Wireless Communications (ITC731)

Lecture Note 11

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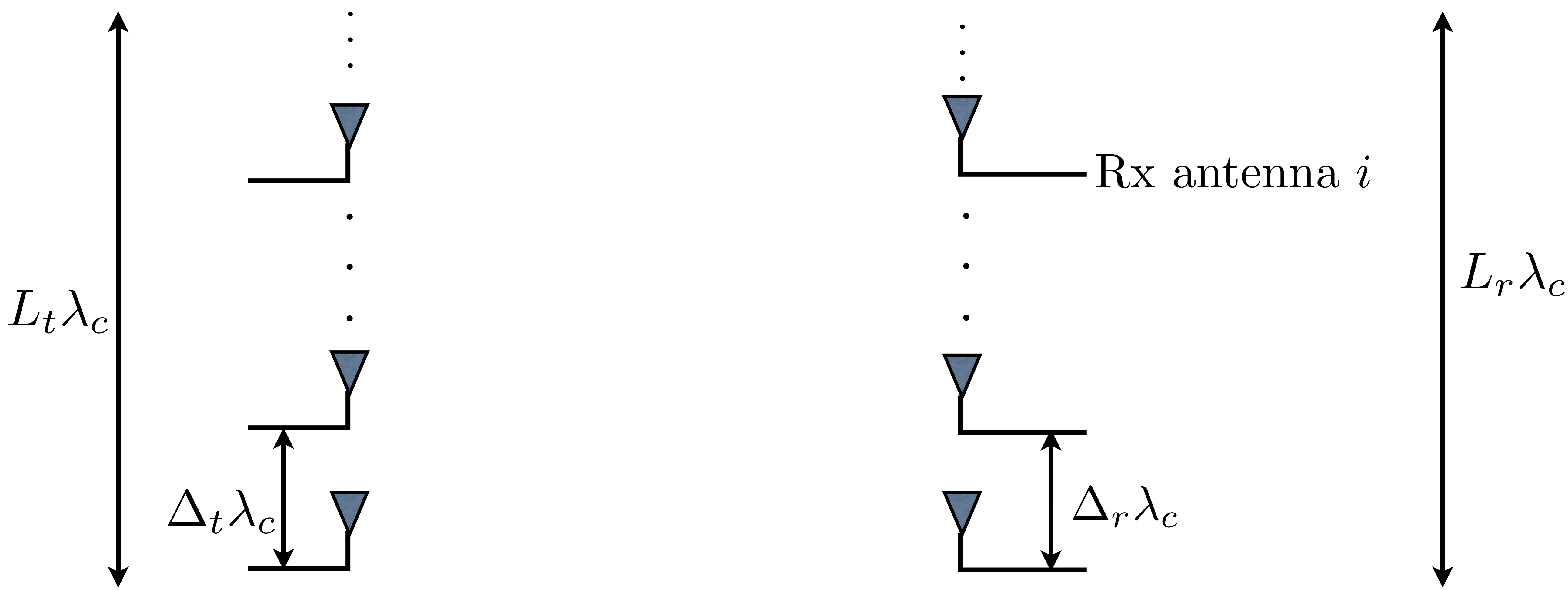
Summary

- MIMO Channels
- Layered space-time MIMO architecture

MIMO Channels



$L_t = n_t \Delta_t$

$L_r = n_t \Delta_r$

Resolvable Paths

The transmit and receive antenna array lengths L_t and L_r dictates the degree of resolvability in the angular domain.

# of resolvable paths at the transmit side $\leq \frac{1}{L_t}$

# of resolvable paths at the receive side $\leq \frac{1}{L_r}$

MIMO Multipath Channel

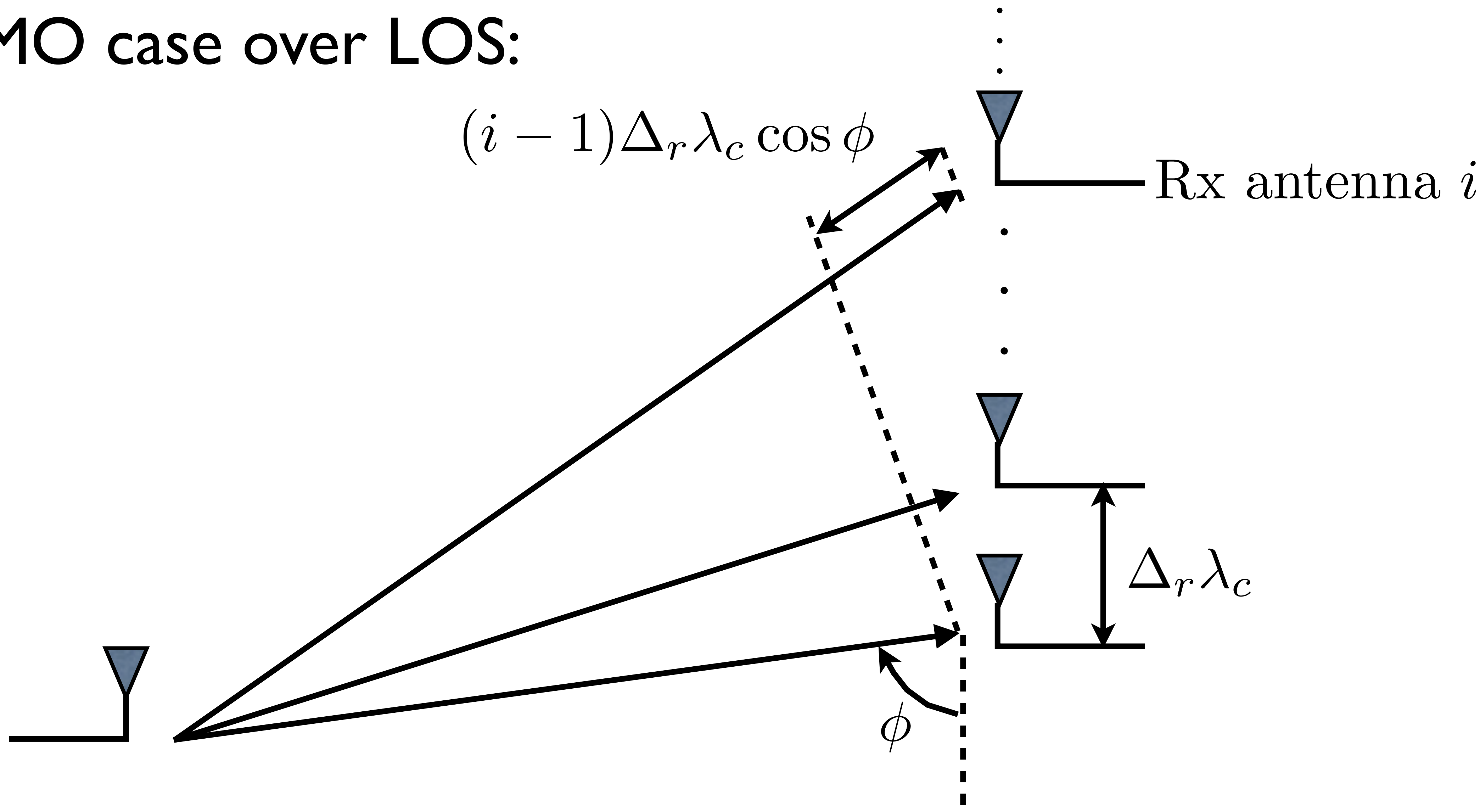
$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{w}$$

Normalized separation

$$\Delta_t = \frac{L_t}{n_t} \qquad \Delta_r = \frac{L_r}{n_r}$$

Now consider the fixed channel (later consider the time-varying channel)

SIMO case over LOS:



Continuous time impulse response $h_i(\tau)$ between the transmit antenna and the i-th receive antenna is

$$h_i(\tau) = \alpha\delta(t - d_i/c), \quad i = 1, ..., n_r$$

attenuation

Baseband channel gain

$$h_i = \alpha \exp\left(-\frac{j2\pi f_c d_i}{c}\right) = \alpha \exp\left(-\frac{j2\pi d_i}{\lambda_c}\right)$$

$$d_i \approx d + (i-1)\Delta_r \lambda_c \cos\phi, \quad i = 1, ..., n_r$$

angle of line-of-sight

distance from the transmit antenna to the first

receive antenna

Directional cosine

$$\mathbf{h} = \alpha \exp\left(-\frac{j2\pi d}{\lambda_c}\right) \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \exp(-j2\pi 2\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r-1)\Delta_r\Omega) \end{bmatrix} = \alpha^b \mathbf{e}_r(\Omega)$$

where

$$\alpha^b = \sqrt{n_r}\alpha \exp\left(-\frac{j2\pi d}{\lambda_c}\right)$$

$$\mathbf{e}_r(\Omega) = \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \exp(-j2\pi 2\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r-1)\Delta_r\Omega) \end{bmatrix}$$

$$\Omega = \cos\phi$$

MIMO Channel over LOS

Assume there are arbitrary number of resolvable paths.

The i-th path has an attenuation of α_i with the angle:

$$\begin{aligned}\Omega_{ti} &= \cos\phi_{ti} \\ \Omega_{ri} &= \cos\phi_{ri}\end{aligned}$$

The the channel matrix is given by

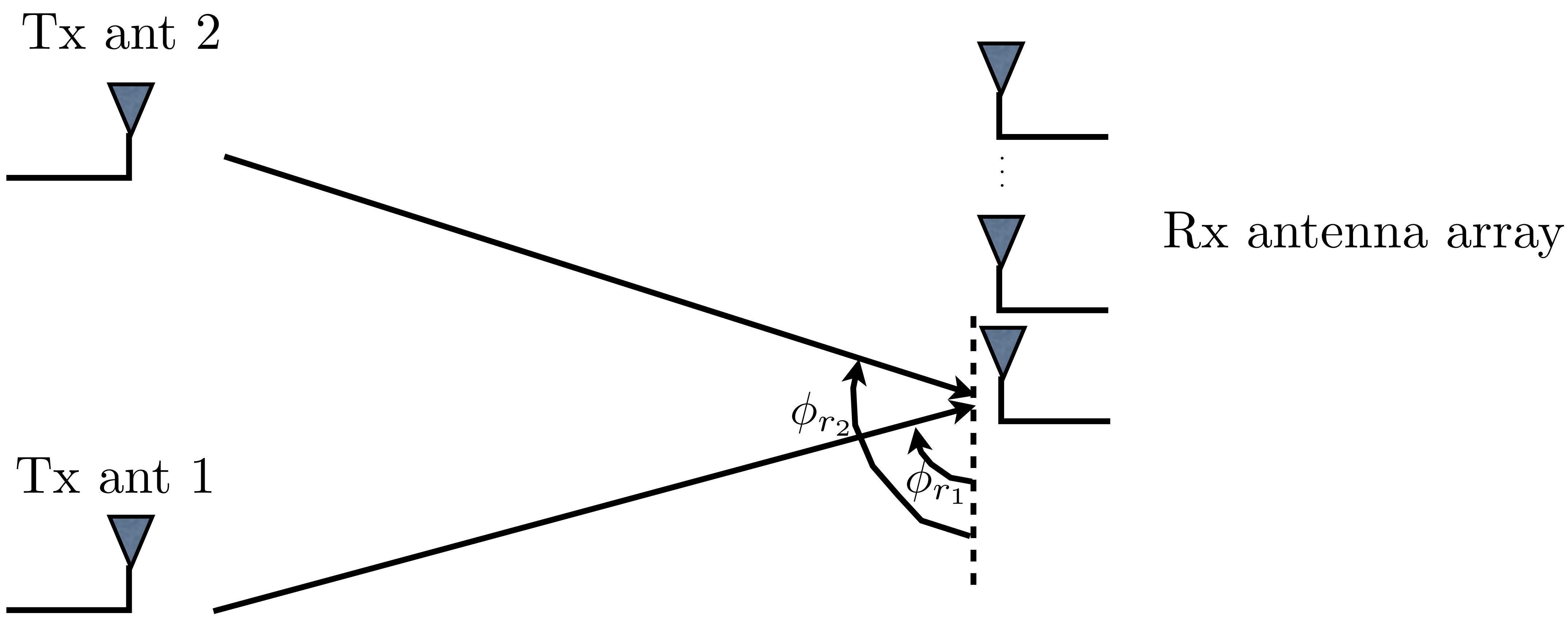
$$\mathbf{H} = \sum_i \alpha_i^b \mathbf{e}_r(\Omega_{ri})\mathbf{e}_t(\Omega_{ti})^*$$

$$\alpha_i^b = \alpha_i \sqrt{n_t n_r} \exp\left(-\frac{j2\pi d^{(i)}}{\lambda_c}\right)$$

$$\mathbf{e}_t(\Omega) = \frac{1}{\sqrt{n_t}} \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_t\Omega) \\ \exp(-j2\pi 2\Delta_t\Omega) \\ \vdots \\ \exp(-j2\pi(n_t-1)\Delta_t\Omega) \end{bmatrix}$$

$$\mathbf{e}_r(\Omega) = \frac{1}{\sqrt{n_r}} \begin{bmatrix} 1 \\ \exp(-j2\pi\Delta_r\Omega) \\ \exp(-j2\pi 2\Delta_r\Omega) \\ \vdots \\ \exp(-j2\pi(n_r-1)\Delta_r\Omega) \end{bmatrix}$$

Geographically Separated Two TX Antennas (Review)



$$\mathbf{h}_k = \alpha_k \sqrt{n_r} \exp\left(-\frac{j2\pi d_{1k}}{\lambda_c}\right) \mathbf{e}_r(\Omega_{rk}), \quad k = 1, 2$$

$$\Omega_{r1} = \cos\phi_{r1} \qquad \Omega_{r2} = \cos\phi_{r2}$$

We can prove that the spatial signature $\mathbf{e}_r(\Omega)$ is a periodic function of Ω with period $1/\Delta_r$, and within one period it never repeats itself.

Thus, the channel matrix $\mathbf{H} = [\mathbf{h}_1 \;\; \mathbf{h}_2]$ has distinct and linearly independent columns as long as the separation in the directional cosines

$$\Omega_r = \Omega_{r2} - \Omega_{r1} \neq 0 \mod \frac{1}{\Delta_r}$$

Resolvability in the angular domain

The angle θ between the two spatial signature:

$$|\cos\theta| = |\mathbf{e}_r(\Omega_{r1})^* \mathbf{e}_r(\Omega_{r2})|.$$

Note that $\mathbf{e_r}(\Omega_{r1})^* \mathbf{e_r}(\Omega_{r2})$ depends only on the difference:

$$\Omega_r = \Omega_{r2} - \Omega_{r1}$$

Define then:

$$f_r(\Omega_{r2} - \Omega_{r1}) = \mathbf{e_r}(\Omega_{r1})^* \mathbf{e_r}(\Omega_{r2}).$$

Then we can show by direct computation as

$$f_r(\Omega_r) = \frac{1}{n_r} \exp(j\pi\Delta_r\Omega_r(n_r - 1)) \frac{\sin(\pi L_r \Omega_r)}{\sin(\pi L_r \Omega_r / n_r)}$$

where $L_r = n_r \Delta_r$

Hence,

$$|\cos\theta| \quad = \quad \left| \frac{\sin(\pi L_r \Omega_r)}{n_r \sin(\pi L_r \Omega_r / n_r)} \right|$$

The conditioning of the matrix $\mathbf{H}$ depends directly on $\cos\theta$.

Assume $\alpha_1 = \alpha_2$.

The singular values of $\mathbf{H}\mathbf{H}^H$ are

$$\begin{aligned} \lambda_1^2 &= \alpha^2 n_r (1 + |\cos\theta|), \\ \lambda_2^2 &= \alpha^2 n_r (1 - |\cos\theta|). \end{aligned}$$

The condition number is $\dfrac{\lambda_1}{\lambda_2} = \sqrt{\dfrac{1 + |\cos\theta|}{1 - |\cos\theta|}}$

The matrix is ill-conditioned whenever $|\cos\theta| \approx 1$, and well conditioned otherwise.

Properties of $f_r(\cdot)$:

- $f_r(\Omega_r)$ is periodic with period $n_r/L_r = 1/\Delta_r$
- $f_r(\Omega_r)$ peaks at $\Omega_r = 0$; $f(0) = 1$
- $f_r(\Omega_r) = 0$ at $\Omega_r = k/L_r$, $k = 1, ..., n_r - 1$.

The channel matrix is ill-conditioned whenever

$$|\Omega_r - \frac{m}{\Delta_r}| << \frac{1}{L_r} \text{ for some integer } m.$$

Now that $|\Omega_r| \leq 2$, this condition reduces to $|\Omega_r| << \dfrac{1}{L_r}$

whenever the antenna spacing $\Delta_r \leq 1/2$.

Angular Domain Representation of MIMO Signals

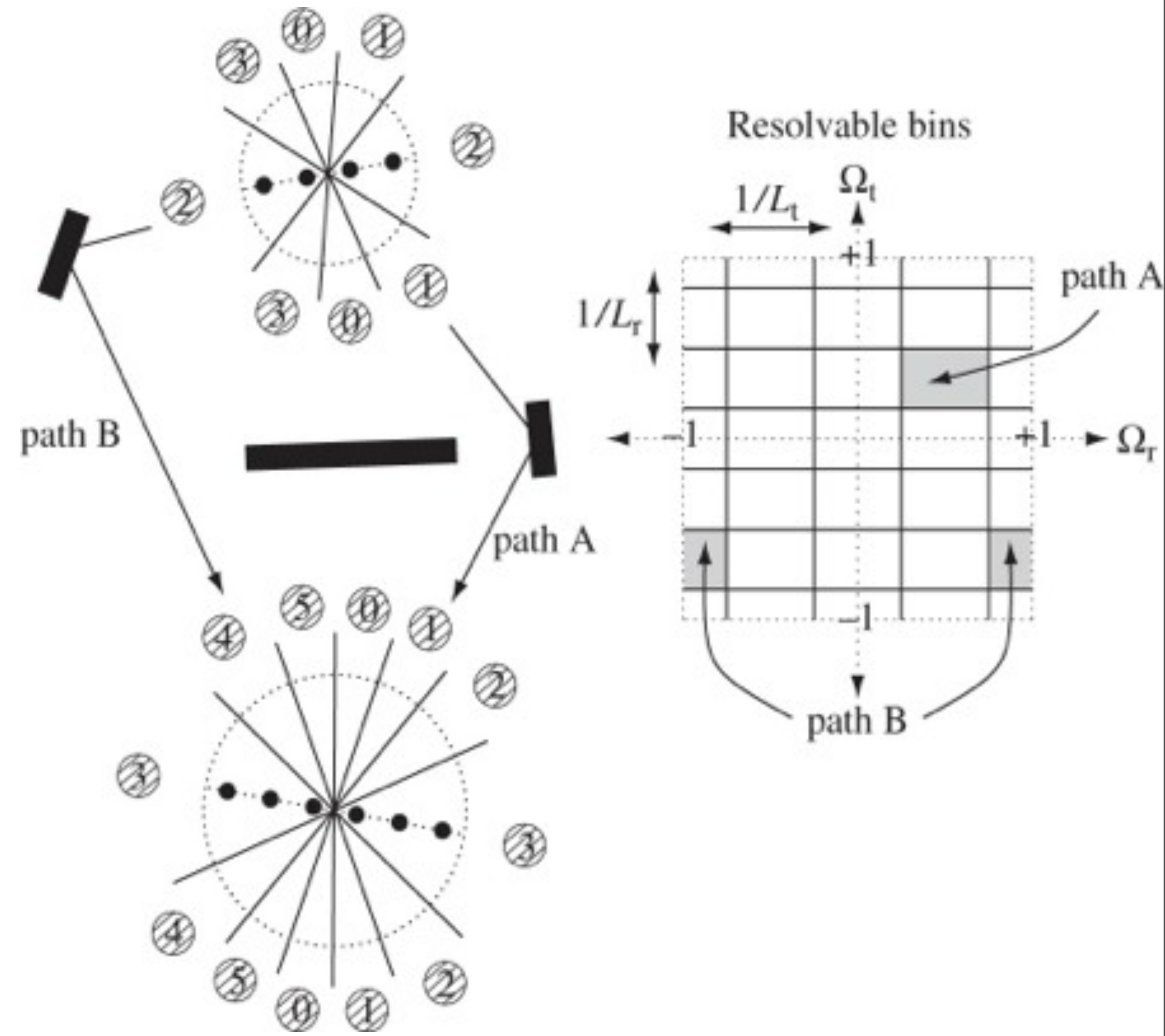
The outgoing paths are grouped into resolvable bins $\{\mathcal{T}_l\}$ of angular width $1/L_t$

The incoming paths are grouped into resolvable bins $\{\mathcal{R}_k\}$ of angular width $1/L_r$.

The $(k,l)^{th}$ entry of $\mathbf{H}^a$ is (approximately) the aggregation of paths in $\mathcal{T}_l \cap \mathcal{R}_k$.

Can statistically model each entry as independent and Gaussian.

Bins that have no paths will have zero entries in $\mathbf{H}^a$.



[Tse and Viswanathan, *Fundamentals of Wireless Communication*, Cambridge press]

Spatial-Angular Domain Transformation

Note

$$f_r(\Omega) = \mathbf{e_r}(\Omega)^* \mathbf{e_r}(\Omega) = \frac{1}{n_r} \exp(j\pi\Delta_r\Omega(n_r - 1)) \frac{\sin(\pi L_r \Omega)}{\sin(\pi L_r \Omega / n_r)}.$$

In particular

$$f_r\left(\frac{k}{L_r}\right) = 0, \text{ and } f_r\left(\frac{-k}{L_r}\right) = f_r\left(\frac{n_r - k}{L_r}\right), \quad k = 1, ..., n_r - 1$$

Hence, n_r fixed vectors form an orthonormal basis for the received signal space.

$$\mathbf{U}_r = \left[\mathbf{e_r}(0),\ \mathbf{e_r}\left(\frac{1}{L_r}\right),\ ...,\ \mathbf{e_r}\left(2 - \frac{1}{L_r}\right)\right]$$

We similarly define the angular domain representation of the transmitted signal.

$$\mathbf{U}_t = \left[\mathbf{e_t}(0),\ \mathbf{e_t}\left(\frac{1}{L_t}\right),\ ...,\ \mathbf{e_t}\left(2-\frac{1}{L_t}\right)\right]$$

What is the relationship between angular $\mathbf{H}^a$ and spatial $\mathbf{H}$?

Transmit angular basis matrix (orthonormal):

$$\mathbf{U}_t = \left[\mathbf{e_t}(0),\ \mathbf{e_t}\left(\frac{1}{L_t}\right),\ ...,\ \mathbf{e_t}\left(2-\frac{1}{L_t}\right)\right]$$

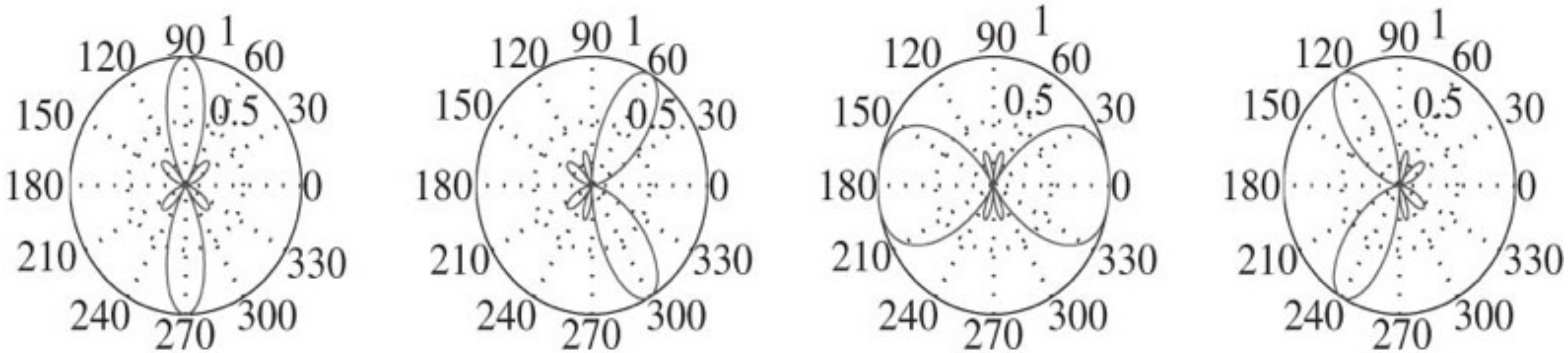
Receive angular basis matrix (orthonormal):

$$\mathbf{U}_r = \left[\mathbf{e_r}(0),\ \mathbf{e_r}\left(\frac{1}{L_r}\right),\ ...,\ \mathbf{e_r}\left(2-\frac{1}{L_r}\right)\right]$$

Input, output in angular domain: $\mathbf{x} = \mathbf{U_t}x^a,\ \ \mathbf{y} = \mathbf{U_r}\mathbf{y}^a$

so

$$\mathbf{H}^a = \mathbf{U_r}^*\mathbf{H}\mathbf{U_t}$$



(a) $L_r = 2$, $n_r = 4$

Antennas are critically spaced at half the wavelength ($\Delta_r = 1/2$). In this case, each basis vector $\mathrm{e}_{\mathbf{r}}(k/L_t)$ has a single pair of main lobes around the angles $\pm\mathrm{arcos}(k/L_t)$.

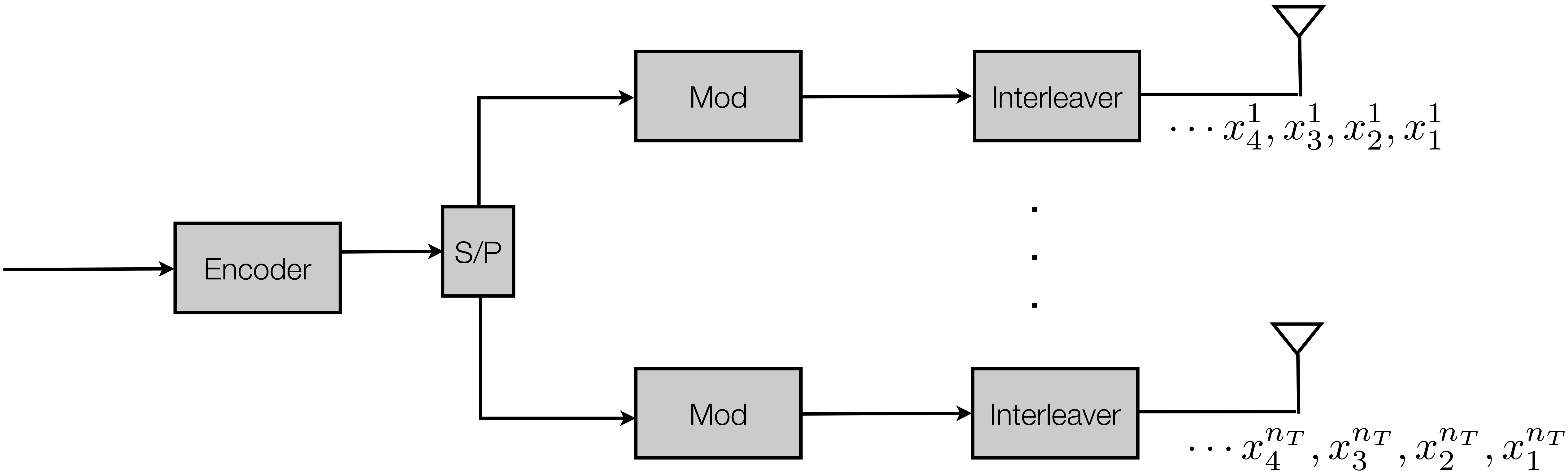
[Tse and Viswanathan, *Fundamentals of Wireless Communication*, Cambridge press]

Analogy with Time-Frequency Channel

	Time-Frequency	Spatial-Angular
Domains	Time ⟷ Angular Frequency ⟷ Spatial	Angular Spatial
Resources	signal duration T bandwidth W	angular spreads Ω_t, Ω_r antenna array lengths L_t,L_r
Resolution of multipaths	into delay bins of $1/W$	into angular bins of $1/L_t$ by $1/L_r$
d.o.f.	WT	$\min(L_t\Omega_t, L_r\Omega_r)$
Diversity	# of non-zero delay bins	# of non-zero angular bins

Layered Space-Time (LST) Architecture for Spatial Multiplexing MIMO

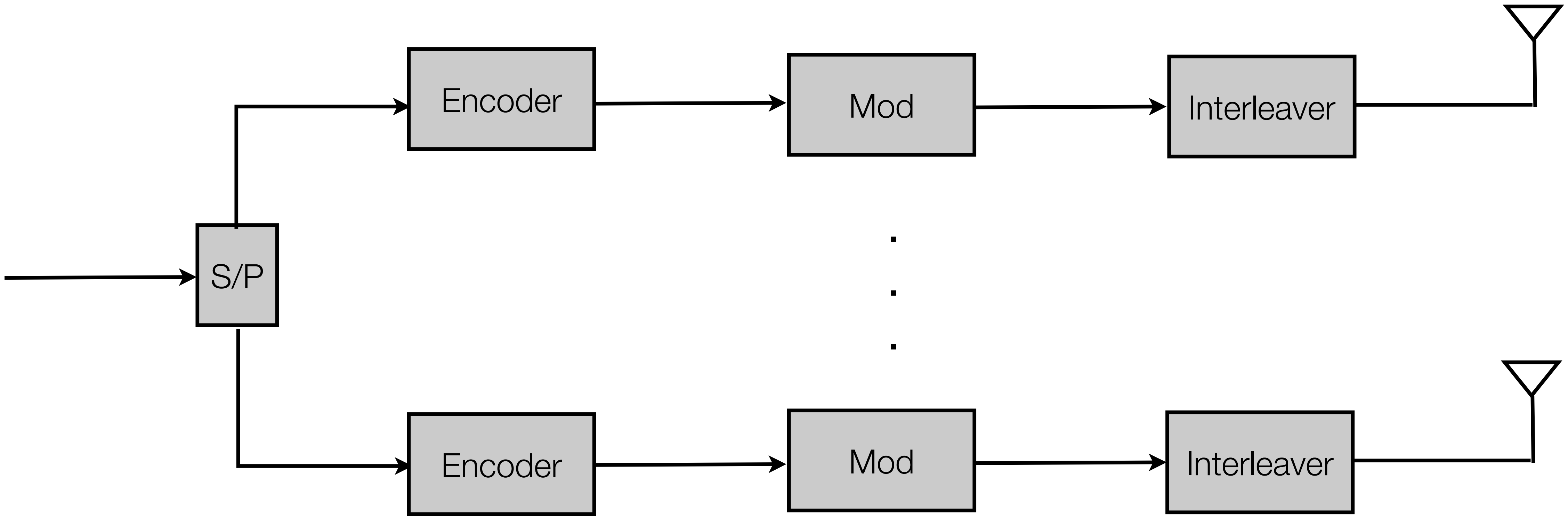
- Horizontal layered space-time (HLST) architecture



- Transmission matrix

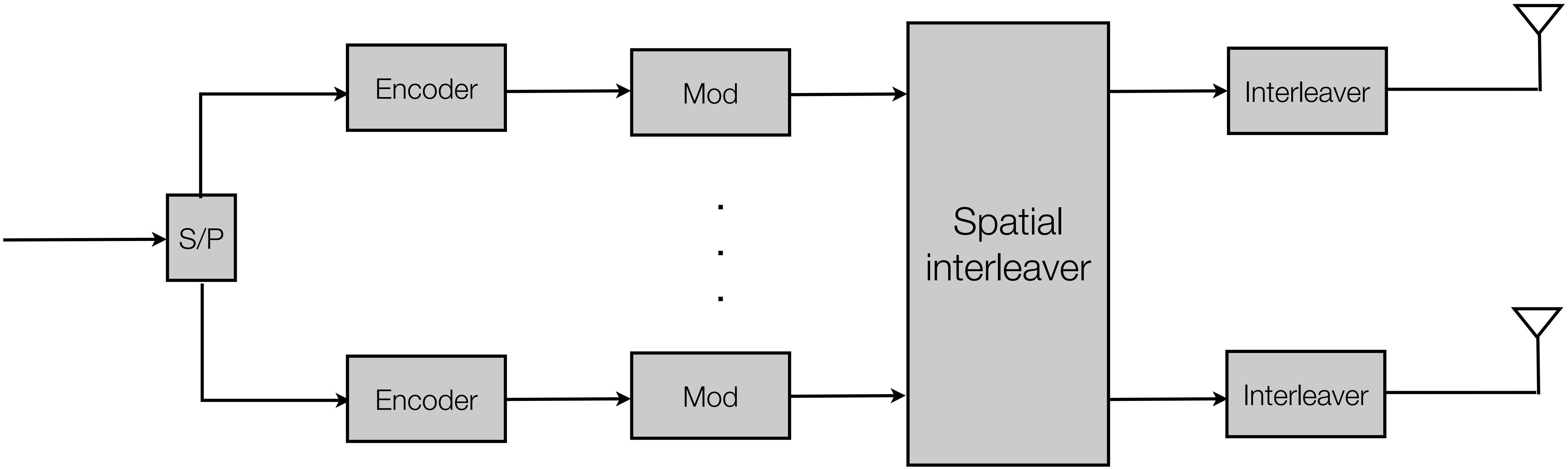
$$\mathbf{X} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & x_4^1 & \cdots \\ x_1^2 & x_2^2 & x_3^2 & x_4^2 & \cdots \\ x_1^3 & x_2^3 & x_3^3 & x_4^3 & \cdots \end{bmatrix}$$

- HLST with encoder at each branch



- Different coding in each sub-stream can be used.

Diagonal layered space-time (DLST) architecture



Spatial interleaving

$$\begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & x_4^1 & x_5^1 & x_6^1 & \cdots \\ 0 & x_1^2 & x_2^2 & x_3^2 & x_4^2 & x_5^2 & \cdots \\ 0 & 0 & x_1^3 & x_2^3 & x_3^3 & x_4^3 & \cdots \end{bmatrix} \rightarrow \begin{bmatrix} x_1^1 & x_1^2 & x_1^3 & x_4^1 & x_4^2 & x_4^3 & \cdots \\ 0 & x_2^1 & x_2^2 & x_2^3 & x_5^1 & x_5^2 & \cdots \\ 0 & 0 & x_3^1 & x_3^2 & x_3^3 & x_6^1 & \cdots \end{bmatrix}$$

The diagonal layering introduces space diversity and thus achieves a better performance but with loss of spectral efficiency.

LST Receiver

- The signals transmitted from various antennas interfere with each other upon reception at the receiver.

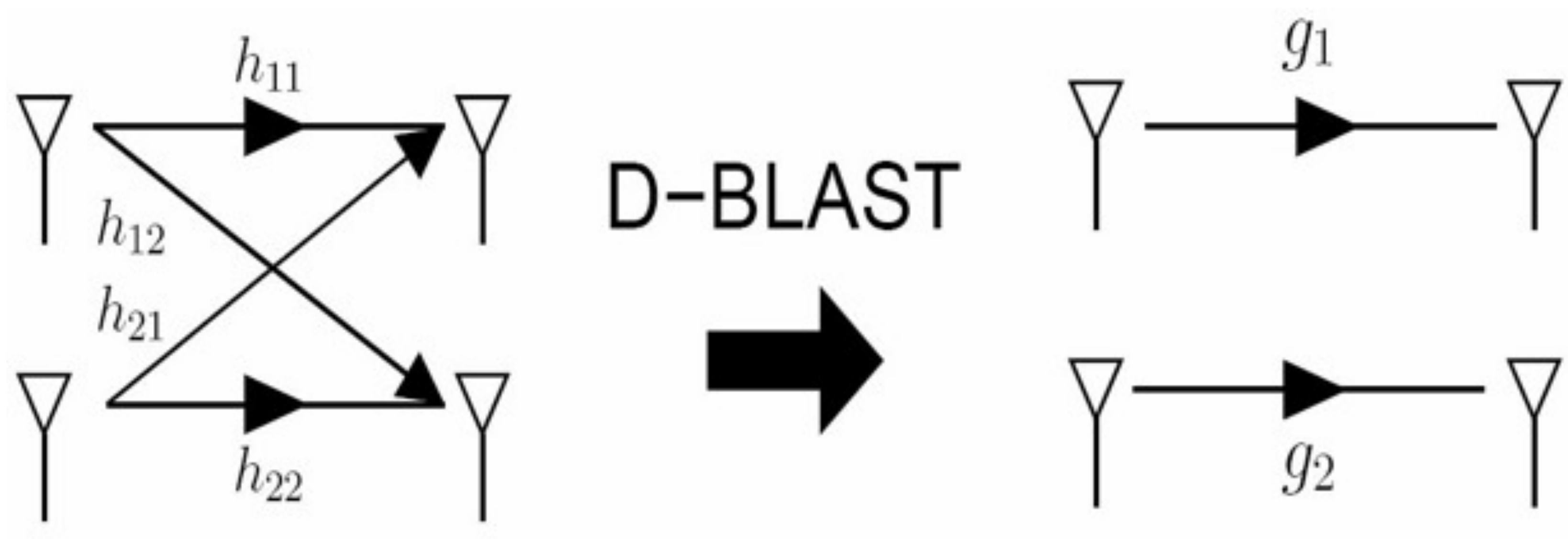
$$
\begin{aligned}
r_1 &= h_{11}x_1 + h_{12}x_2 + \cdots + h_{1n_T}x_{n_T} \\
r_2 &= h_{21}x_1 + h_{22}x_2 + \cdots + h_{2n_T}x_{n_T} \\
&\vdots
\end{aligned}
$$

$$\mathbf{r}_t = \mathbf{H}\mathbf{x}_t + \mathbf{n_t}$$

- At the receiver, we want to suppress and cancel the interference for the detection.

LST Architecture

- Parallel channel conversion



[Ref: Tse and Viswanathan, "Fundamentals of Wireless Communications", Cambridge Press]

QR decomposition

- Any $n_R \times n_T$ matrix $\mathbf{H}$, where $n_R \geq n_T$, can be decomposed as

$$\mathbf{H} = \mathbf{UR}$$

where

$$\mathbf{U} \quad : \quad n_R \times n_T \text{ unitary matrix}$$

$$\mathbf{R} \quad : \quad n_T \times n_T \text{ upper triangular matrix given as}$$

$$\mathbf{R} = \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,n_T} \\ 0 & R_{2,2} & \cdots & R_{2,n_T} \\ 0 & 0 & \cdots & R_{2,n_T} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & R_{n_T,n_T} \end{bmatrix}$$

$$\mathbf{U}^T\mathbf{U} = \mathbf{I}$$

- Let us introduce n_T-component column matrix $\mathbf{y}$ defined as

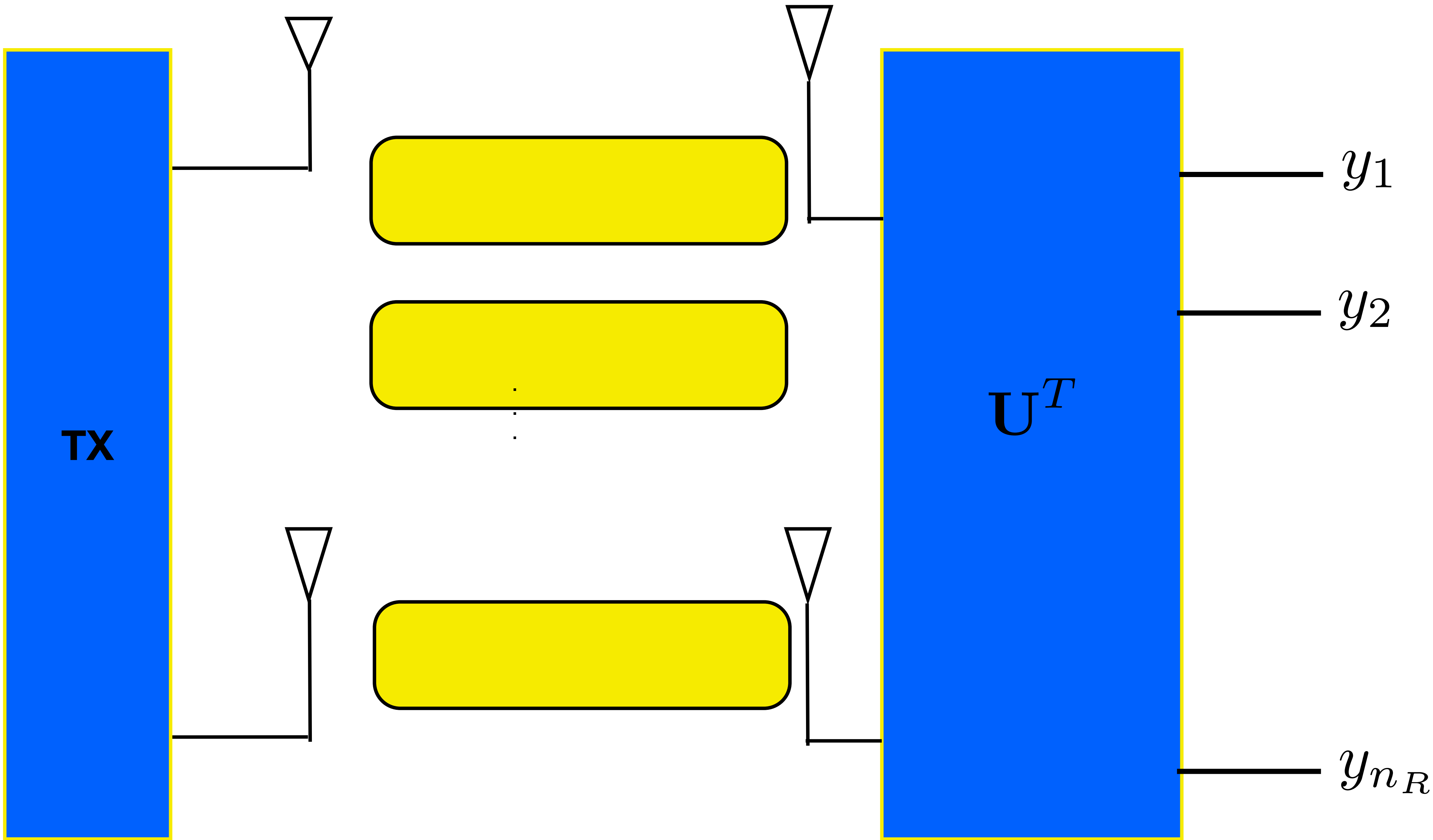
$$\mathbf{y} = \mathbf{U}^T \mathbf{r}$$

or

$$\mathbf{y} = \mathbf{U}^T \mathbf{H} \mathbf{x} + \mathbf{U} \mathbf{n}$$

$$= \mathbf{U}^T \mathbf{Q} \mathbf{R} \mathbf{x} + \mathbf{U} \mathbf{n}$$

$$= \mathbf{R} \mathbf{x} + \mathbf{n}'$$


TX
$\mathbf{U}^T$
y_1
y_2
y_{n_R}

$$
\begin{aligned}
\mathbf{y} &= \mathbf{U}^T\mathbf{r} = \mathbf{U}^T(\mathbf{Hx}+\mathbf{n}) \\
&= \mathbf{U}^T(\mathbf{URx}+\mathbf{n})
\end{aligned}
$$

$$
= \begin{bmatrix} R_{1,1} & R_{1,2} & \cdots & R_{1,n_T} \\ 0 & R_{2,2} & \cdots & R_{2,n_T} \\ 0 & 0 & \cdots & R_{2,n_T} \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & R_{n_T,n_T} \end{bmatrix} \left(\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n_T} \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ \vdots \\ n_{n_R} \end{bmatrix} \right)
$$

$$
\begin{aligned}
y_{n_T} &= R_{n_T,n_T} x_{n_T} + n'_{n_T} \\
y_{n_T-1} &= R_{n_T-1,n_T-1} x_{n_T-1} + R_{n_T-1,n_T} x_{n_T} + n'_{n_T-1} \\
&\vdots \\
y_1 &= R_{1,1} x_1 + R_{1,2} x_2 + \cdots + R_{1,n_T} x_{n_T} + n'_1
\end{aligned}
$$

or simply

$$
y_k = R_{k,k} x_k + \sum_{j=k+1}^{n_T} R_{k,j} x_j + n'_k \quad k = 1, 2, \ldots, n_T
$$

Self-Interference

Decision statistics

$$\hat{x}_k = q\left(\frac{y_k - \sum_{j=k+1}^{n_T} R_{k,j}x_j}{R_{k,k}}\right), \qquad i = 1, 2, \ldots, n_T$$

where $q(\cdot)$ is the hard decision operation.

- Example for 3 by 3 antennas with the channel matrix given as

$$\hat{x}_3 = q\left(\frac{y_3}{R_{3,3}}\right)$$

$$\begin{aligned} y_1 &= R_{1,1}x_1 + R_{1,2}x_2 + R_{1,3}x_3 + n'_1 \\ y_2 &= R_{2,2}x_2 + R_{2,3}x_3 + n'_3 \\ y_3 &= R_{33}x_3 + n'_3 \end{aligned}$$

$$\hat{x}_2 = q\left(\frac{y_2 - R_{2,3}\hat{x}_3}{R_{2,2}}\right)$$

$$\hat{x}_1 = q\left(\frac{y_1 - R_{1,3}\hat{x}_3 - R_{1,2}\hat{x}_2}{R_{1,1}}\right.$$