Mobile Communications (KECE425)

Lecture Note 22

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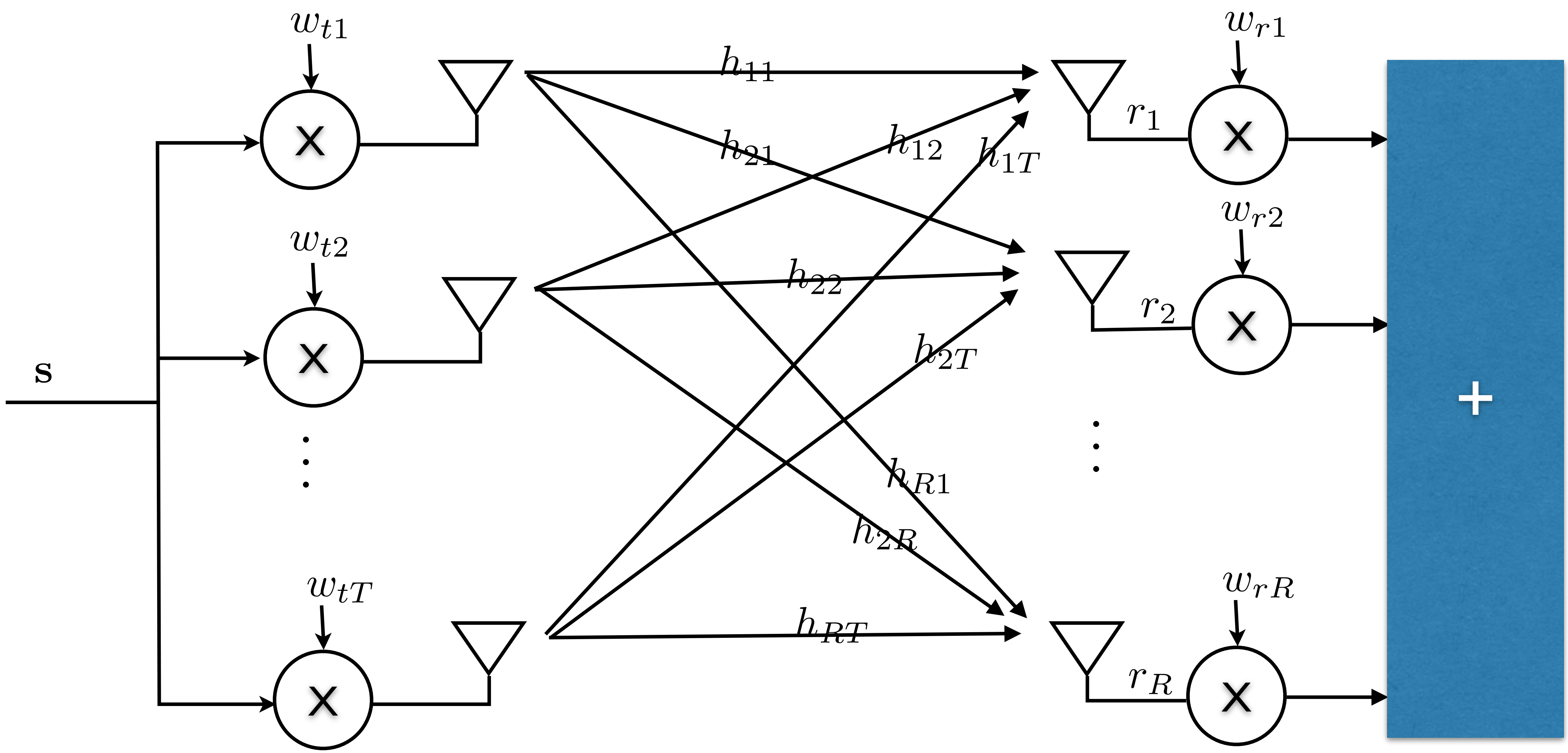
Summary

- Complexity issues of diversity systems
    - ADC and Nyquist sampling theorem
- Transmit diversity
    - Channel is known at the transmitter (Closed-loop transmit diversity: CLTD)
    - Channel is unknown at the transmitter (Space-time block coding: STBC)
- Transmit-Receive diversity (Maximal ratio transmission)
- Multi-user opportunistic diversity
- MIMO channel capacity

Maximal Ratio Transmission (MRT)

- MRT is also called multiple input multiple output (MIMO)-MRC.



- MIMO channel can be represented in matrix form:

$$\mathbf{H} = \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1T} \\ h_{21} & h_{22} & \cdots & h_{2T} \\ \vdots & \vdots & \vdots & \vdots \\ h_{R1} & h_{R2} & \cdots & h_{RT} \end{bmatrix}$$

- Vector representation

$$\mathbf{w}_t = [w_{t1} \;\; w_{t2} \;\; \cdots \;\; w_{tT}]^T$$

$$\mathbf{w}_r = [w_{r1} \;\; w_{r2} \;\; \cdots \;\; w_{rR}]^T$$

$$\mathbf{n} = [n_1 \;\; n2 \;\; \cdots \;\; n_R]^T$$

- Received signal:

$$\begin{aligned} r_1 &= (w_{t,1}h_{11} + w_{t,2}h_{12} + \cdots + w_{t,T}h_{1T})s + n_1 \\ r_2 &= (w_{t,1}h_{21} + w_{t,2}h_{22} + \cdots + w_{t,T}h_{2T})s + n_2 \\ &\vdots \\ r_R &= (w_{t,1}h_{R1} + w_{t,2}h_{R2} + \cdots + w_{t,T}h_{RT})s + n_R \end{aligned}$$

  - Received signal in vector form:

$$\mathbf{r} = \mathbf{H}\mathbf{w}_t s + \mathbf{n}$$

- Combined signal:

$$r_t = \mathbf{w}_r \mathbf{r}$$

- Optimal receive weight vector $\mathbf{w}_r$ can be easily shown to be given as

$$\mathbf{w}_r = c\left(\mathbf{H}\mathbf{w}_t\right)^H = c\mathbf{w}_t^H \mathbf{H}^H$$

where $(\cdot)^H$ denote the Hermitian operation.

  - In this case, the received signal can be written as

$$\begin{aligned} r_t &= \mathbf{w}_r \mathbf{r} \\ &= \mathbf{w}_r(\mathbf{H}\mathbf{w}_t s + \mathbf{n}) \\ &= c\mathbf{w}_t^H \mathbf{H}^H \mathbf{H}\mathbf{w}_t s + c\mathbf{w}_t^H \mathbf{H}^H \mathbf{n} \end{aligned}$$

- SNR of the received signal
  - Received signal can be written as

$$r_t = c\mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t s + c\mathbf{w}_t^H \mathbf{H}^H \mathbf{n}$$

  - SNR of r_t

$$\gamma_t = \frac{1}{\sigma_n^2} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t$$

- Optimal transmit weight vector, $\mathbf{w}_t^{\text{opt}}$

$$\begin{aligned} \mathbf{w}_t^{\text{opt}} &= \max_{\mathbf{w}_t} \gamma_t \\ &= \max_{\mathbf{w}_t} \frac{1}{\sigma_n^2} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t \\ &= \max_{\mathbf{w}_t} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t \end{aligned}$$

- Find the optimal weight vector $\mathbf{w}_t$ to maximize the SNR γ_t.

$$\mathbf{w}_t^{\mathrm{opt}} = \max_{\mathbf{w}_t} \mathbf{w}_t^H \mathbf{H}^H \mathbf{H} \mathbf{w}_t$$

  - We can solve this problem by making use of Rayleigh-Ritz theorem.

- Rayleigh-Ritz theorem

$$\mathbf{x}^H \mathbf{A} \mathbf{x} \leq ||\mathbf{x}|| \lambda_{\max}$$

where $\mathbf{A}$ is the Hermitian matrix, $\mathbf{x}$ is an y non-zero complex vector and $\lambda_{\max}$ is the largest eigenvalue of $\mathbf{A}$.

  - Equality holds if and only if $\mathbf{x}$ is the eigenvector corresponding to $\lambda_{\max}$.

- Based on Rayleigh-Ritz theorem, we can find the optimal weight vector $\mathbf{w}_t^{\mathrm{opt}}$, we can find the optimal weight vector as

$$\mathbf{w}_t^{\mathrm{opt}} = \sqrt{\Omega} \mathbf{U}_{\max}$$

where $\mathbf{U}_{\max}$ is the eigenvector corresponding to the largest eigenvalue of the quadratic form $\mathbf{F} = \mathbf{H}^H \mathbf{H}$ and $\mathbf{U}_{\max}^H \mathbf{U}_{\max} = \mathbf{I}$

- Combined SNR with the optimum weight vector

$$\gamma_t = \frac{\Omega \lambda_{\max}}{\sigma_n^2}$$

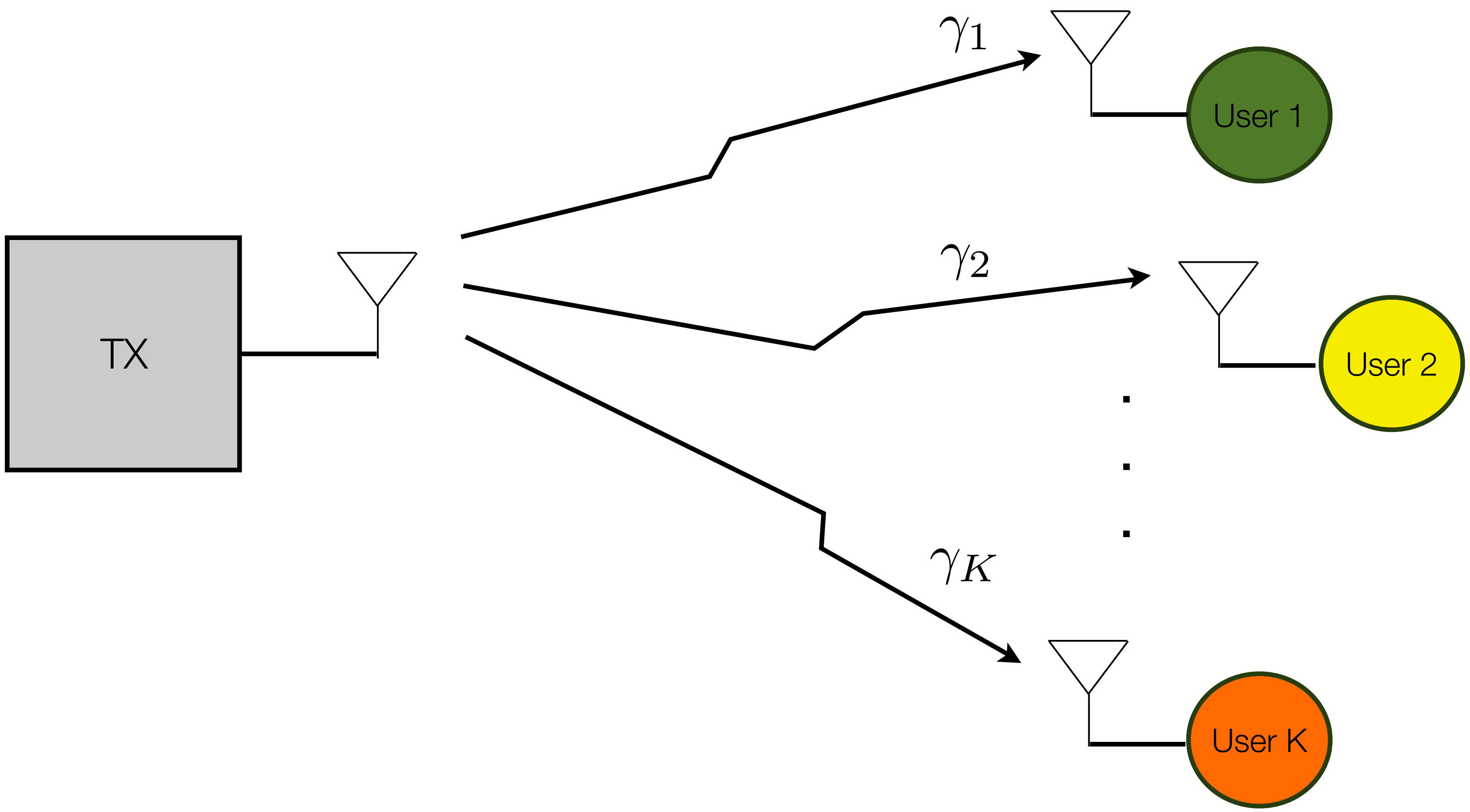
Multi-User Opportunistic Diversity

- We often need to select users if there are more than users to support the service, for a certain limited frequency (or/and time) resource.

- Example:
  - There are 50 MHz bandwidth for the service and each user takes 5 MHz bandwidth. In this case, we can support 10 users for a given time.
  - However, more than 50 users, saying 100 users, are willing to communicate at the same time, what is the best way to select users among 100 users?

- Multi-user opportunistic diversity scheme is simply to select the users with the strongest SNRs.

- Schematic concept of multi-user diversity (MUD).



- Choose the user which has the largest SNR among K users.

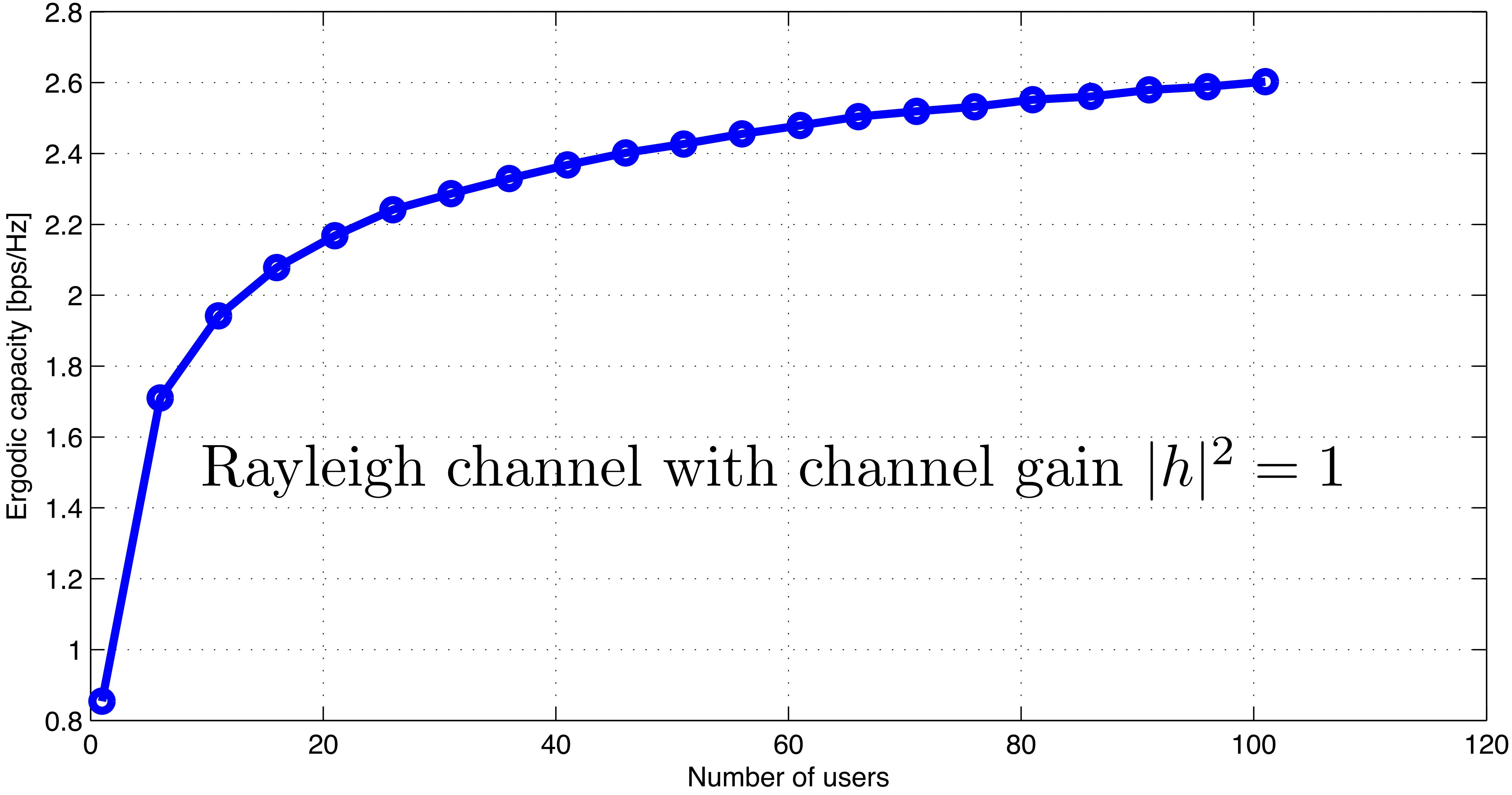
- If one user is selected out of K users at every selection period, the selected user k^* can be written as

$$k^* = \max_k (\gamma_1, \gamma_2, \cdots, \gamma_K)$$

- By doing this, we can improve the channel capacity such as

$$\begin{aligned} C &= E\left[\log_2(1+\gamma_k^*)\right] \\ &= \int_0^{\infty} \log_2\left(1+\gamma_k^*\right) p_{\gamma_{k^*}}\left(\gamma_{k^*}\right) d\gamma_{k^*} \end{aligned}$$

- Multi-user diversity gain



Channel Capacity in Diversity MIMO

$$C = \log_2 \left(1 + \gamma_t\right) \quad [\text{bps/Hz}]$$

Channel capacity is logarithmically increasing versus SNR which is very slow rate of increasing.

Degree of freedom is 1.

$$C = \log_2\left(1 + \gamma_t\right) \quad [\text{bps/Hz}]$$