Quantum Mechanics II

Assignment 2

Due: October 8 (Tuesday), 2013

1. We considered the relativistic and the spin-orbit corrections to the hydrogen atom. Neglecting the relativistic correction for the time being, find the energy splitting of the $n = 2$ unperturbed states of the hydrogen atome due to the spin-orbit interaction only. Reproduce the diagram in the middle in Figure 12-1 in Gasiorowicz.

2. For an isotropic harmonic oscillator in three dimensions, compute the energy shift of the ground state for the relativistic expression of the kinetic energy.

3. If the nucleus is taken to be a uniformly charged sphere of radius R, then the Coulomb potential in the atom is modified to

$$V(r) = \begin{cases} \dfrac{Ze^2}{2R}\left(\dfrac{r^2}{R^2} - 3\right), & r < R \\ -\dfrac{Ze^2}{r}, & r > R \end{cases} \tag{1}$$

   (a) Identify the perturbing potential which differs from the unperturbed Coulomb potential of a point charge.

   (b) Calculate the energy shift produced by this perturbation for the $1S$ and $2S$ levels of hydrogen.

4. Consider the spin Hamiltonian given by

$$H = As_z^2 + B(S_x^2 - S_y^2), \quad |B| \ll |A|, \tag{2}$$

   for a system of spin-1. This Hamiltonian is obtained for a spin-1 ion located in a crystal with rhombic symmetry.

   (a) In the basis of S_z, express H in terms of a 3×3 matrix. That is, compute $\langle 1, m|H|1, m'\rangle$ for $m, m' = 1, 0, -1$.

   (b) Find the eigenvalues of this Hamiltonian using degeneratepertur-bation theory.

5. For two particles systems, we defined the relative coordinates $\mathbf{r}$ and momentum $\mathbf{p}$ in class. Starting from $[r_i^a, p_j^b] = i\hbar\delta_{ij}\delta^{ab}$, where $i = 1, 2$ are particle indices and $a, b = x, y, z$ are cartesian components, show that the relative coordinates and momentum satisfy the canonical commutation relations.