Chapter 2

Conservation Force and Potential Energy

2.1 Work

2.1.1 Definition

1. The work W in the object done by the external force $\boldsymbol{F}$ form point A to point B along a path C is defined by

$$W_{A\to B} = \int_{x_A}^{x_B} \boldsymbol{F}\cdot d\boldsymbol{x}. \tag{2.1}$$

2.2 Kinetic energy

2.2.1 definition

1. The kinectic energy defined by

$$T = \frac{1}{2}mv^2. \tag{2.2}$$

or using the momentum $\boldsymbol{p} = m\boldsymbol{v}$,

$$T = \frac{p^2}{2m}. \tag{2.3}$$

2.2.2 Theorems

1. The first-order time derivative of the kinetic energy is

$$\frac{dT}{dt} = \boldsymbol{F}\cdot\boldsymbol{v}. \tag{2.4}$$

2. The work done on an object is the same as the difference in kinetic engergy.

$$W_{A\to B} = T_B - T_A. \tag{2.5}$$

2.3 Potential Energy

2.3.1 Definition

1. The potential energy U of particle is defined by

$$U_A = U_0 - \int_0^A \boldsymbol{F}\cdot d\boldsymbol{x}. \tag{2.6}$$

2.4 Conservative Force

2.4.1 Definition

1. If the potential energy at arbitrary point A is independent of the path C, the force $\boldsymbol{F}$ is called conservative force.

$$U_A = U_0 - \int_C \boldsymbol{F} \cdot d\boldsymbol{x}. \tag{2.7}$$

2.4.2 Theorems

1. If the force is conservative, for any closed path C, The following integral vanishes.

$$\oint \boldsymbol{F} \cdot d\boldsymbol{x} = 0. \tag{2.8}$$

2. If the force is conservative, there exists a scalar function U, which independent of t,

$$\boldsymbol{F} = -\nabla U. \tag{2.9}$$

3. If there exists a scalar function U,

$$\boldsymbol{F} = -\nabla U, \quad \frac{\partial U}{\partial t} = 0, \tag{2.10}$$

$\boldsymbol{F}$ is conservative.

2.5 Total Mechanical Energy

2.5.1 Definition

1. The total mechanical energy E is defined by

$$E = T + U \tag{2.11}$$

, where T is the kinetic energy and U is the potential energy.

2.5.2 Theorems

1. If the force is conservative, the total mechanical energy is conserved.

2. Under one-dimensional conservative force field,

$$F = -\frac{\partial}{\partial x} U(x), \tag{2.12}$$

The time elapsed for the particle to move from x_1 to x_2 is

$$\triangle t = \pm \int_{x_1}^{x_2} \frac{dx}{\sqrt{\frac{2}{m}[E - U(x)]}}. \tag{2.13}$$

3. If

$$\nabla \times \boldsymbol{F} = 0, \tag{2.14}$$

$\boldsymbol{F}$ is conservative. (If an arbitrary vector field $\boldsymbol{A}$ satisfies $\nabla \times \boldsymbol{A} = 0$, we called that $\boldsymbol{A}$ is irrotational.)