# Quantum Mechanics 1

## Assignment 2

Due: April 2 (Tuesday), 2013

1. Consider a wave function of the form

$$\psi(x) = A\exp(-ax^2), \tag{1}$$

where $A$ and $a$ are positive constants.

(a) Express $A$ in terms of $a$.

(b) Compute the wave function $\phi(p)$ in momentum space corresponding to $\psi(x)$.

(c) Compute $\langle x^2\rangle$ and $\langle x\rangle^2$ using $\psi(x)$ and $\phi(p)$.

(d) Compute $\langle p^2\rangle$ and $\langle p\rangle^2$ using $\psi(x)$ and $\phi(p)$.

(e) Verify the uncertainty relation $\Delta x \cdot \Delta p \geq \hbar/2$.

(N.b. The uncertainty is defined as $\Delta x = \sqrt{\langle x^2\rangle - \langle x\rangle^2}$, and likewise for $\Delta p$.)

2. Find the Fourier tranforms of the following functions:

(a)

$$f(x) = \begin{cases} x, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases} \tag{2}$$

(b)

$$f(x) = \begin{cases} |x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases} \tag{3}$$

(c)

$$f(x) = \begin{cases} 1 - |x|, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases} \tag{4}$$

3. Consider the wave packet at $t = 0$ $\psi(x, 0) = A \exp[-(|x|/L) + ip_0 x/\hbar]$.

   (a) Normalize $\psi(x, 0)$.

   (b) Calculate $\phi(p, 0)$ and $\phi(p, t)$. Verify that each is normalized.

   (c) Examine the width of the wave packets in configuration space and momentum space at $t = 0$ and verify that the uncertainty relations are satisfied.

4. Consider a state function which is real, $\psi(x) = \psi^*(x)$.

   (a) Show that $\langle p \rangle = 0$. What about $\langle p^2 \rangle$? $\langle x \rangle$?

   (b) Under what conditions of $\psi(x)$, is $\phi(p)$ real, and what then is $\langle x \rangle$?

5. In class, I argued that the commutation relation $[x, p] = i\hbar$ is fundamental, and the form of the operators $x$ and $p$ in coordinate, and momentum spaces are devised to satisfy the commutation relation. For example, in coordinate space, we put $x_{\text{op}} = x$, and $p_{\text{op}} = -i\hbar\partial/\partial x$.

   Let us define, in coordinate space, $x_{\text{op}} = x + x_0$, $p_{\text{op}} = -i\hbar\partial/\partial x + p_0$, where $x_0$ and $p_0$ are real numbers. These operators also satisfy the above commutation relation. Then what is the meaning of these new operators? (Hint: Consider expectation values.)

   And what are the forms of the newly defined operators in momentum space?