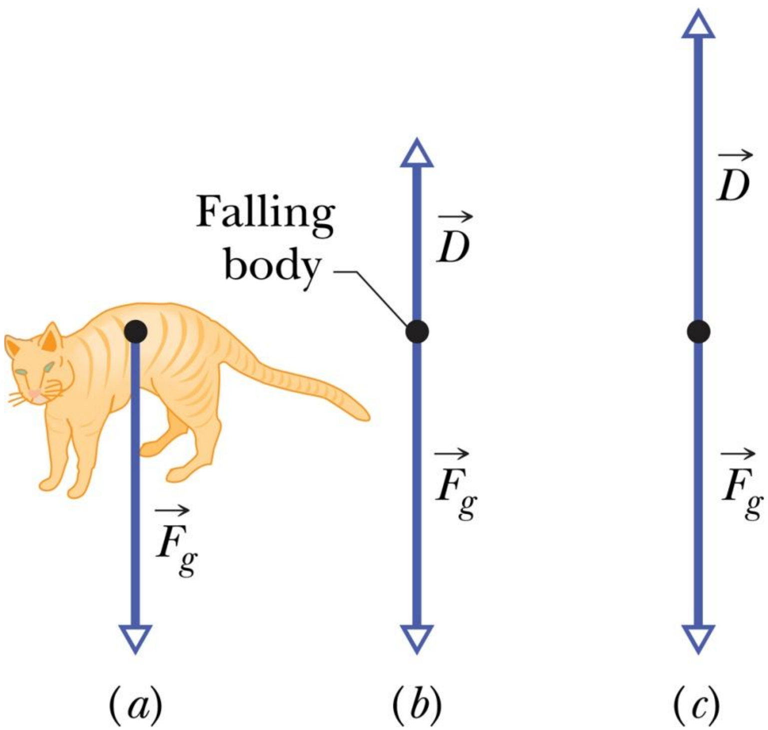
Copyright statement

- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

drag force

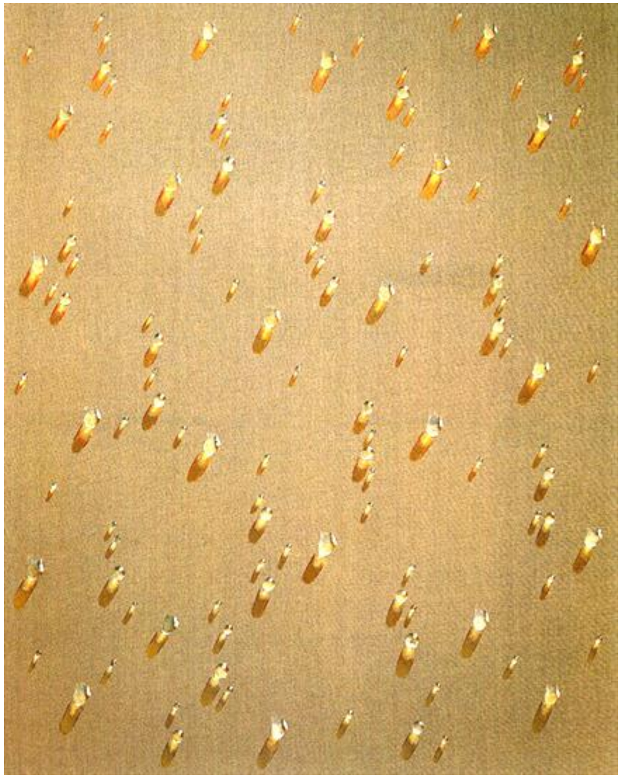


$$D \propto v^2$$

$$D = \frac{1}{2} C \rho A v^2$$

$$D_t = \frac{1}{2} C \rho A v_t^2 = mg \longrightarrow v_t = \sqrt{\frac{2mg}{C \rho A}}$$

Sample problem



$$m = \tfrac{4}{3}\pi R^3 \rho_w$$

$$A = \pi R^2$$

$$v_t = \sqrt{\frac{2mg}{C\rho A}} = 7.4\text{m/s}$$

$R = 1.5\text{mm},\ h = 1200\text{m},\ \text{C} = 0.60$

$\rho_w = 1000\text{kg/m}^2,\ \rho_{air} = 1.2\text{kg/m}^3$

$v_t = ?$

$$D = 0 \rightarrow v = \sqrt{2gh} = 150\text{m/s}$$

Chap. 5 Kinetic Energy and Work

Kinetic energy

definition $$K = \frac{1}{2}mv^2$$

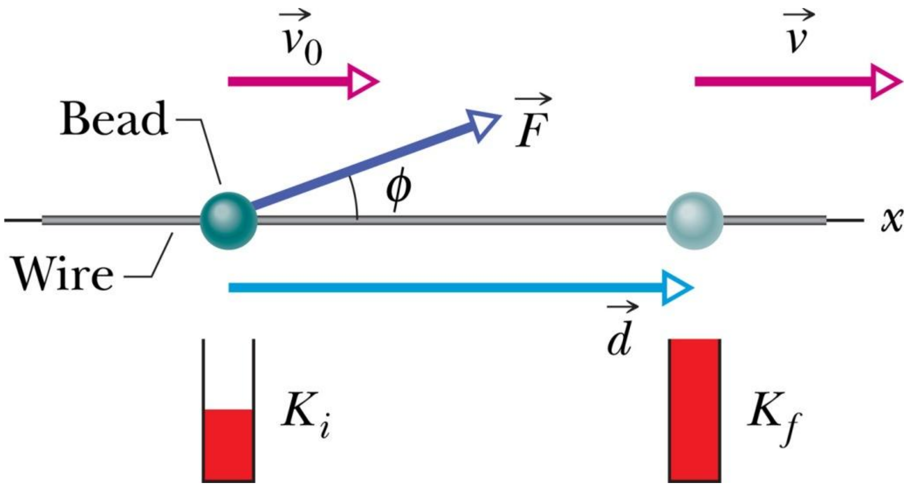
unit $$1 \text{ joule} = 1 \text{ J} = 1 \text{ kg} \cdot \text{m}^2/\text{s}^2$$

dimension $$[E] = ML^2T^{-2}$$

work

- work: 물체에 힘을 가하여 물체가 얻는 에너지.
- 물체에 힘을 가하여 물체의 에너지가 줄면 음(negative)의 일을 했다고 한다.
- work: 물체에 에너지를 전달하는 과정.

Work and kinetic energy



$$F_x = ma_x$$

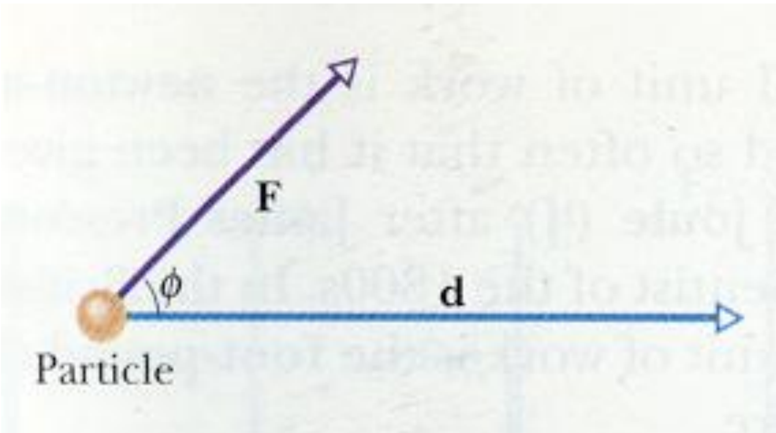
$$v^2 - v_0^2 = 2a_x d$$

$$\frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 = F_x d \equiv W$$

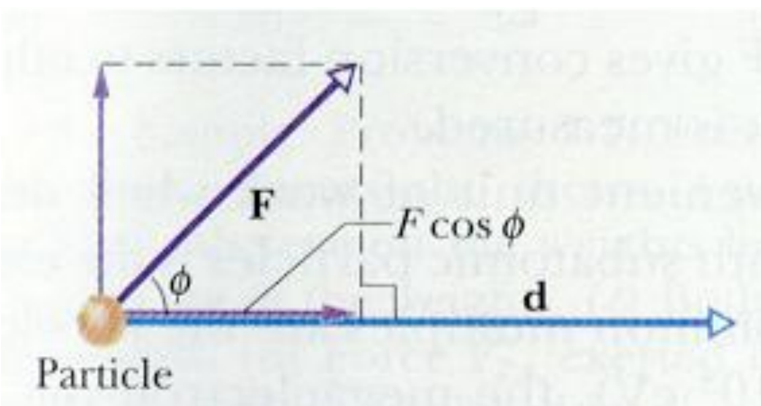
일반적으로 3차원의 경우 일정한 힘에 대해서는

$$W = Fd\cos\phi = \mathbf{F}\cdot\mathbf{d}$$

예: 중력이 한 일



$$W \equiv \mathbf{F} \cdot \mathbf{d}$$



$$W = Fd\cos\phi$$

Work-kinetic energy theorem

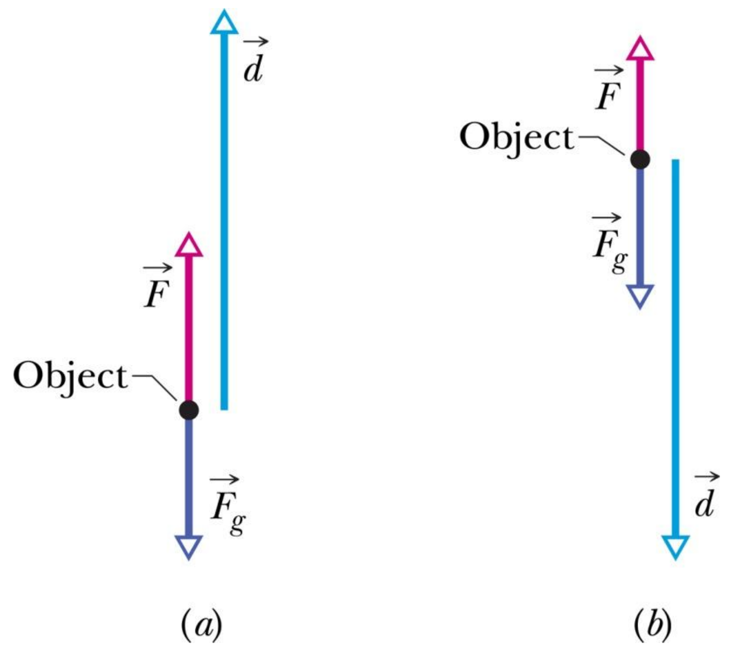
(change in kinetic energy) =
(work done on the body)

$$\Delta K \equiv K_f - K_i = W$$

(kin. Energy after work)
= (kin. Energy before work)+(work)

$$K_f = K_i + W$$

Work done by gravity



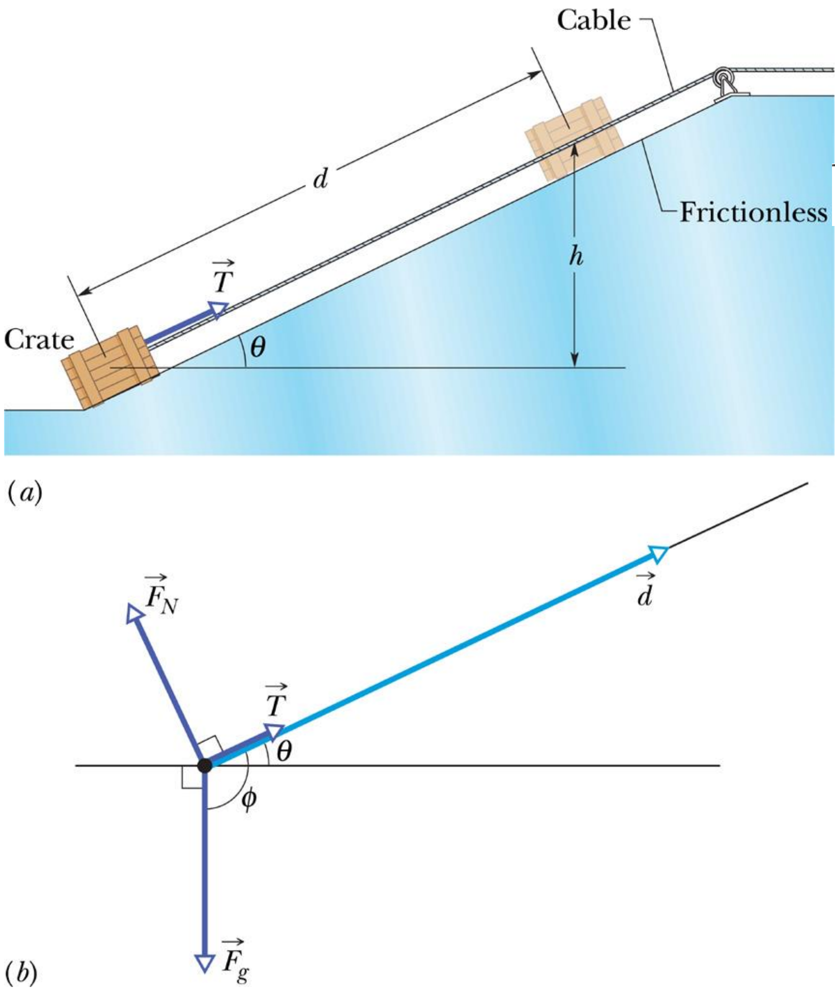
올라갈 때

$$W_g = mgd\cos\phi = +mgd\cos 180^\circ = -mgd$$

내려갈 때

$$W_g = mgd$$

Sample problem



$$W_g = -mg\,d\sin\theta = -mgh = -368 \text{ J}$$

$$mg\cos\left(\frac{\pi}{2}+\theta\right)$$

$$\Delta K = 0 = W_g + W_T + W_N$$

$$W_T = -W_g = 368 \text{ J}$$

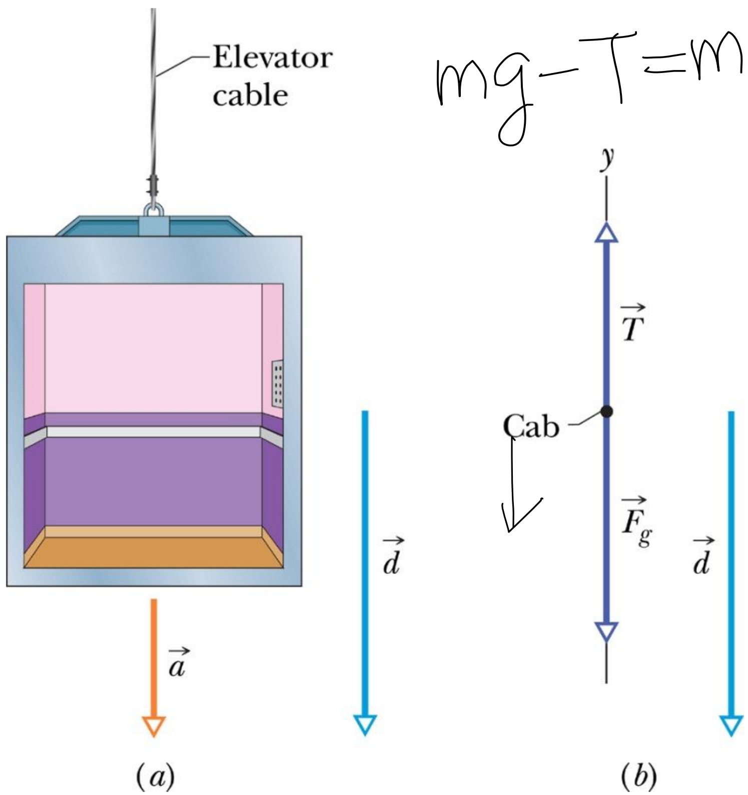
$m = 15.0\text{kg}, \;\; L = 5.70\text{m}, \;\; h = 2.50\text{m}$

Sample problem

$F_g = mg$

$T = m(\frac{4}{5}g)$

$mg - T = ma$



$m = 500\text{kg}$

$\mathbf{a} = \frac{1}{5}\mathbf{g}$

$v_i = 4.0\text{m/s}$

(1) d=12m, 중력이 한 일

$$W_1 = mgd\cos 0^\circ = 5.88\times10^4\,\text{J}$$

(2) d=12m, 장력이 한 일

$$\sout{T - mg = ma}$$

$$\therefore T = m(g+a) = 3920\text{N}$$

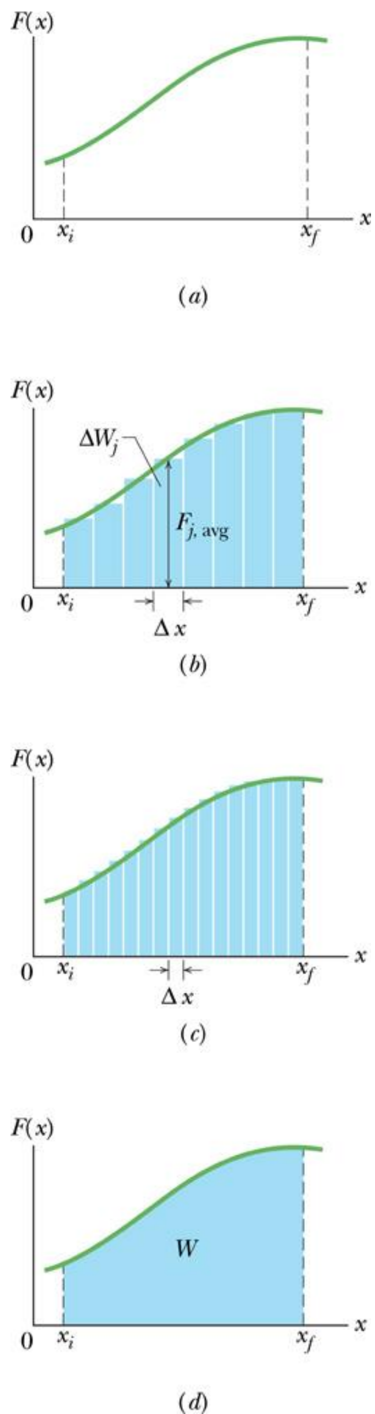
$$W_2 = \mathbf{T}\bullet\mathbf{d} = -Td = -4.7\times10^4\,\text{J}$$

(3) d=12m, 운동에너지

$$K_i = \tfrac{1}{2}mv_i^2 = 4000\text{J}$$

$$K_f = K_i + W = K_i + W_1 + W_2 = 1.6\times10^4\,\text{J}$$

Work done by varied force



Δx 동안 한 미소량의 일

$$\Delta W_j = F_{j,\text{avg}} \Delta x$$

전체 일 $$W = \sum \Delta W_j = \sum F_{j,\text{avg}} \Delta x$$

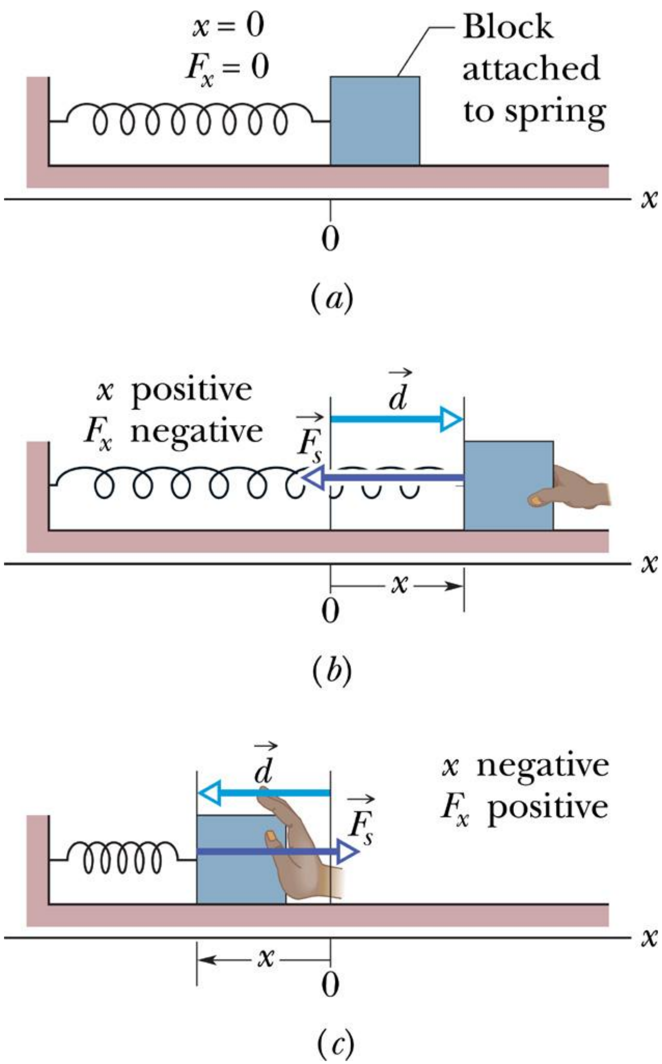
$$W = \lim_{\Delta x \to 0} \sum F_{j,\text{avg}} \Delta x = \int_{x_1}^{x_2} F(x) dx$$

3차원: $\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j} + F_z \mathbf{j}, \; d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$

$$dW = \mathbf{F} \cdot d\mathbf{r} = F_x dx + F_y dy + F_z dz$$

$$W = \int_{r_1}^{r_f} dW = \int_{x_1}^{x_2} F_x dx + \int_{y_1}^{y_2} F_y dy + \int_{z_1}^{z_2} F_z dz$$

Work done by a spring



Hooke's law

$$\mathbf{F} = -k\mathbf{d}$$

$$F = -kx$$

Work done by a spring

$$W_s = \int_{x_i}^{x_f} (-kx)dx = -\frac{1}{2}kx_f^2 + \frac{1}{2}kx_i^2$$

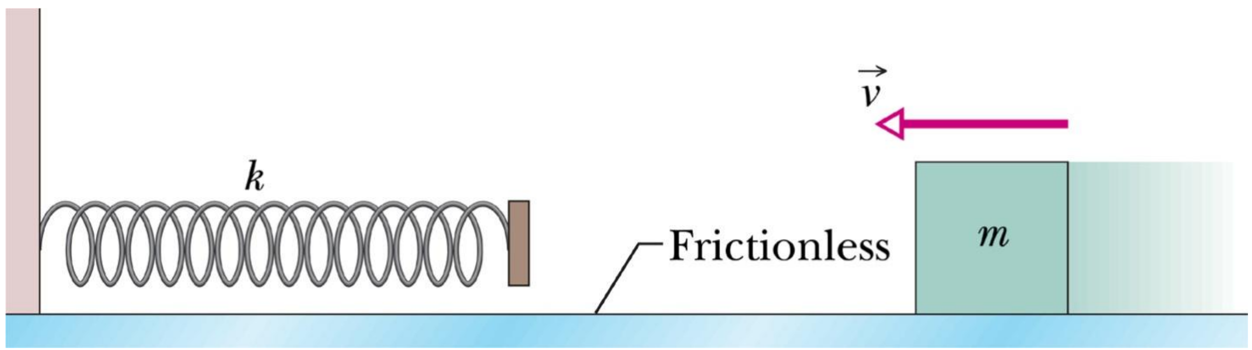
$$x_i = 0,\ x_f = x \longrightarrow W_s = -\frac{1}{2}kx^2$$

Work done by external force

$$\Delta K = K_f - K_i = W_a + W_s = 0$$

$$W_a = -W_s = \frac{1}{2}kx^2 \quad (\Delta K = 0)$$

Sample problem



$$m = 0.4 \text{ kg}, \;\; v = 0.50 \text{ m/s}, \;\; k = 750 \text{ N/m}, \;\; d = ?$$

$$K_f - K_i = -\frac{1}{2}kd^2 \qquad\qquad 0 - \frac{1}{2}mv^2 = -\frac{1}{2}kd^2$$

$$d = v\sqrt{\frac{m}{k}} = 1.2 \times 10^{-2} \text{ m}$$

Work-kinetic energy theorem using calculus

$$\frac{d}{dt} f(t)$$

$$\frac{df}{dt}$$

$$\begin{aligned}
W &= \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f} madx \\
&= \int_{x_i}^{x_f} m\frac{dv}{dt}dx = \int_{x_i}^{x_f} m\frac{dx}{dt}dv \\
&= \int_{v_i}^{v_f} mvdv = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = \Delta K
\end{aligned}$$

$$\frac{\sin x}{i} = \sin x$$

$$W = K_f - K_i = \Delta K$$

power

힘이 한 일의 시간 변화율

평균일률 $$P_{\text{avg}} = \frac{\Delta W}{\Delta t}$$

순간일률 $$P = \frac{dW}{dt}$$

$$1 \text{ watt} = 1\ W = 1 \text{ J/s} \qquad 1 \text{ hp} = 746\ W$$

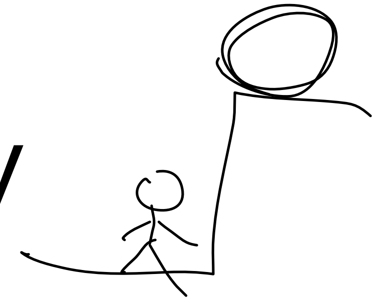
dimension: $$[P] = (MLT^{-2}L)T^{-1} = ML^2T^{-3}$$

힘이 일정할 경우: $$P = \frac{dW}{dt} = \frac{F\cos\phi\, dx}{dt} = F\cos\phi\frac{dx}{dt}$$

$$P = \mathbf{F}\cdot\mathbf{v}$$

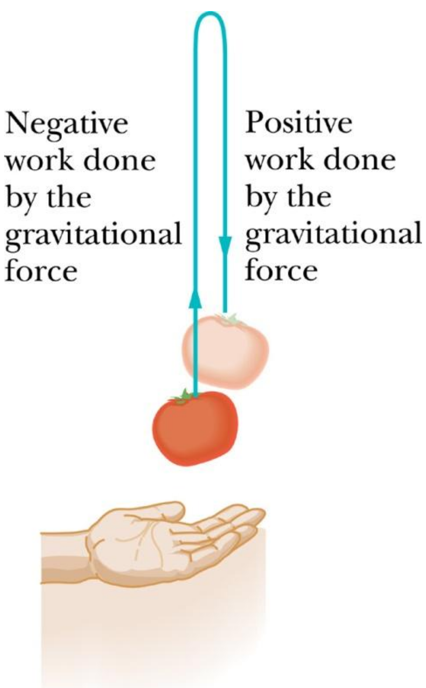
Ch. 6 Potential energy and energy conservation

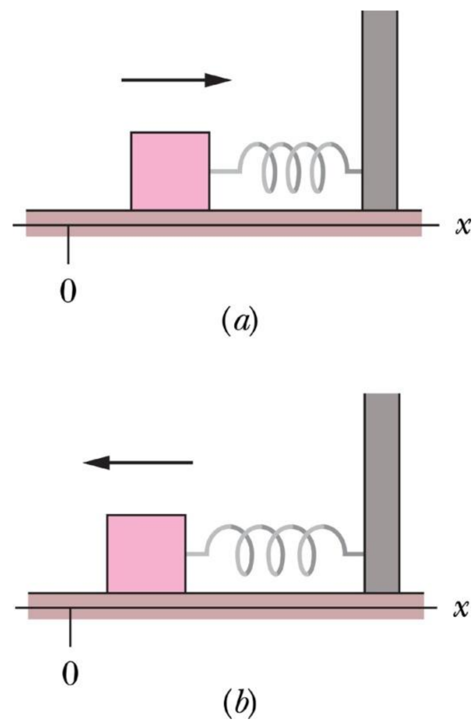
Potential energy



- 물리계 안에서 물체가 배열된 상태에 의해 결정되는 에너지의 형태

예: 중력 potential energy, 탄성퍼텐셜에너지

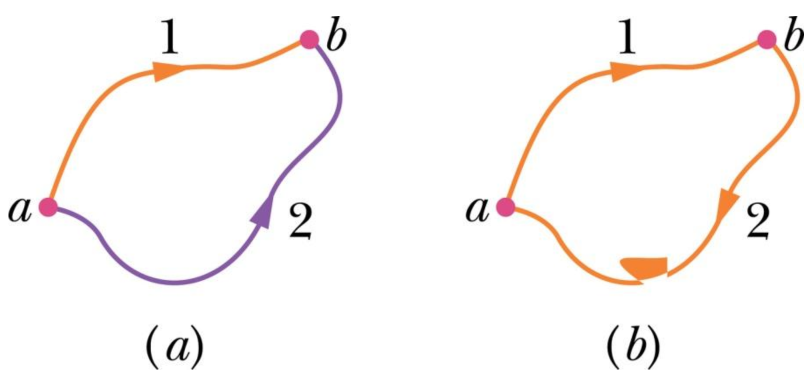




$$\Delta U = -W$$

conservative force

정의1: 임의의 닫힌 폐곡선을 따라 입자가 움직일 때 한 일이 0이면 이 힘을 conservative force라고 한다.

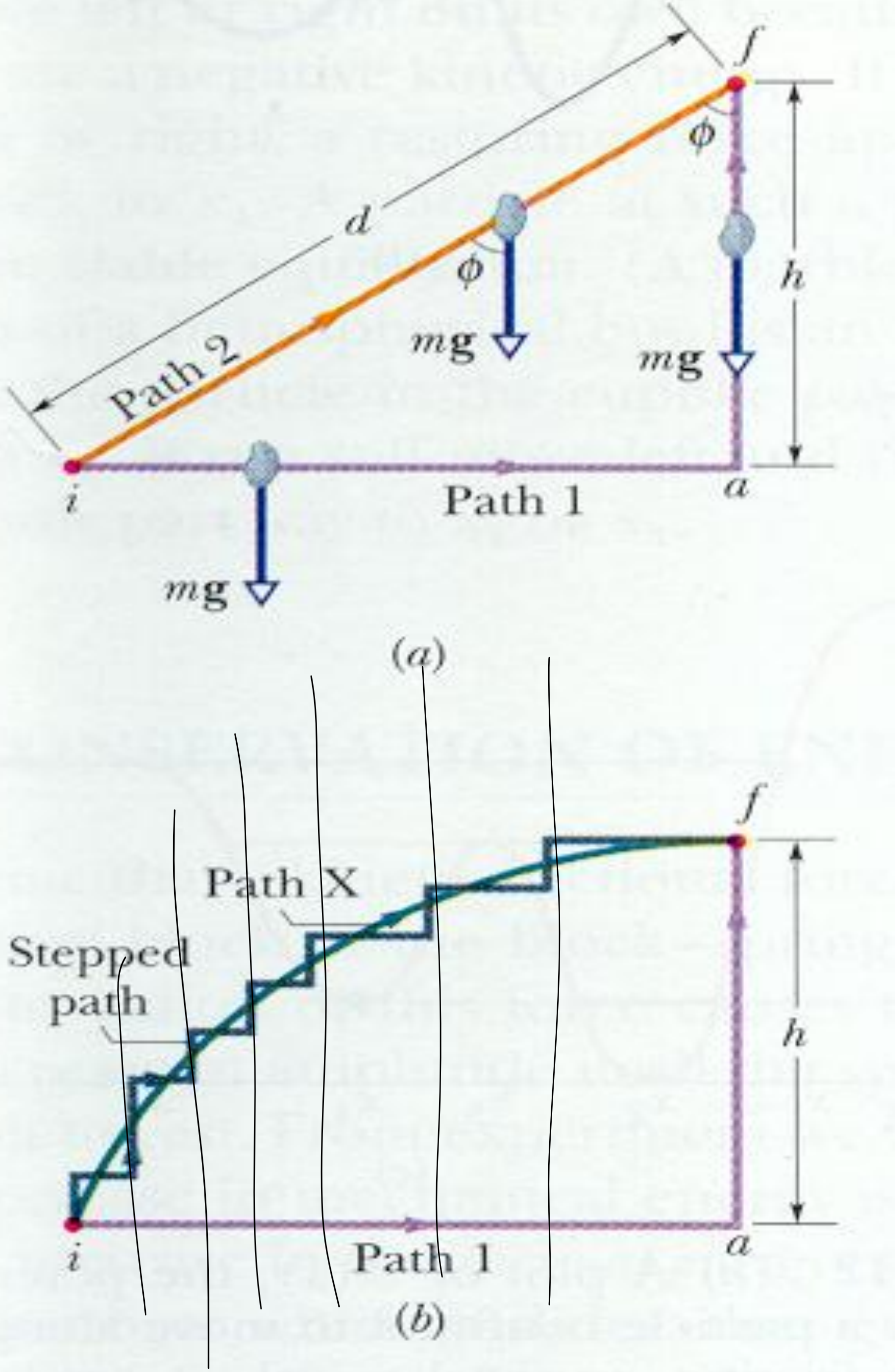


$$W_{aba} = 0$$

$$W_{aba} = W_{ab}^{1} + W_{ba}^{2} = 0$$

$$\therefore W_{ab}^{1} = -W_{ba}^{2} = W_{ab}^{2}$$

정의 2: 힘이 한 일이 경로에 무관하면 이 힘을 conservative force라고 한다.



중력이 한 일

$$W_1 = W_{ia} + W_{af}$$
$$= m\mathbf{g} \bullet \mathbf{d}_{ia} + m\mathbf{g} \bullet \mathbf{d}_{af}$$
$$= 0 + (-mgh)$$

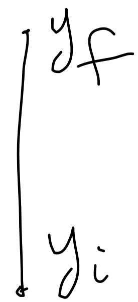
$$W_2 = m\mathbf{g} \bullet \mathbf{d}_{if}$$
$$= mgd\cos(180^\circ - \phi)$$
$$= -mgd\cos\phi$$
$$= -mgh$$

$$\therefore\ W_1 = W_2$$

중력은 conservative force이다.

Potential energy 결정

$$\Delta U = -W$$



$$W = \int_{x_i}^{x_f} F(x)dx$$

$$\Delta U = -\int_{x_i}^{x_f} F(x)dx$$

중력

$$\Delta U = -\int_{y_i}^{y_f} (-mg)dy = mg(y_f - y_i) = mg\Delta y$$

$$U(y) = mgy$$

용수철

$$\Delta U = -\int_{x_i}^{x_f} (-kx)dx = \frac{1}{2}kx_f^2 - \frac{1}{2}kx_i^2$$

$$U(x) = \frac{1}{2}kx^2$$

Conservation of mechanical energy

일과 운동에너지 정리 $\Delta K = W$

$$\Delta K = -\Delta U$$

Potential energy의 정의 $\Delta U = -W$

$$K_2 - K_1 = -(U_2 - U_1)$$

$$K_1 + U_1 = K_2 + U_2$$

$$\Delta E_{\mathrm{mech}} \equiv \Delta K + \Delta U = 0$$

$$\Delta K + \Delta U = \Delta(K + U)$$

Mechanical energy

한 일 ➔ 일을 할 수 있는 능력 🡐 potential energy

$$W = \Delta K \;\Rightarrow\; \Delta K + \Delta U = 0 \;\Leftarrow -\Delta U$$

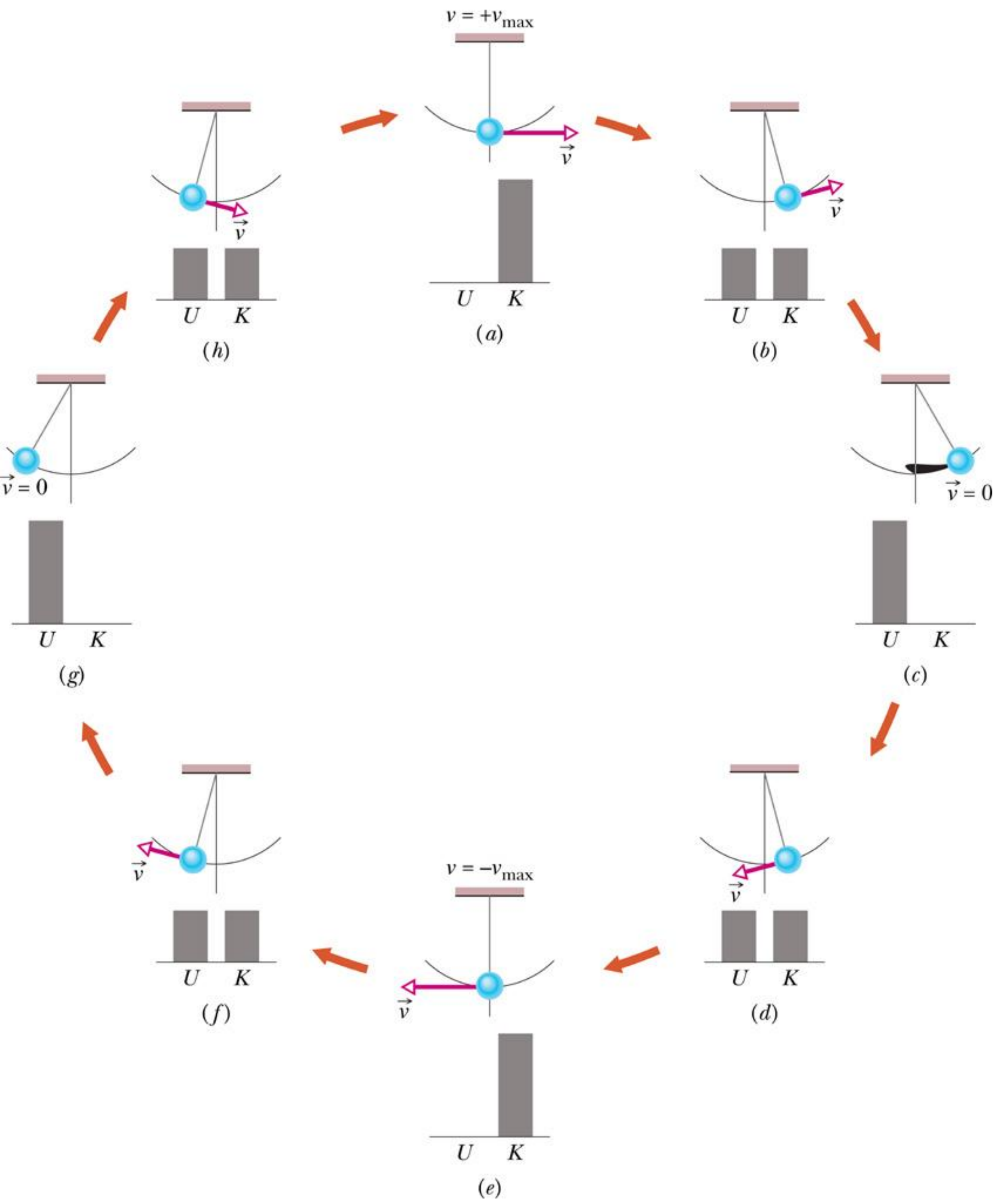
$$\Delta U = -\Delta K = -W = -\int_i^f F(x)dx$$

Mechanical energy

$$E \equiv K + U$$

conservation

$$\Delta E = 0$$



중력 potential energy

$$U(x) = -\int_0^x F(x)dx$$

$$= -\int_0^h (-mg)dx = mgh$$

$$E = K + U(x)$$

$$= \frac{1}{2}mv^2 + mgh = const.$$

Conservative force

정의1: 임의의 닫힌 폐곡선을 따라 입자가 움직일 때 한 일이 0이면 이 힘을 conservative force라고 한다.

$$W = -\Delta U(x)$$

$$\Downarrow$$

$$F(x)\Delta x$$

$$\therefore F(x) = -\frac{dU(x)}{dx}$$

역학에너지가 보존되는 형태로 potential energy 함수를 정의할 수 있을 때의 힘을 conservative force 라고 부른다.

이때 운동에너지와 potential energy 사이의 전환이 양 방향 모두 가능하므로 보존력이 한 일은 항상 가역적이다.