KECE321 Communication Systems I

(Haykin Sec. 3.6)

Lecture #10, April 16, 2012

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Summary

Summary

- Amplitude modulation
  - Hilbert transform
  - Single-Sideband (SSB) Modulation
    - Generation of SSB modulation
    - Coherent detection
    - Non-coherent detection

Hilbert Transform

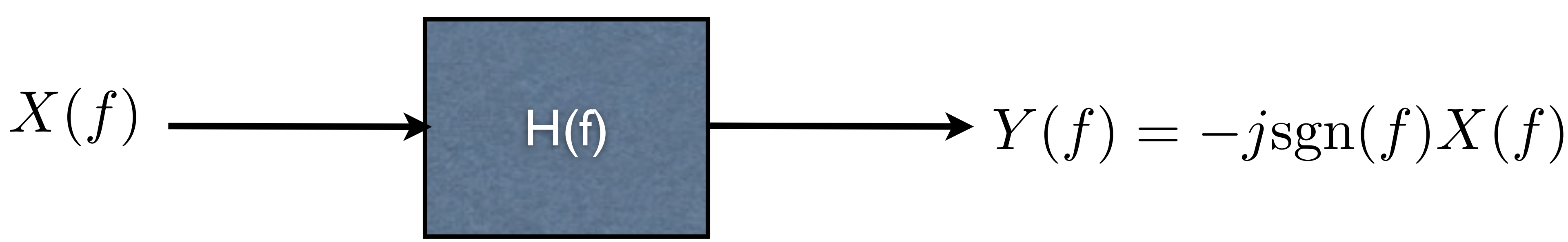
- Consider a filter that simply phase shifts all frequency components of its input by $-\pi/2$ radians, that is, its transfer function is

$$H(f) = -j\text{sgn} f$$

- Note that

$$|H(f)| = 1, \text{ and } \angle H(f) = \begin{cases} -\pi/2 & f > 0, \\ \pi/2 & f < 0 \end{cases}$$

- Input-Hilbert filter-Output signals



- Let us denote

$$\hat{x}(t) = \mathcal{F}^{-1}[Y(f)]$$

- Then

$$\hat{x}(t) = \mathcal{F}^{-1}[-j\mathrm{sgn}(f)X(f)] = h(t) * x(t)$$

- Now let us calculate the inverse transform of h(t).
    - Recall $\mathcal{F}[\mathrm{sgn}(t)] = \dfrac{1}{j\pi f}$, then using the duality property we have

$$\mathcal{F}^{-1}[\mathrm{sgn}(f)] = \frac{1}{j\pi(-t)} = \frac{j}{\pi t}$$

- We get the Fourier transform pair

$$\frac{j}{\pi t} \Longleftrightarrow \mathrm{sgn}(f) \qquad \text{or} \qquad \frac{1}{\pi t} \Longleftrightarrow -j\mathrm{sgn}(f)$$

- Now we obtain the output of the filter

$$\hat{x}(t) = x(t) * h(t) = \int_{-\infty}^{\infty} \frac{x(\lambda)}{\pi(t-\lambda)}\, d\lambda$$

- The function $\hat{x}(t)$ is defined as the Hilbert transform of $x(t)$.

- Remarks

  - The Hilbert transform corresponds to a phase shift of $-\pi/2$.

  - The Hilbert transform of $\hat{x}(t)$

$$\hat{\hat{x}}(t) = -x(t)$$

Properties of Hilbert Transform

1. Energies are equal

$$|\hat{X}(f)|^2 = |-j\mathrm{sgn}(f)|^2|X(f)|^2 = |X(f)|^2$$

2. A signal and its Hilbert transform are orthogonal;

$$\int_{-\infty}^{\infty} x(t)\hat{x}(t)\,dt = 0 \qquad \int_{-\infty}^{\infty} X(f)\hat{X}(f)\,df = 0$$

Analytic Signals

- Definition of the analytic signal $x_p(t)$

$$x_p(t) = x(t) + j\hat{x}(t)$$

- Fourier transform of the analytic signal

$$X_p(f) = X(f) + j[-j\text{sgn}(f)X(f)] = X(f)[1 + \text{sgn}(f)]$$

or

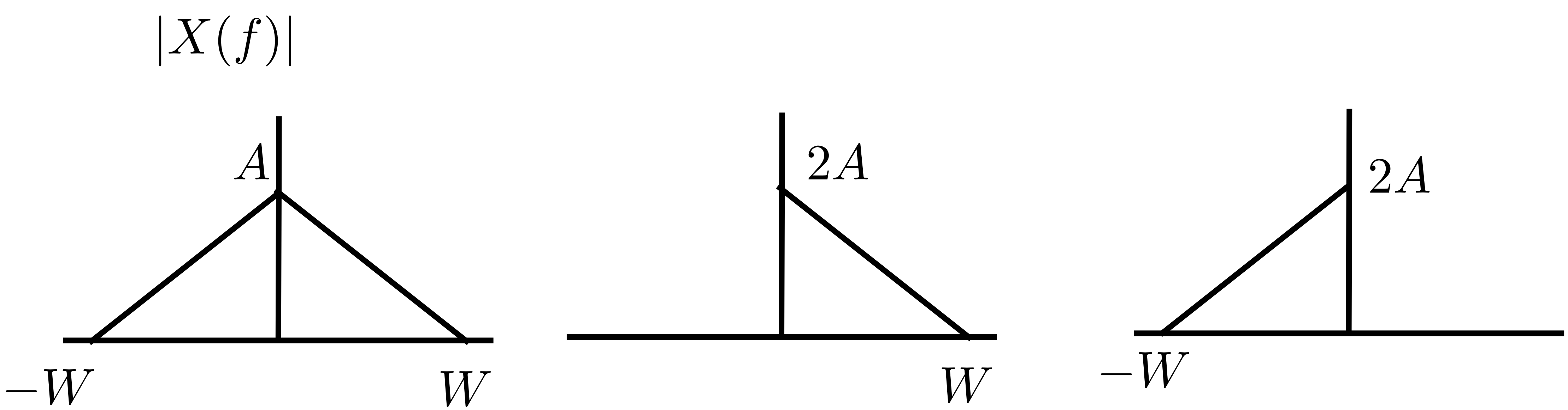
$$X_p(f) = \begin{cases} 2X(f), & f > 0 \\ 0, & f < 0 \end{cases}$$

- We can also show that

$$x_q(t) = x(t) - j\hat{x}(t)$$

and its Fourier transform

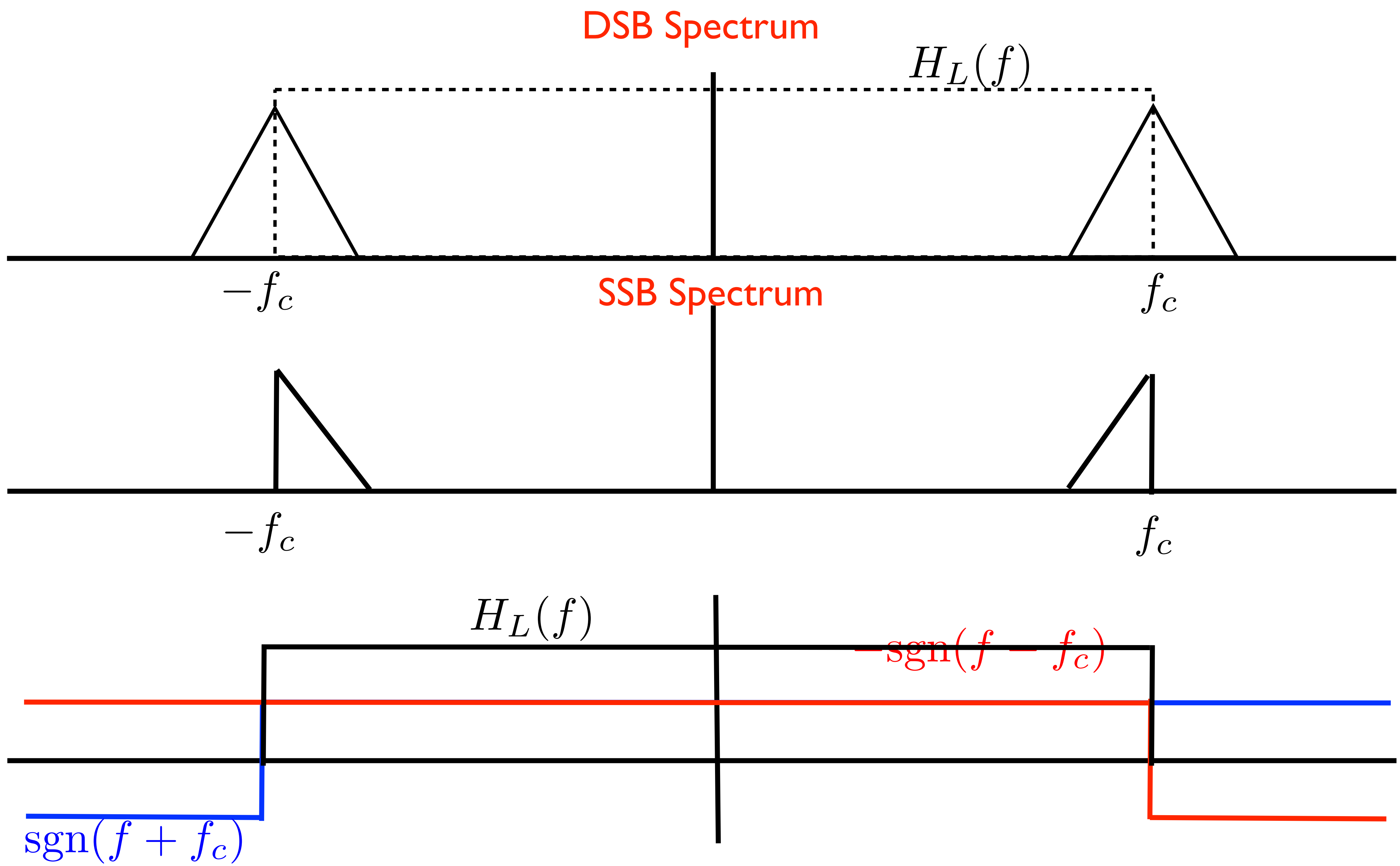
$$X_q(f) = X(f)\left[1 - \mathrm{sgn}(f)\right]$$
$$= \begin{cases} 0, & f > 0 \\ 2X(f), & f < 0 \end{cases}$$



Single-Sideband (SSB) Modulation

- SSB modulation
  - Suppress one of the two sidebands in the DSB-SC modulated wave prior to transmission
- Method of SSB Modulations
  - Time domain expression
    - SSB signal from DSB-SC is derived using the Hilbert transform.
  - Frequency domain expression
    - SSB signal from DSB-SC is generated Analytic signal is derived using the analytic signal.

Generation of LSB SSB



- Sideband filter

$$H_L(f) = \frac{1}{2}\left[\text{sgn}(f+f_c) - \text{sgn}(f+f_c)\right]$$

- Fourier transform of DSB-SC signal

$$S(f) = \frac{1}{2}A_c\left[M(f-f_c) + M(f+f_c)\right]$$

- Lower-Sideband SSB signal

$$= M(f-f_c)$$

$$S_{LSB}(f) = \frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c) + M(f-f_c)\text{sgn}(f+f_c)\right]$$
$$-\frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f-f_c) + M(f-f_c)\text{sgn}(f-f_c)\right]$$

$$= -M(f+f_c)$$

or

$$S_{LSB}(f) = \frac{1}{4}A_c\left[M(f+f_c) + M(f-f_c)\right]$$
$$+\frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c) - M(f-f_c)\text{sgn}(f-f_c)\right]$$

- From our study of DSB

$$\frac{1}{2}A_c m(t)\cos(2\pi f_c t) \Longleftrightarrow \frac{1}{4}A_c\left[M(f+f_c)+M(f-f_c)\right]$$

- Also recall the Hilbert transform

$$\hat{m(t)} \Longleftrightarrow -j\text{sgn}(f)M(f),\ \ \hat{m}(t)e^{\pm j2\pi f_c t} \Longleftrightarrow -jM(f\mp f_c)\text{sgn}(f\mp f_c)$$

- Thus

$$\begin{aligned}\mathcal{F}^{-1}&\left\{\frac{1}{4}A_c\left[M(f+f_c)\text{sgn}(f+f_c)-M(f-f_c)\text{sgn}(f-f_c)\right]\right\}\\ &= -A_c\frac{1}{4j}\hat{m}(t)e^{-j2\pi f_c t}+A_c\frac{1}{4j}\hat{m}(t)e^{+j2\pi f_c t}\\ &= \frac{1}{2}A_c\hat{m}(t)\sin 2\pi f_c t\end{aligned}$$

- General form of a lower-sideband SSB signal

$$s_{LSB}(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) + \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

- Similarly, we can obtain the general form of a upper-sideband SSB signal

$$s_{USB}(t) = \frac{1}{2}A_c m(t)\cos(2\pi f_c t) - \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

- Block diagram for implementation

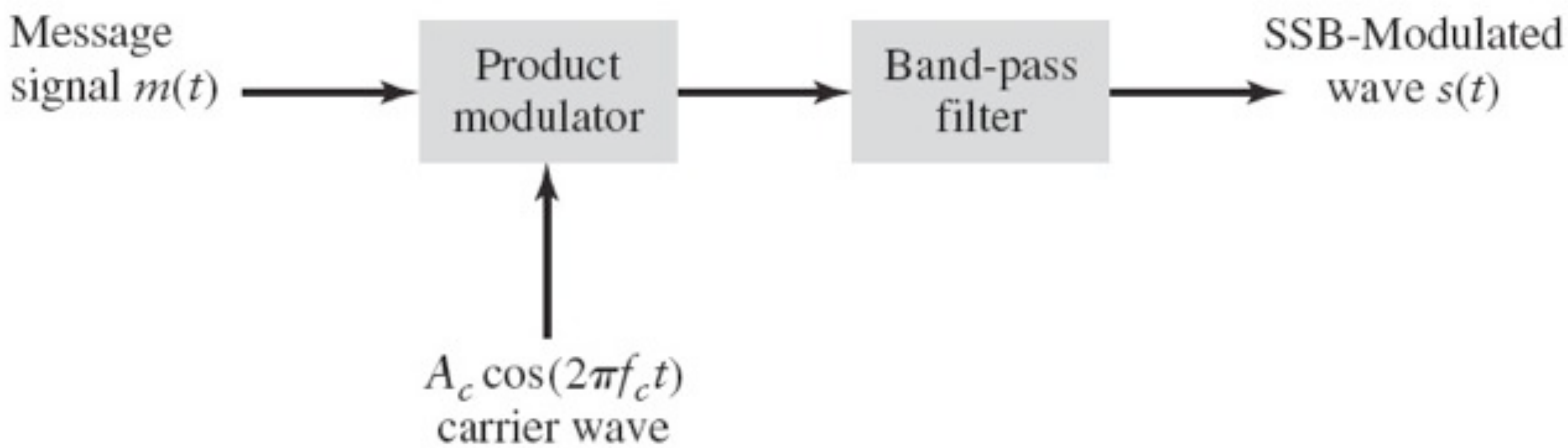


FIGURE 3.19 Frequency-discrimination scheme for the generation of a SSB modulated wave.

[Ref: Haykin & Moher, Textbook]

SSB Generation using Analytic Signal

- The positive-frequency portion of M(f)

$$M_p(f) = \frac{1}{2}\mathcal{F}\left[m(t) + j\hat{m}(t)\right]$$

- The negative-frequency portion of M(f)

$$M_n(f) = \frac{1}{2}\mathcal{F}\left[m(t) - j\hat{m}(t)\right]$$

- Upper-sideband SSB signal in the frequency domain

$$S_{USB}(f) = \frac{1}{2}A_c M_p(f - f_c) + \frac{1}{2}A_c M_n(f + f_c)$$

- Inverse Fourier transform

$$s_{USB}(t) = \frac{1}{4}A_c[m(t) + j\hat{m}(t)]e^{j2\pi f_c t} + \frac{1}{4}A_c[m(t) - j\hat{m}(t)]e^{-j2\pi f_c t}$$

or

$$s_{USB}(t) = \frac{1}{4}A_c m(t)[e^{j2\pi f_c t} + e^{-j2\pi f_c t}] - j\frac{1}{4}A_c\hat{m}(t)[e^{j2\pi f_c t} - e^{-j2\pi f_c t}]$$

Block Diagram for Implementation

- Wide-band phase-shifter is designed to produce the Hilbert transform in response to the incoming message signal.

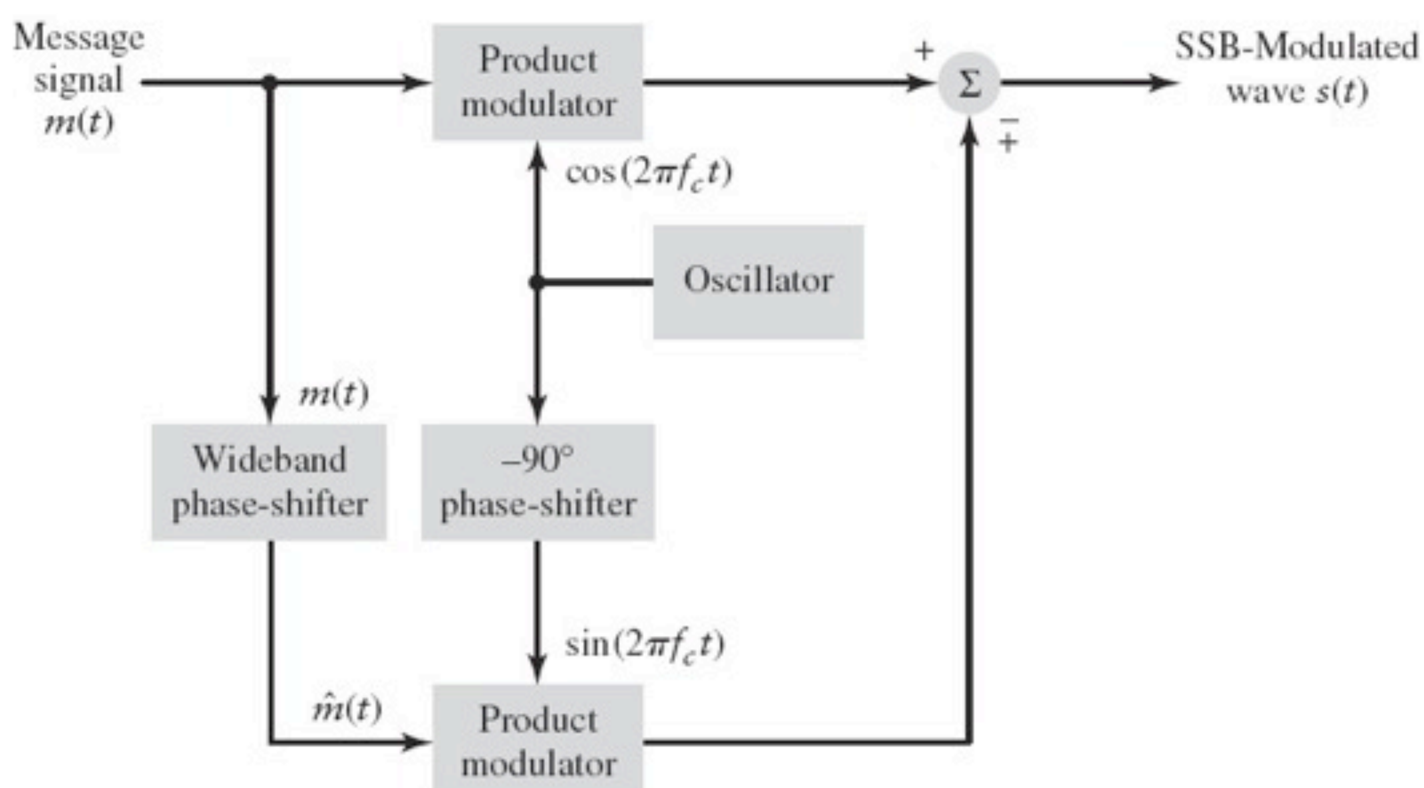


FIGURE 3.20 Phase discrimination method for generating a SSB-modulated wave. Note: The plus sign at the summing junction pertains to transmission of the lower sideband and the minus sign pertains to transmission of the upper sideband.

[Ref: Haykin & Moher, Textbook]

Coherent Detection of SSB

- Synchronization of a local oscillator in the receiver with the oscillator responsible for generating the carrier in the transmitter.
- Assume that the demodulation carrier has a phase error θ

$$
\begin{aligned}
d(t) &= \left[\frac{1}{2}m(t)\cos(2\pi f_c t) \pm \frac{1}{2}\hat{m}(t)\sin(2\pi f_c t)\right] \cdot \cos(2\pi f_c t + \theta) \\
&= \frac{1}{4}\left[m(t)\cos\theta + m(t)\cos(4\pi f_c t + \theta) \mp \hat{m}(t)\sin\theta \pm \hat{m}(t)\sin(4\pi f_c t + \theta)\right]
\end{aligned}
$$

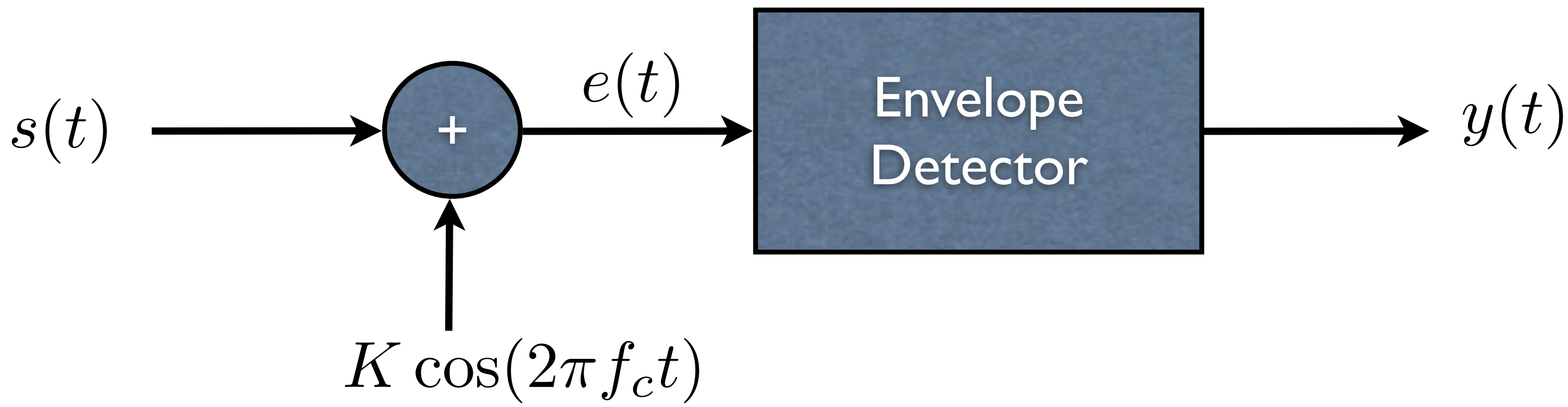
- Low-pass filtering and amplitude scaling yields

$$y(t) = m(t)\cos\theta \mp \hat{m}(t)\sin\theta$$

  - For θ equal to zero, the demodulator output is the desired message signal.

Envelope Detector of SSB Signal (Noncoherent)

- Consider the demodulator as follows:



$$e(t) = \frac{1}{2}[A_c m(t) + K]\cos(2\pi f_c t) \pm \frac{1}{2}A_c \hat{m}(t)\sin(2\pi f_c t)$$

$$y(t) = \sqrt{\left[\frac{1}{2}\left(A_c m(t) + K\right)\right]^2 + \left[\frac{1}{2}A_c \hat{m}(t)\right]^2}$$

- If we set K to be

$$\left[\frac{1}{2}A_c m(t) + K\right]^2 >> \left[\frac{1}{2}A_c \hat{m}(t)\right]$$

- Then,

$$y(t) \approx \frac{1}{2}A_c m(t) + K$$