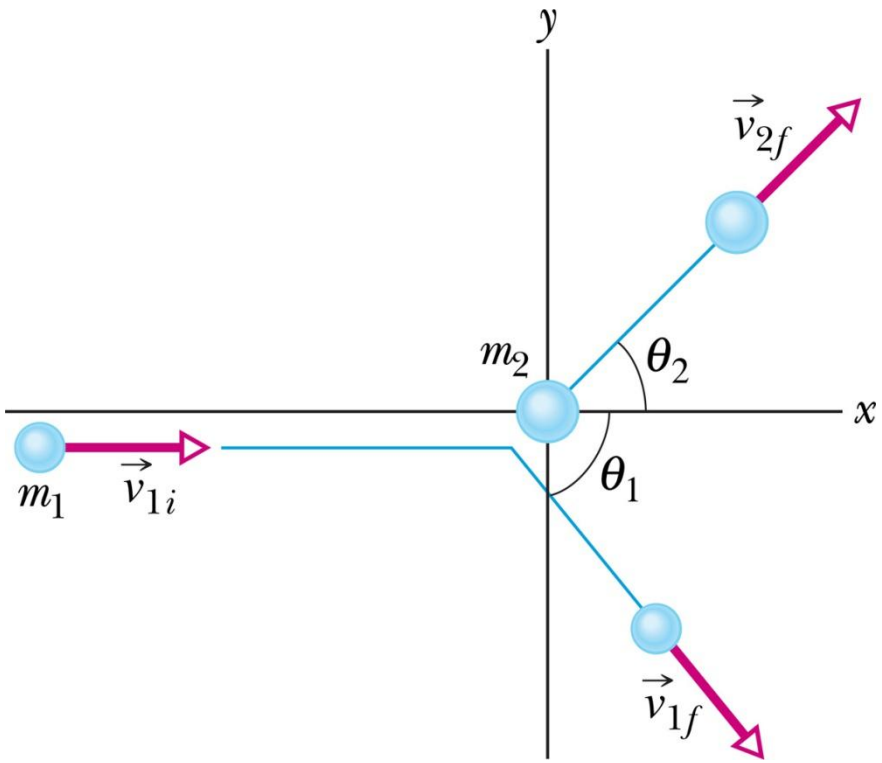
$$v_{1i} = \sqrt{2gh_1} = 1.252\text{m/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = \frac{2m_1}{m_1 + m_2} \sqrt{2gh_1} = m_2 g h_2$$

$$m_1 v_{1i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

2D collision



$$m_1 v_{1i} = m_1 v_{1f} \cos\theta_1 + m_2 v_{2f} \cos\theta_2$$

$$0 = -m_1 v_{1f} \sin\theta_1 + m_2 v_{2f} \sin\theta_2$$

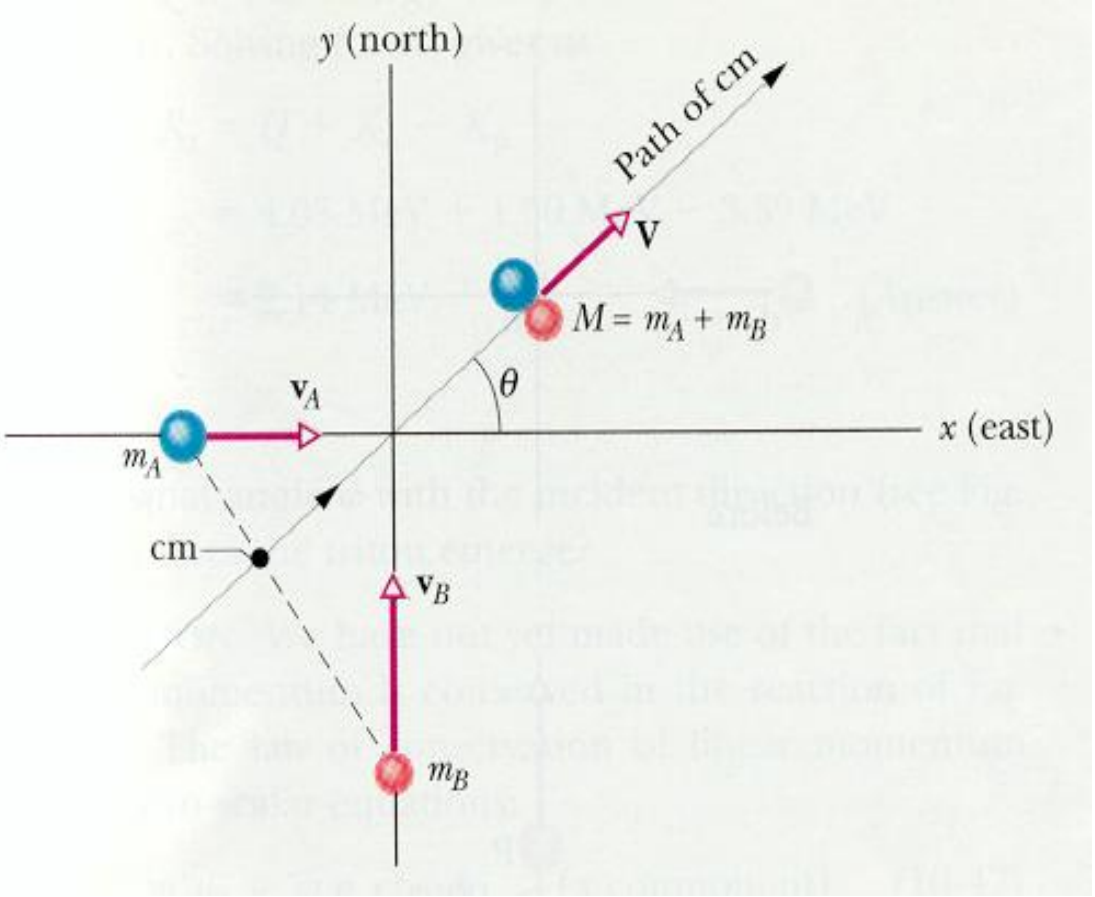
$$\tfrac{1}{2} m_1 v_{1i}^2 = \tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2$$

(탄성충돌의 경우)

미지수의 개수: $v_{1f}, v_{2f}, \theta_1, \theta_2$

독립적인 미지수의 개수: 2 (비탄성충돌), 1 (탄성충돌), 0 (완전비탄성충돌)

Sample problem



$$m_A = 83\text{kg},\ v_A = 6.2\text{km/h}$$

$$m_B = 55\text{kg},\ v_B = 7.8\text{km/h}$$

$$m_A v_A = MV\cos\theta$$

$$m_B v_B = MV\sin\theta$$

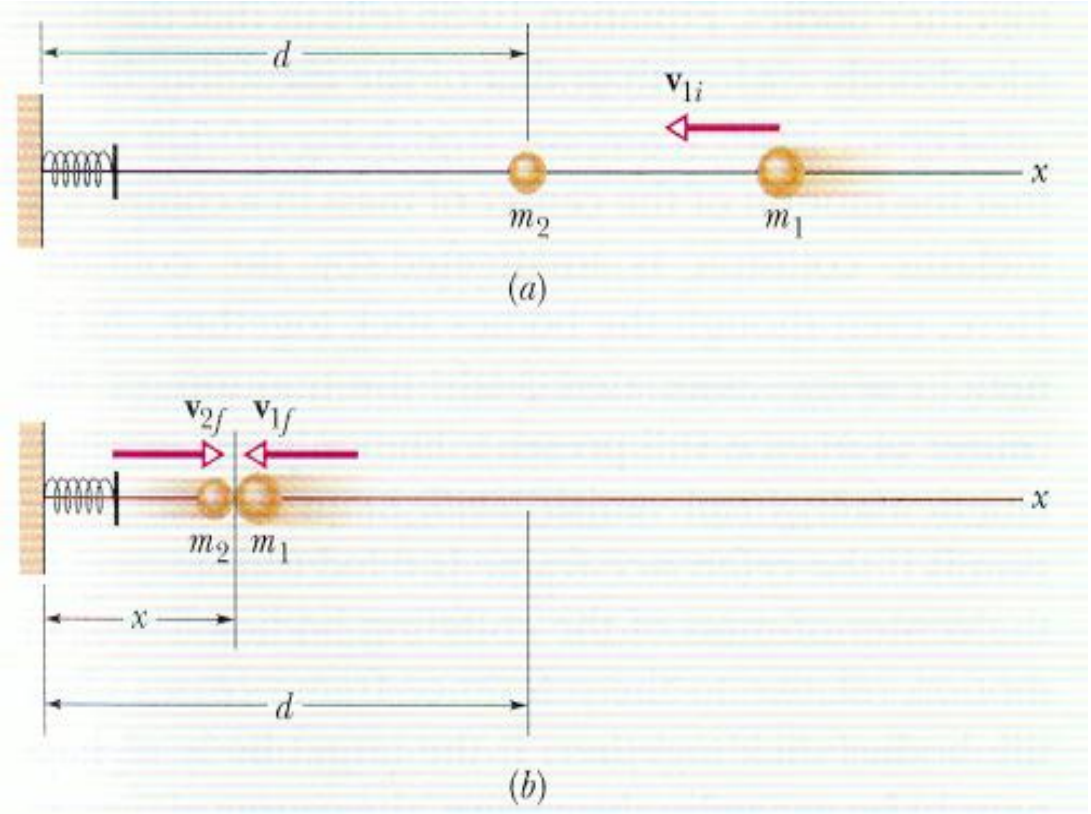
$$\tan\theta = \frac{m_B v_B}{m_A v_A} \Rightarrow \theta = 39.8^\circ$$

$$V = \frac{m_B v_B}{M\sin\theta} = 4.86\text{km/h}$$

$$K_i = \tfrac{1}{2} m_A v_A^2 + \tfrac{1}{2} m_B v_B^2 = 3270$$

$$K_f = \tfrac{1}{2} MV^2 = 1630$$

Sample problem



$$m_2 = 350\text{g},\ d = 53\text{cm}$$

$$m_1 = 590\text{g},\ v_1 = -75\text{cm/s}$$

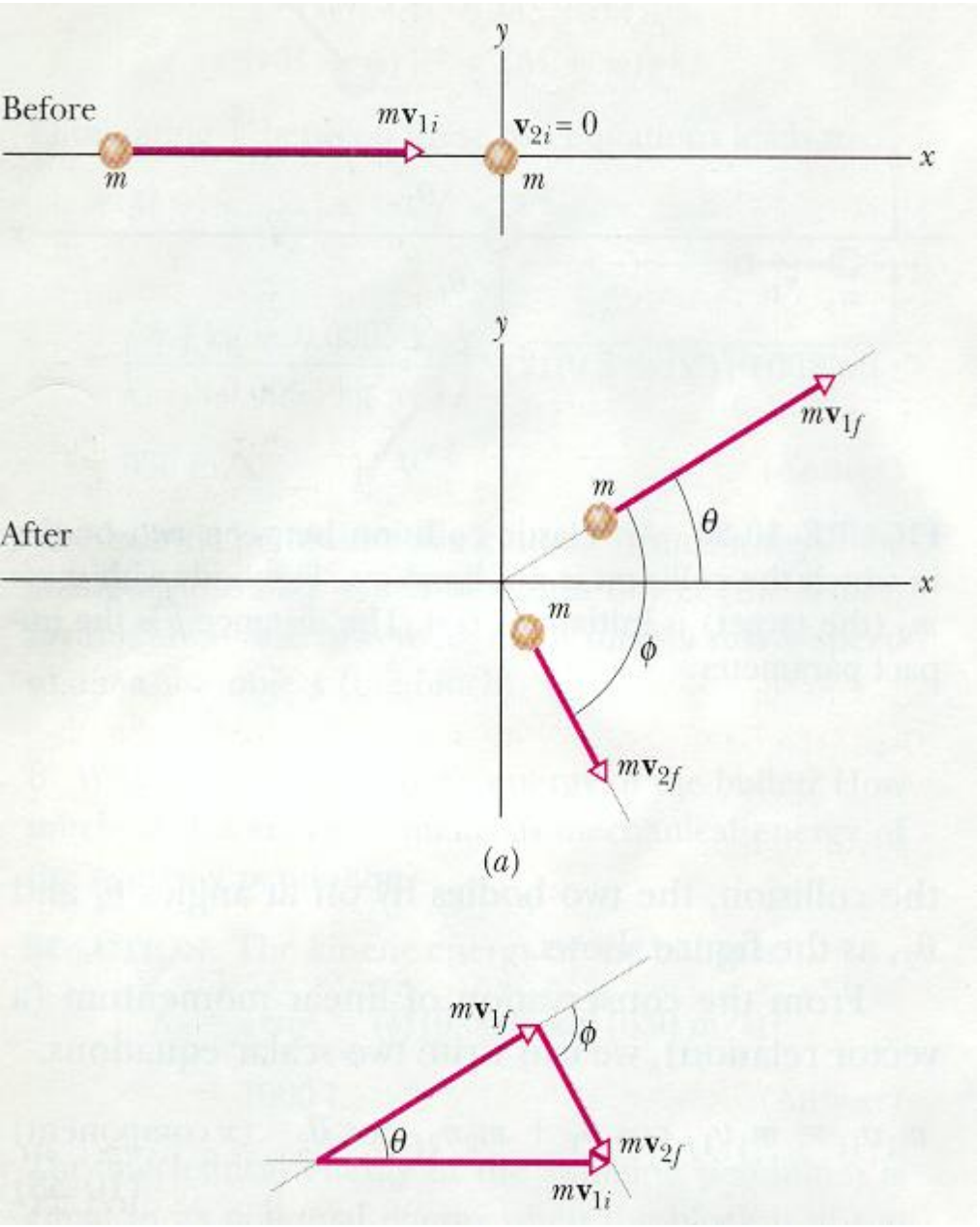
$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} = -19\text{cm/s}$$

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} = -94\text{cm/s}$$

2번째 충돌 거리 => x

$$t = \frac{d - x}{v_{1f}} = \frac{d + x}{v_{2f}}$$

$$\therefore\ x = 35\text{cm}$$



Sample problem

$$m\mathbf{v}_{1i} = m\mathbf{v}_{1f} + m\mathbf{v}_{2f}$$

$$\frac{1}{2}mv_{1i}^2 = \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2$$

$$\mathbf{v}_{1f} \perp \mathbf{v}_{2f}$$