Wireless Communications (ITC731)

Lecture Note 8

23-April-2013

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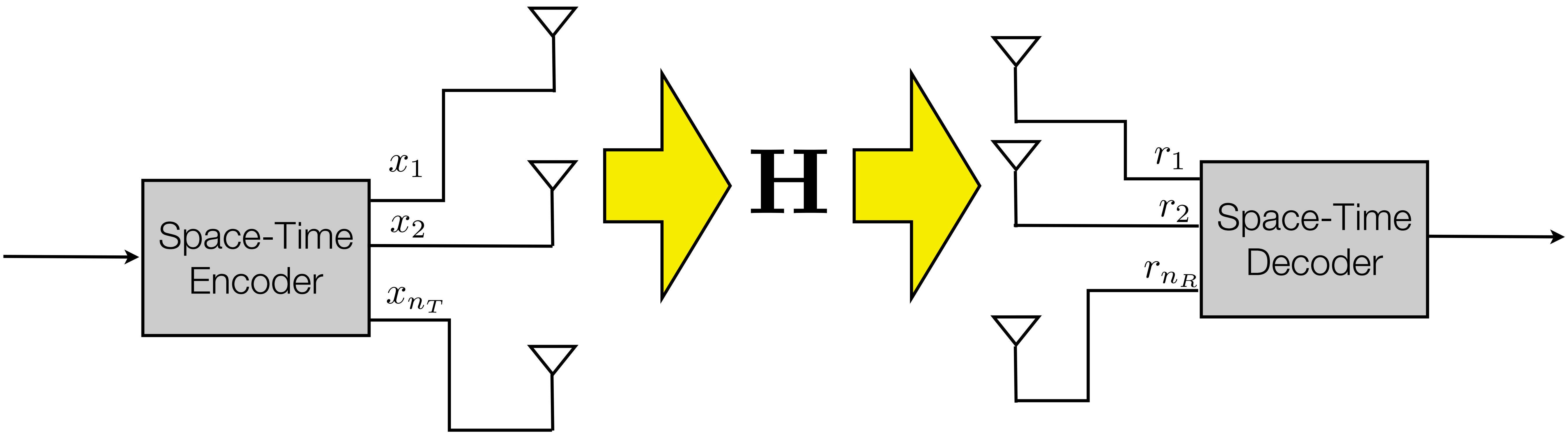
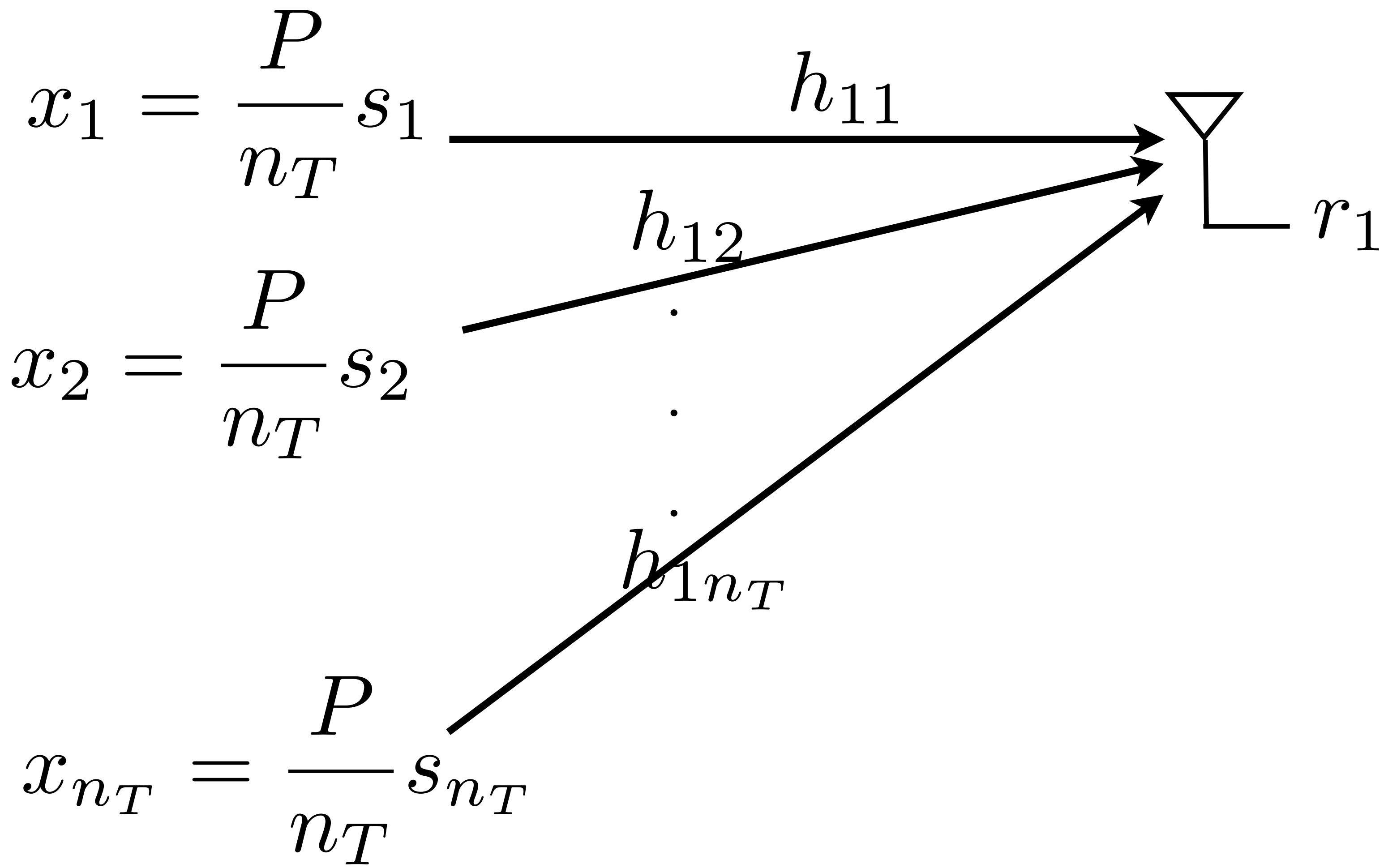
Summary

- Spatial multiplexing MIMO
  - Channel capacity
  - Channel capacity with power allocation
  - Layered space-time MIMO architecture

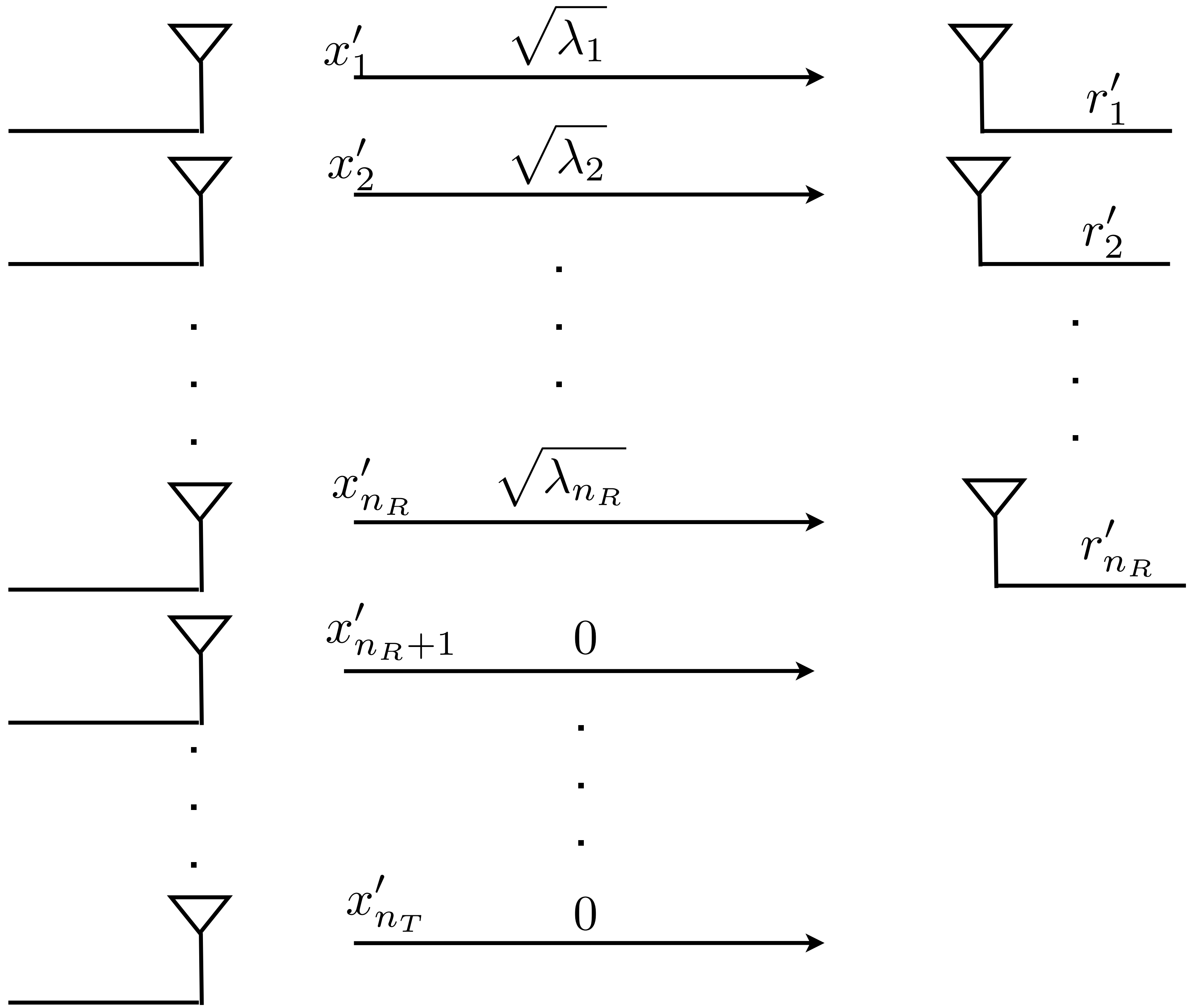
MIMO Systems



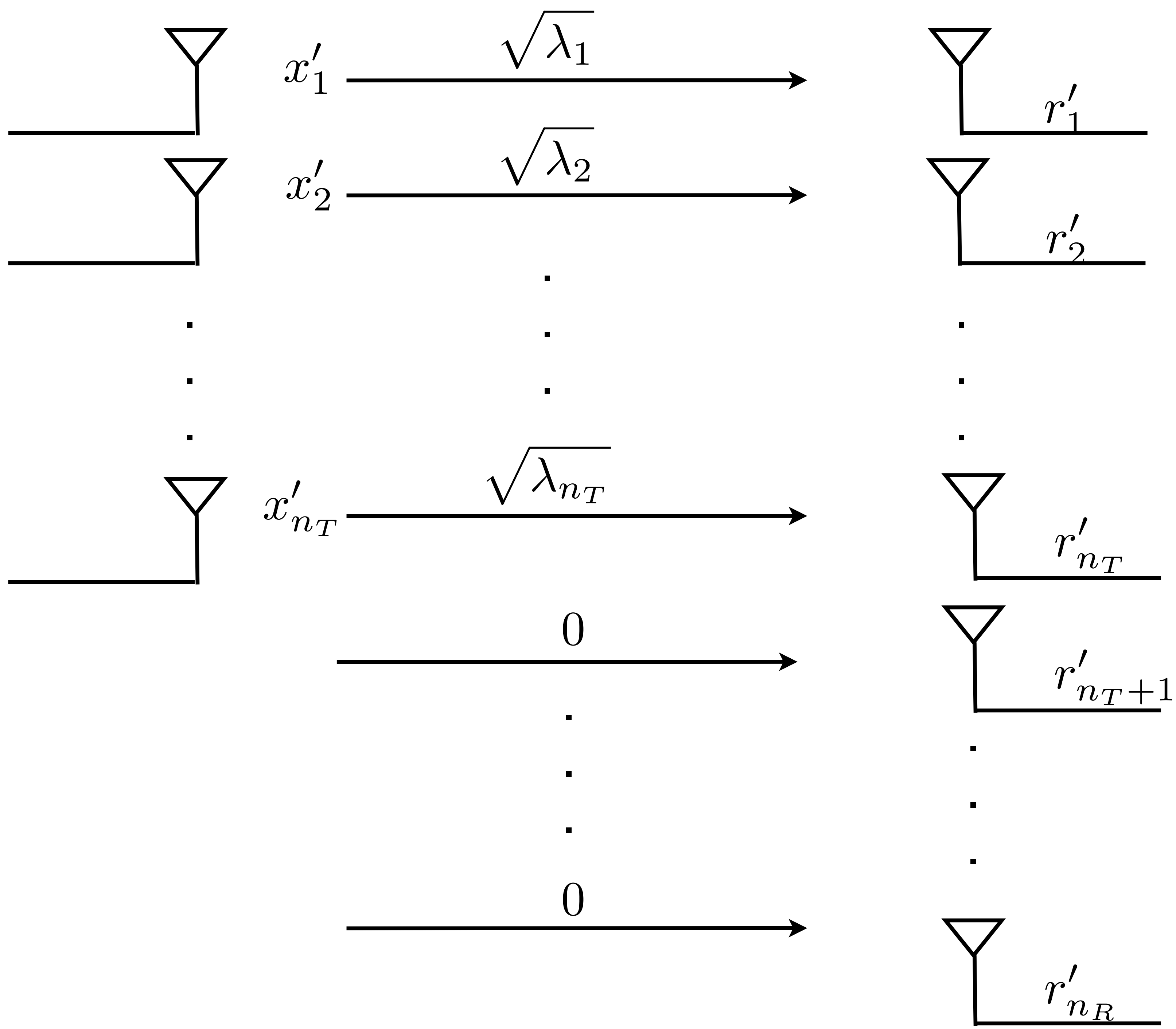
Equal Power MIMO Systems



Block diagram of an equivalent MIMO channel if $n_T > n_R$



Block diagram of an equivalent MIMO channel if $n_R > n_T$

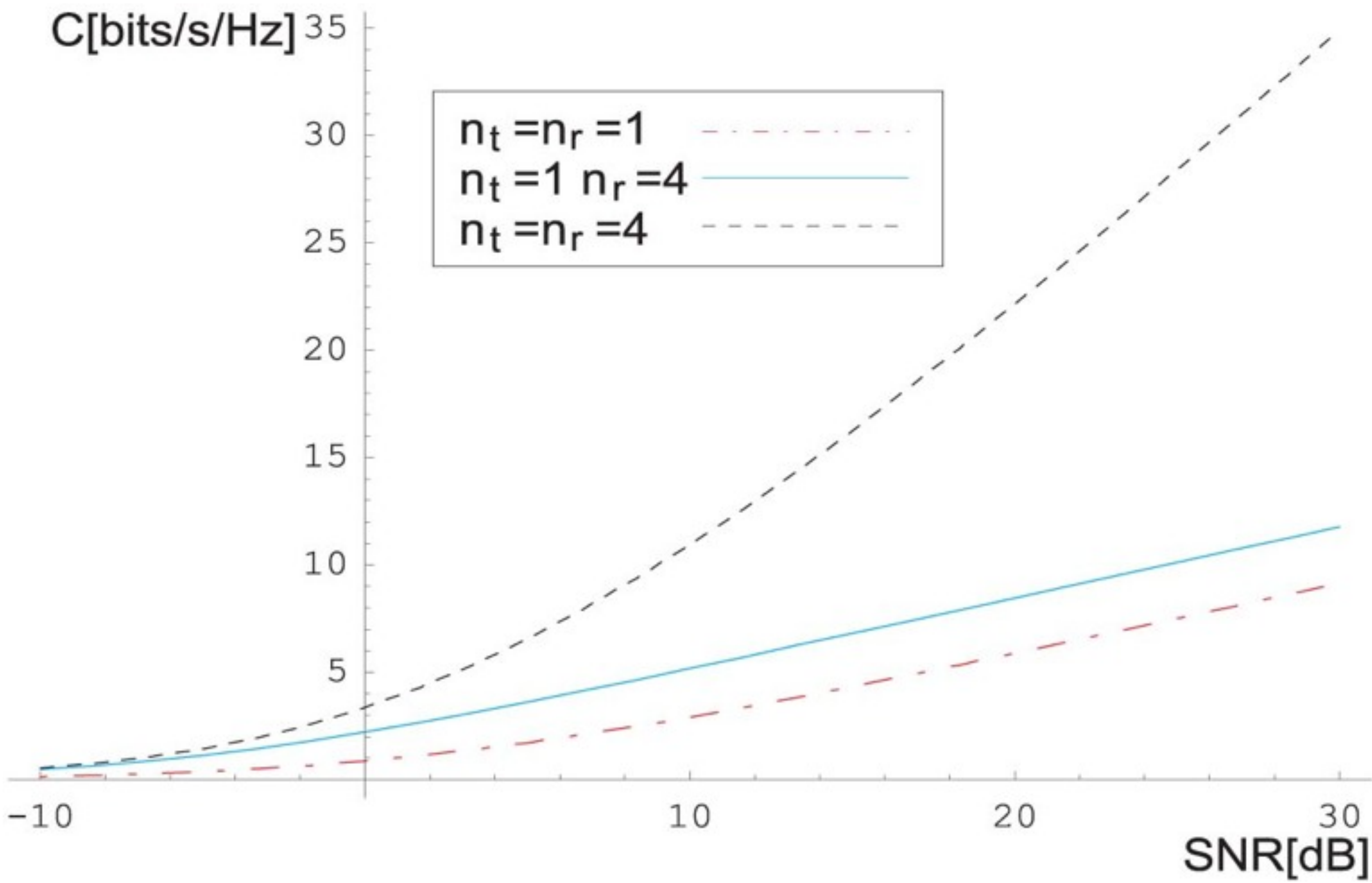


- The capacity formula can be written as

$$
\begin{aligned}
C &= W \log_2 \prod_{i=1}^{r} \left(1 + \frac{\lambda_i P}{n_T \sigma^2}\right) \\
&= W \log_2 \det \left(\mathbf{I}_r + \frac{P}{n_T \sigma^2} \mathbf{Q}\right) \\
&\sim r \log_2 \left(1 + \frac{P}{\sigma^2}\right)
\end{aligned}
$$

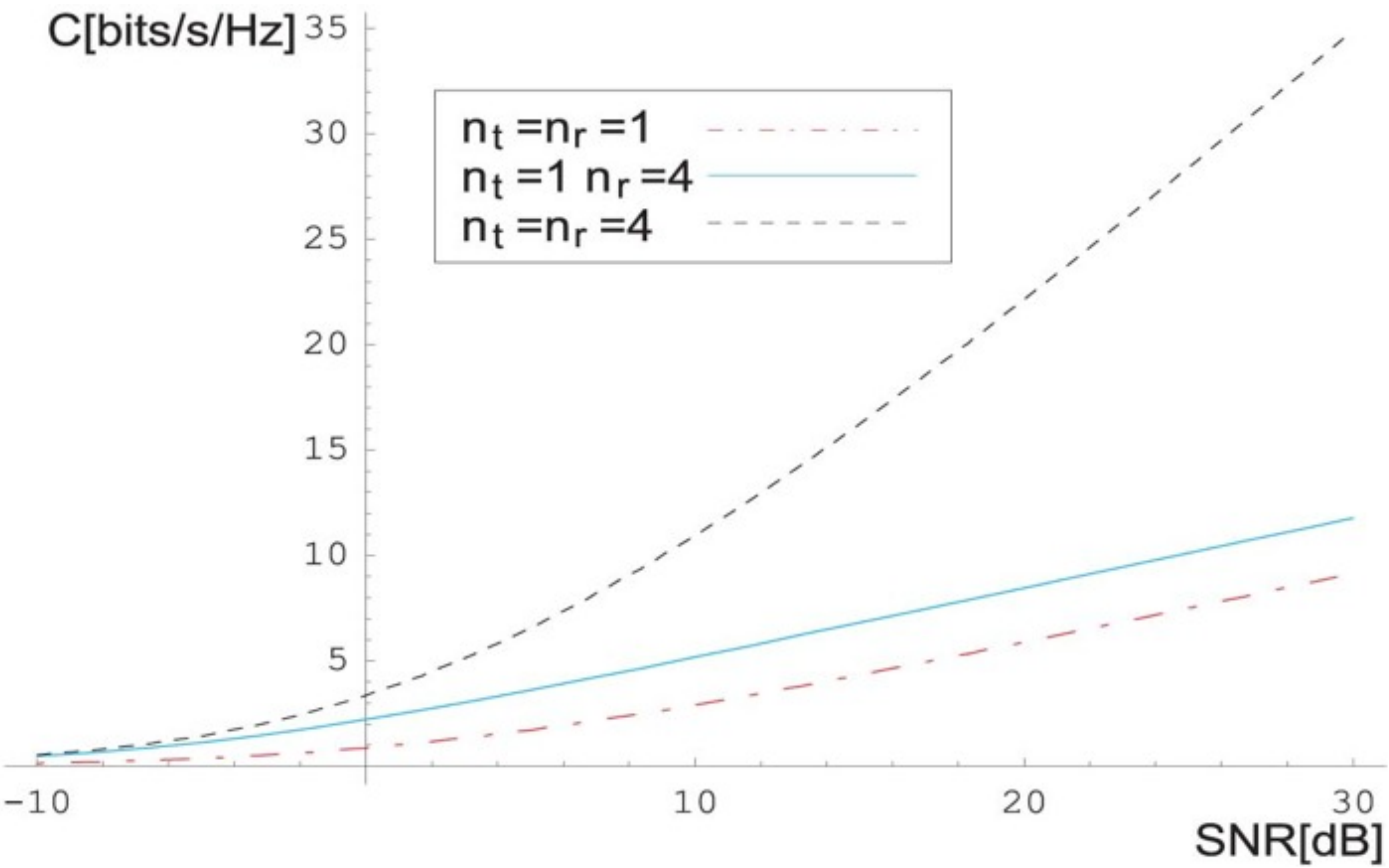
r → degree of freedom

Capacity over I.I.D. Rayleigh fading channels



[Ref: Tse and Viswanathan, "Fundamentals of Wireless Communications", Cambridge Press]

- Degree of freedom (DOF), $\min(n_T, n_R)$, determines the high SNR slope.



[Ref: Tse and Viswanathan, "Fundamentals of Wireless Communications", Cambridge Press]

MIMO Channel Capacity for Adaptive Power Allocation

Power constraint

$$\sum_{i=1}^{n_T} P_i = P, \qquad i = 1, 2, ..., n_T$$

Capacity of MIMO channels

$$\frac{C}{W} = \sum_{i=1}^{n_T} \log_2 \left[1 + \frac{P_i \lambda_i}{\sigma^2} \right]$$

Lagrangian multiplier

$$Z = \frac{C}{W} = \sum_{i=1}^{n_T} \log_2 \left[1 + \frac{P_i \lambda_i}{\sigma^2} \right] + L \left(P - \sum_{i=1}^{n_T} P_i \right)$$

L : Lagrangian multiplier

λ_i : ith channel matrix singular value

σ^2 : noise variance

Solution:

$$\frac{\partial Z}{dP_i} = 0$$

$$\frac{\partial Z}{dP_i} = \frac{1}{\ln 2} \frac{\lambda_i/\sigma^2}{1 + P_i\lambda_i/\sigma^2} - L = 0$$

Thus, we have

$$P_i = \mu - \frac{\sigma^2}{\lambda_i}$$

- Using water-filling rule, we can maximize the channel capacity with the power given as

$$P_i = \left(\mu - \frac{\sigma^2}{\lambda_i}\right)^+, \quad i = 1, 2, \ldots, r$$

where

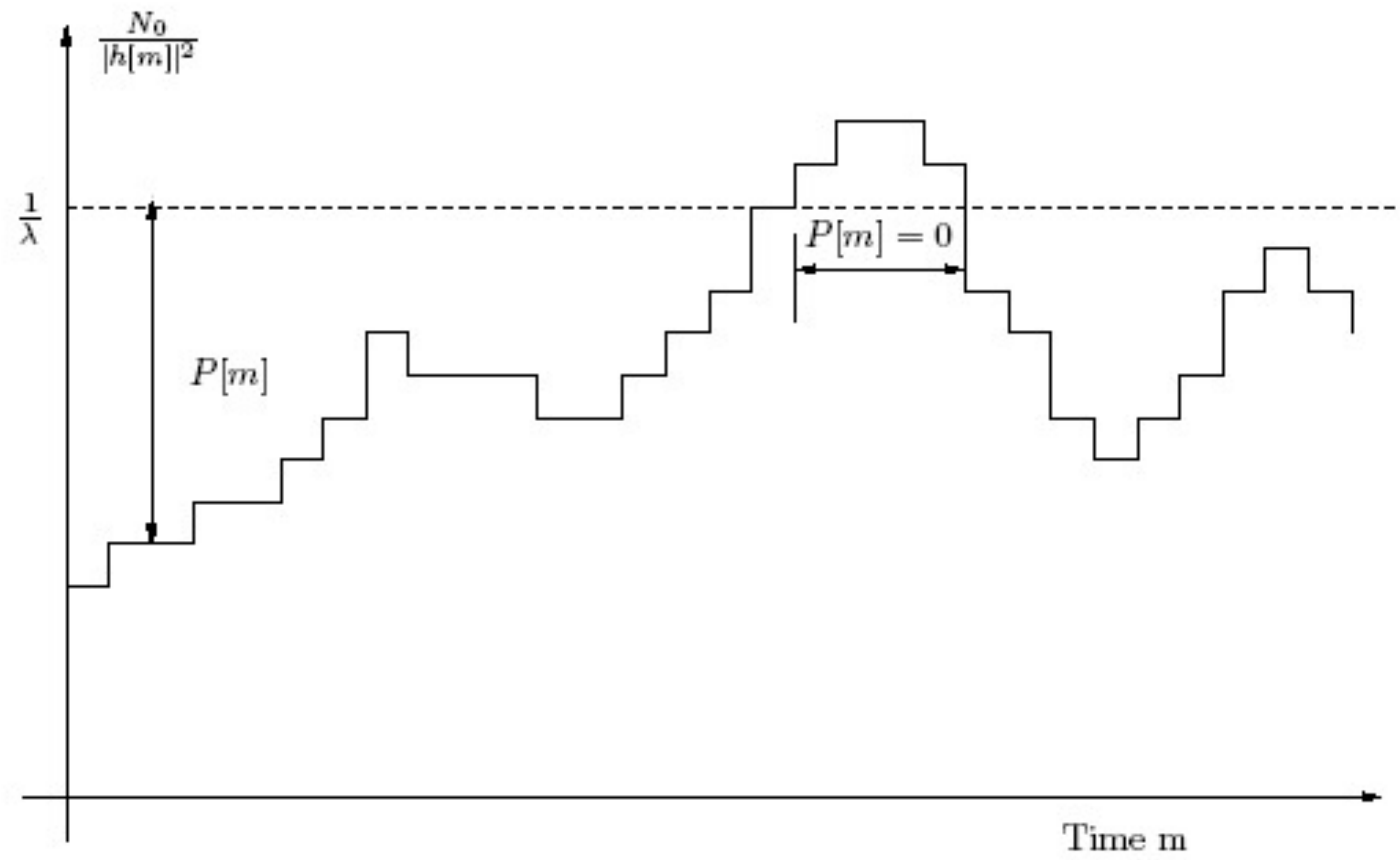
$$a^+ \quad = \quad \max(a, 0)$$

$$\mu \text{ is determined so that } \sum_{i=1}^{r} P_i = P$$

In this case, the channel capacity becomes

$$C = W \sum_{i=1}^{r} \log_2 \left[1 + \frac{1}{\sigma^2} (\lambda_i \mu - \sigma^2)^+ \right]$$

- Idea of Water-filling power allocation
  - Transmit more power when channel is good



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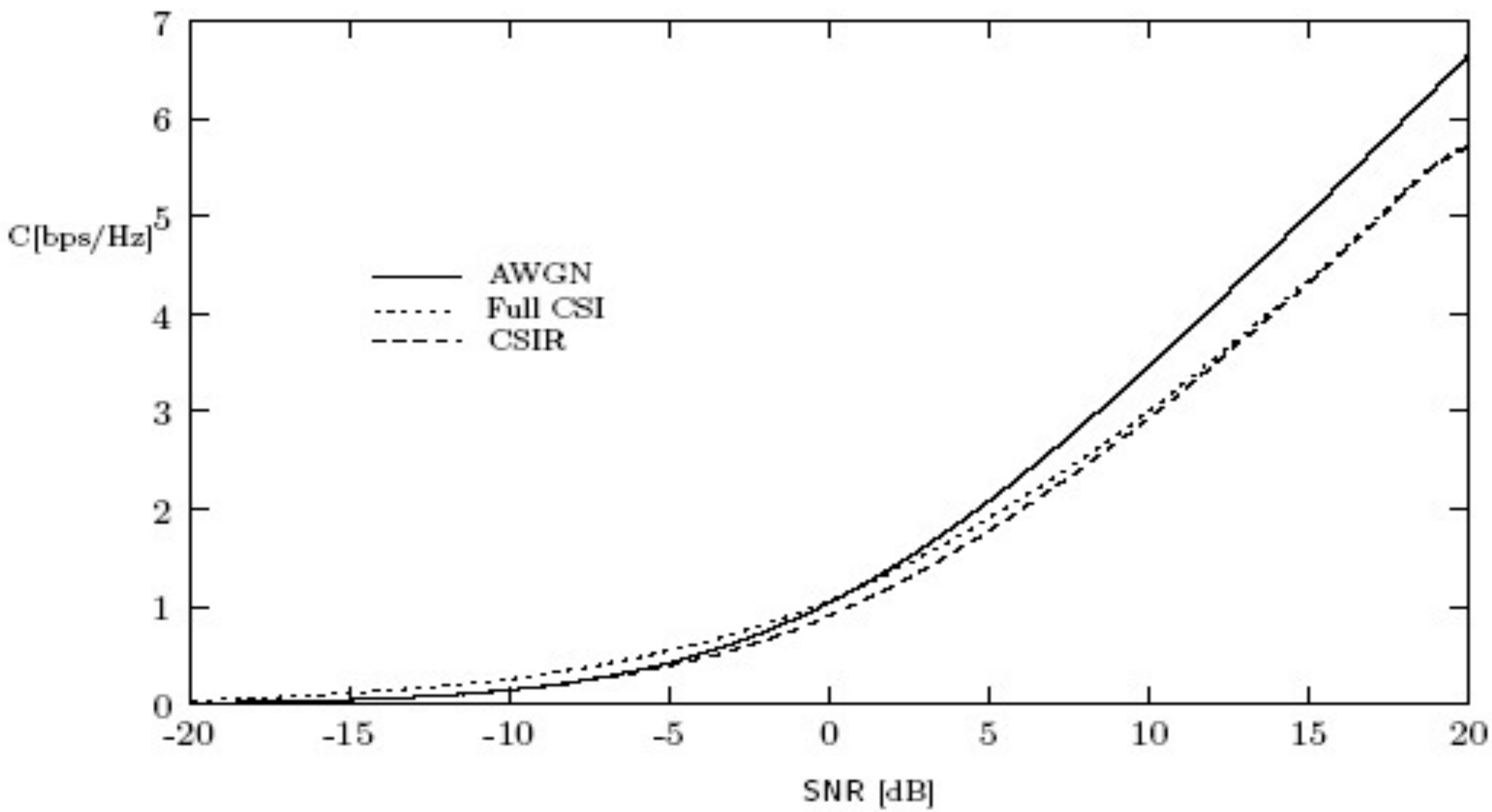
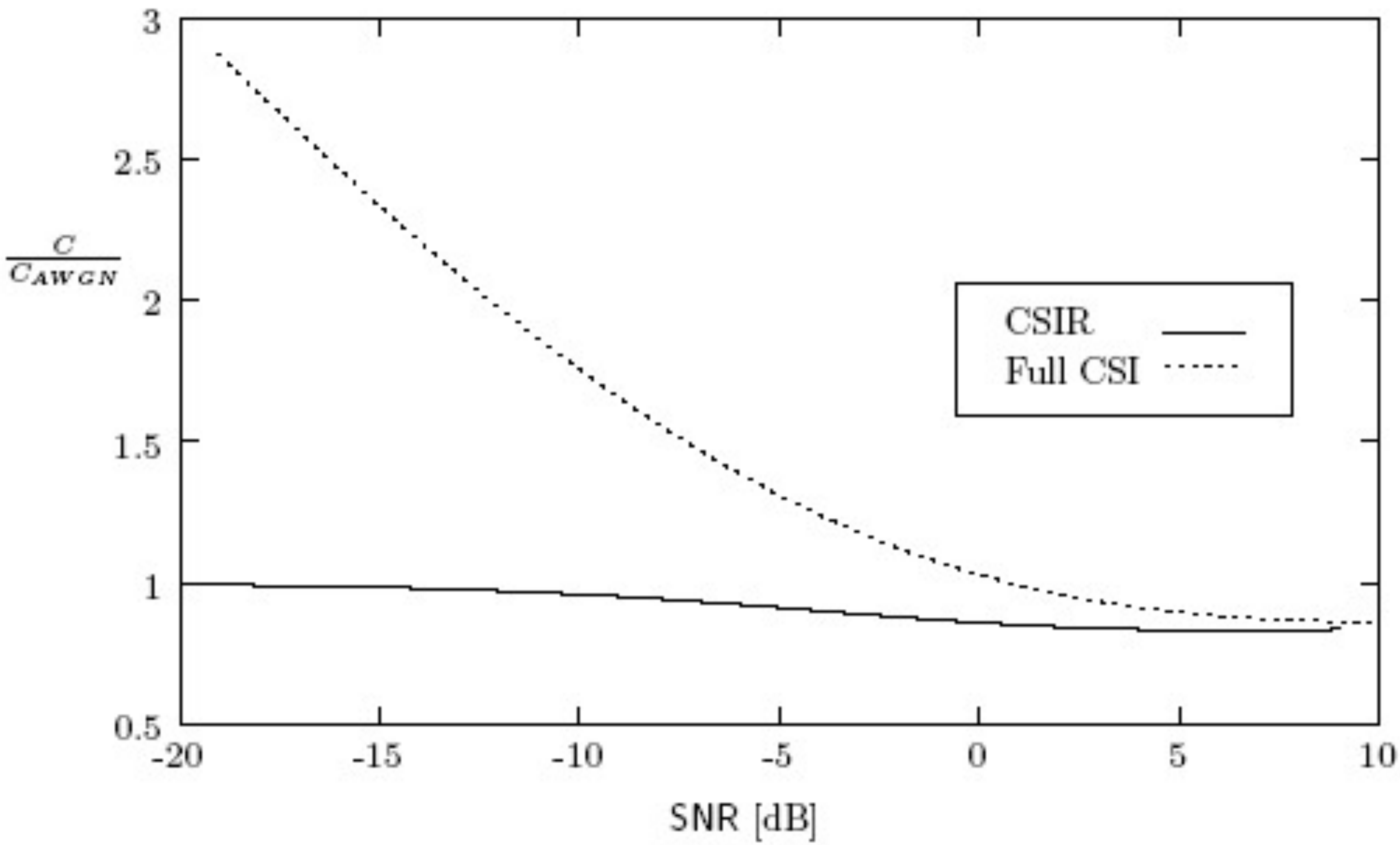
Performance



- At high SNR, water-filling does not provide any gain.

[Ref: Tse and Viswanathan, "Fundamentals of Wireless Communications", Cambridge Press]

Performance at Low SNR



- Water-filling provides a significant power gain at low SNR.

[Ref:Tse and Viswanathan, "Fundamentals of Wireless Communications", Cambridge Press]