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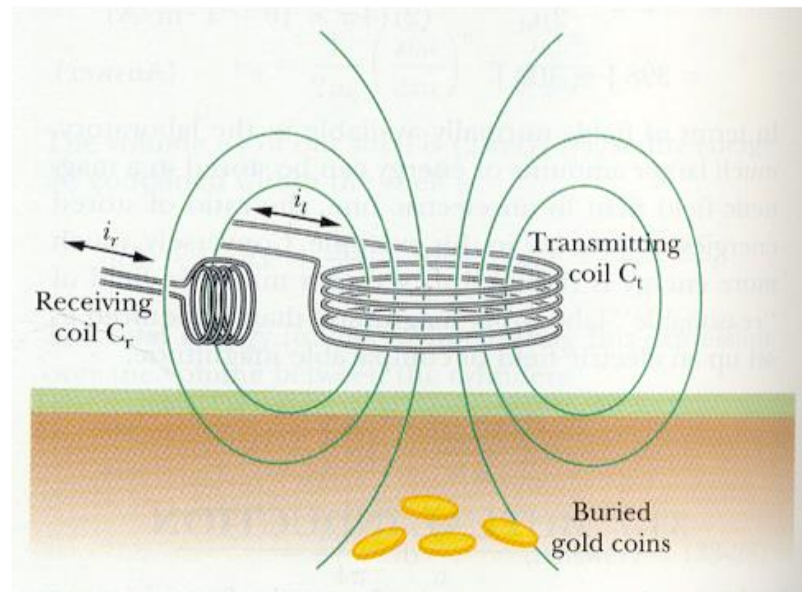
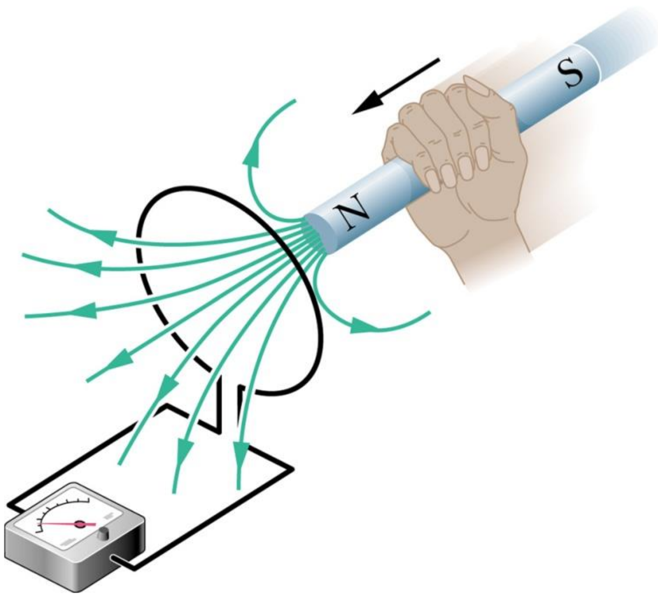
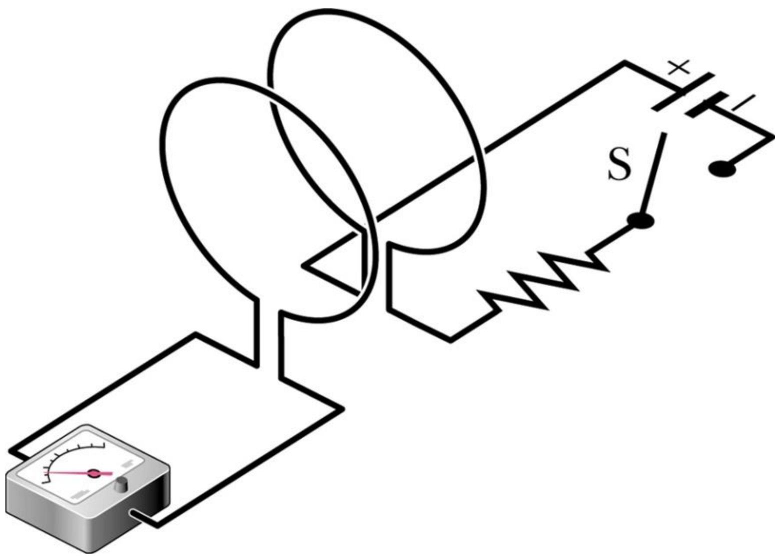
- The images and the pictures in this lecture are provided by the CDs accompanied by the books
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8th and 9th Ed.
- The rest is made by me.

Chapter 29 Electromagnetic induction



Faraday's experiment





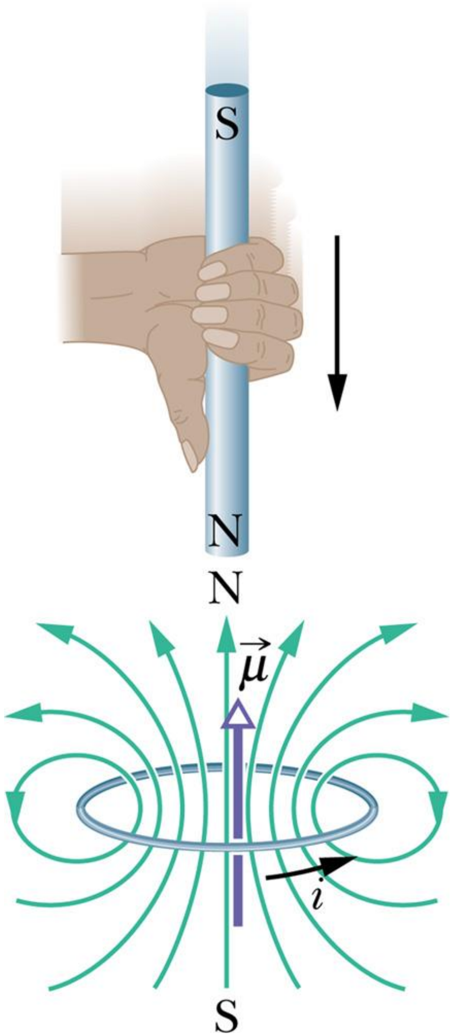
Faraday's induction law

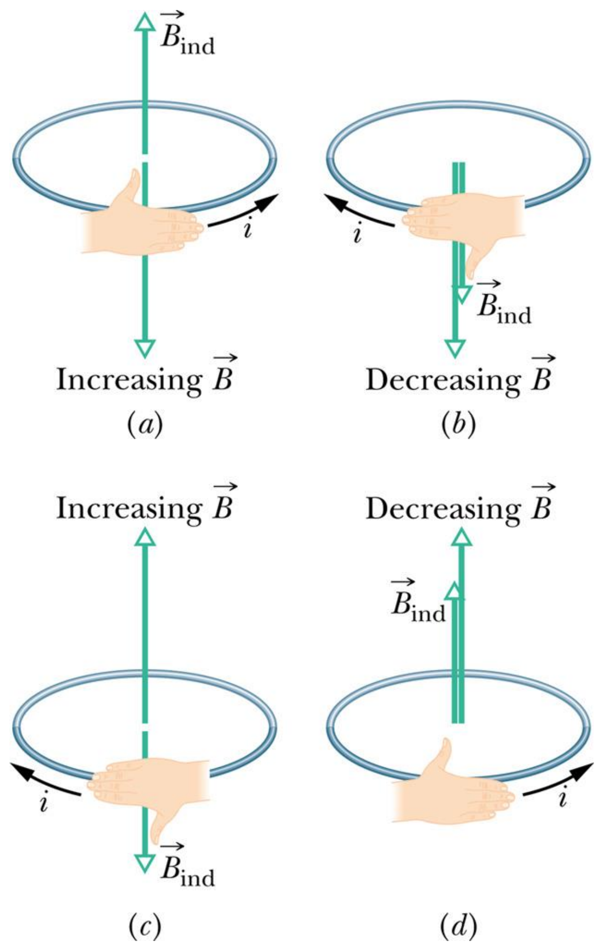
$$\mathcal{E} = -\frac{d\Phi_B}{dt}$$

$$\Phi_B = \int \vec{B} \cdot d\vec{A}$$

SI unit: $1\ \mathrm{Wb} = 1\ \mathrm{T} \cdot \mathrm{m}^2$

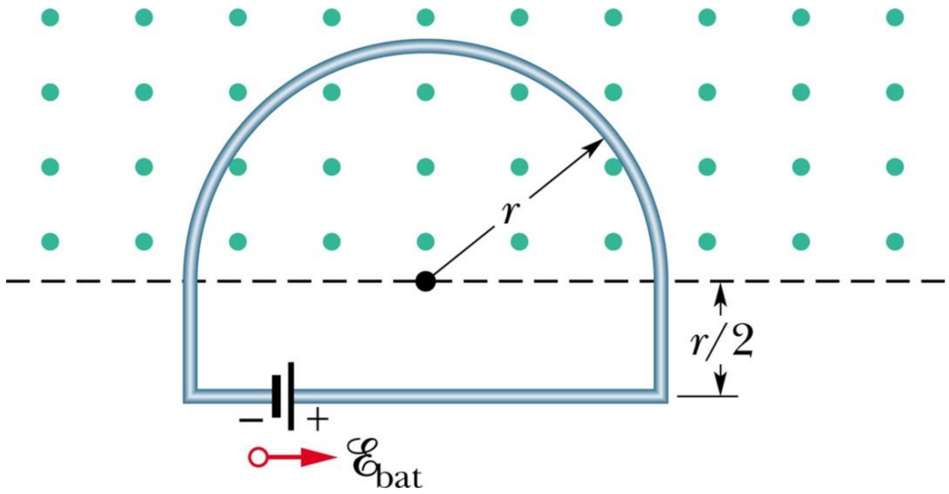
Lenz law





1. 자극의 운동 방해
2. 자기다발의 변화 방해

Example



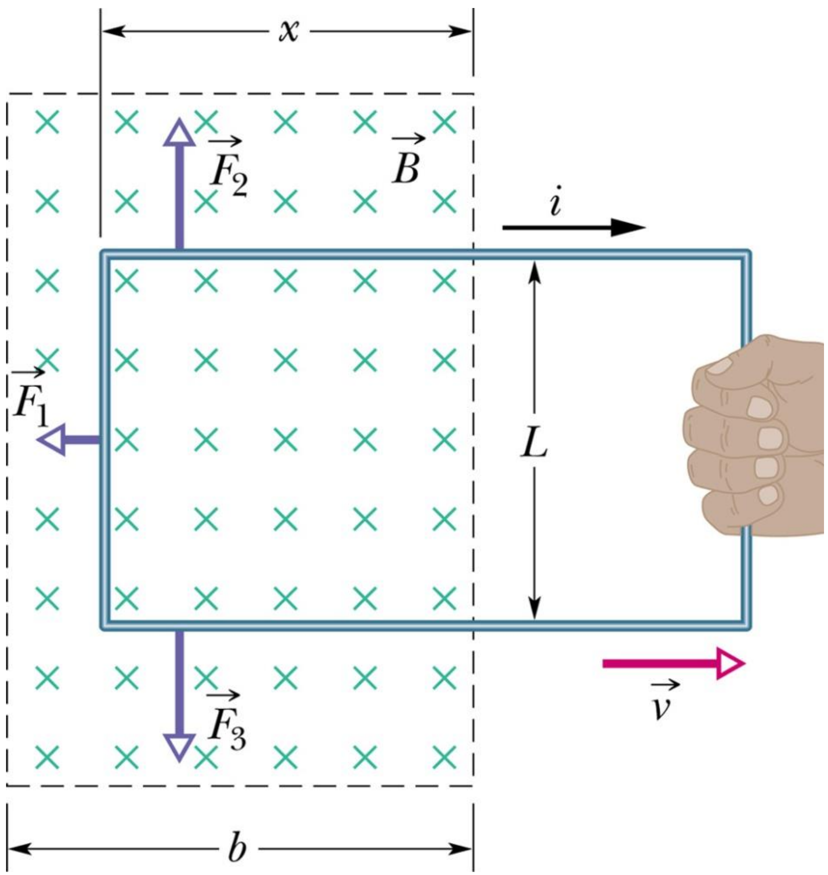
$$B = 4.0t^2 + 2.0t + 3.0$$

$$r = 2.0 \text{ cm}, \ \mathcal{E}_{\text{bat}} = 2.0 \text{ V}, \ R = 2.0 \ \Omega$$

$$\mathcal{E}_{\text{ind}} = \frac{d\Phi_B}{dt} = \frac{d(BA)}{dt} = A\frac{dB}{dt}$$

$$= \frac{\pi r^2}{2}(8.0t + 2.0) = 5.2 \text{ V}$$

Induction and energy transfer



$$P = Fv$$

$$\Phi_B = BA = BLx$$

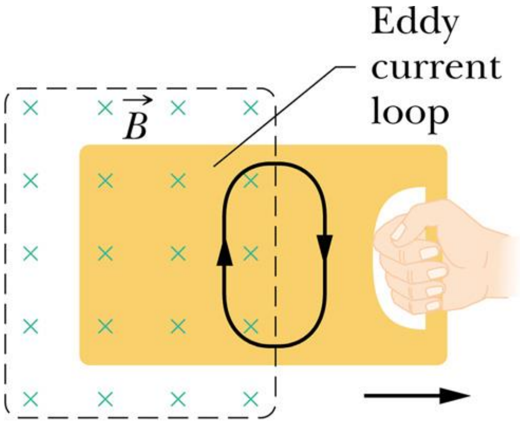
$$\mathcal{E} = \frac{d\Phi_B}{dt} = \frac{d}{dt}BLx = BLv$$

$$i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$$

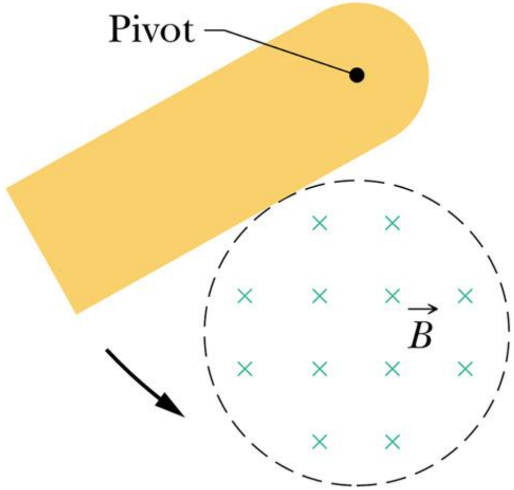
$$F = iLB = \frac{B^2L^2v}{R}$$

$$P = Fv = \frac{B^2L^2v^2}{R} = i^2R$$

Eddy current

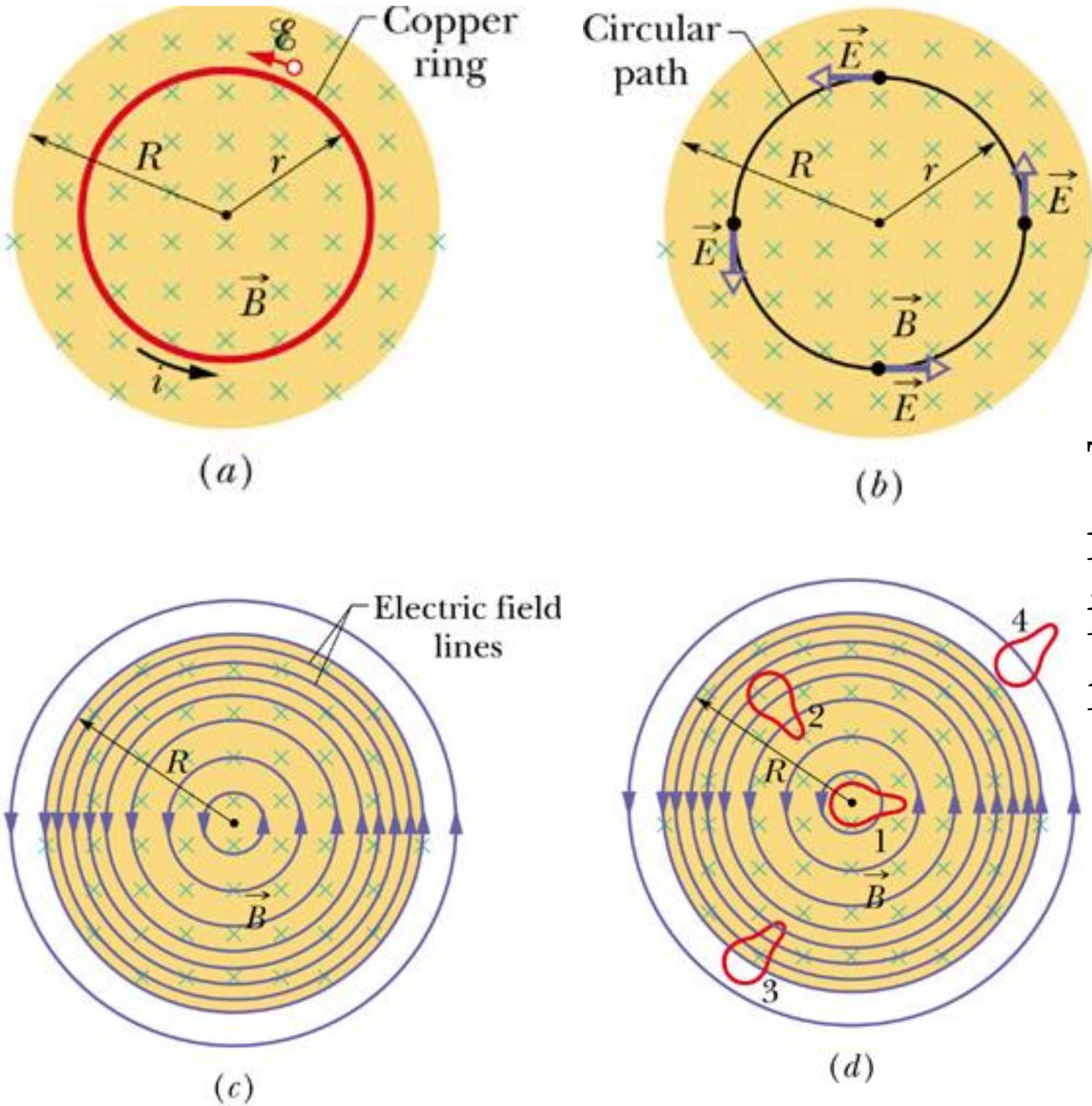


(*a*)



(*b*)

Induced electric field



Time-varying magnetic fields induces electric fields.

Reformulation of Faraday's law

전하 q_0가 반지름 r인 원을 한 바퀴 돌 때 유도된 전기장에 의해 한 일은 $\mathcal{E}q_0$이다. 한편 이 일은

$$\int \mathbf{F} \cdot d\mathbf{s} = q_0 E 2\pi r \longrightarrow \mathcal{E} = 2\pi r E. \tag{1}$$

임의의 닫힌 경로에 대해서는

$$W = \oint \mathbf{F} \cdot d\mathbf{s} = q_0 \oint \mathbf{E} \cdot d\mathbf{s} = \mathcal{E} q_0 \tag{2}$$

$$\mathcal{E} = \oint \mathbf{E} \cdot d\mathbf{s} \tag{3}$$

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \quad (\text{Faraday의 법칙}) \tag{4}$$

A new look at electric potential

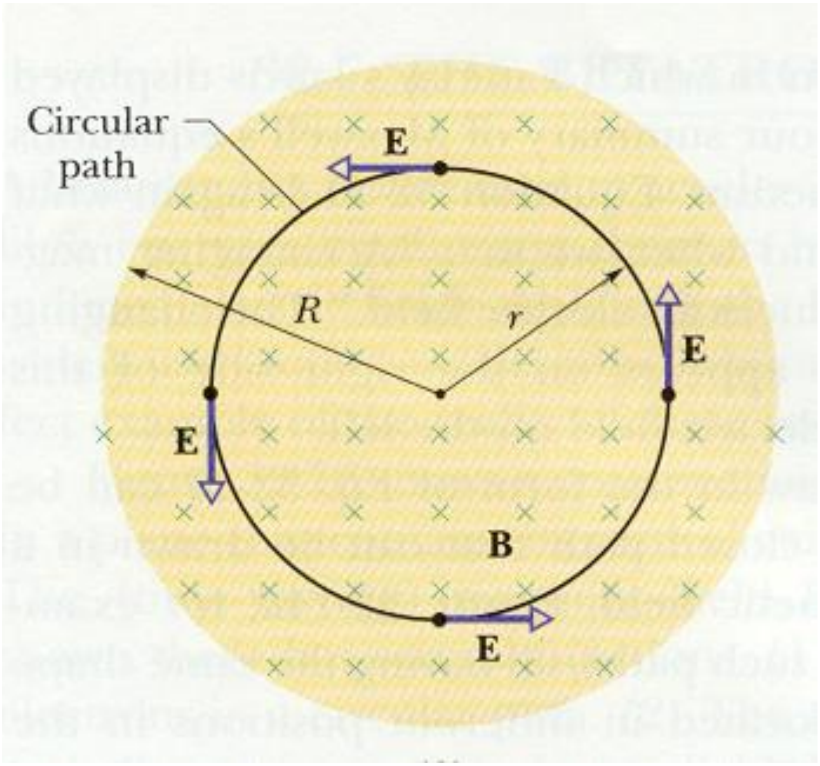
$$\oint \vec{E} \cdot d\vec{s} \neq 0?$$

Well, $\Delta V = -\int_a^b \vec{E} \cdot d\vec{s}$ $\qquad \Delta V = 0 \text{ for } a = b$

Induced electric fields are not conservative.
So we cannot define the electric potential.

Electric potential is meaningful only for static charges.

Example



$$\mathrm{R} = 8.5\mathrm{cm},\ \frac{dB}{dt} = 0.13\mathrm{T/s}$$

$$r = 5.2\mathrm{cm} < R$$

$$-\frac{d\Phi_B}{dt} = \oint \mathbf{E} \bullet d\mathbf{s} = E(2\pi r)$$

$$\Phi_B = B(\pi r^2)$$

$$E = \tfrac{1}{2}(dB/dt)r$$

$$r = 12.5\mathrm{cm} > R$$

$$\Phi_B = B(\pi R^2)$$

$$E = \tfrac{1}{2}(dB/dt)\frac{R^2}{r}$$

Inductor and inductance

인덕턴스의 정의

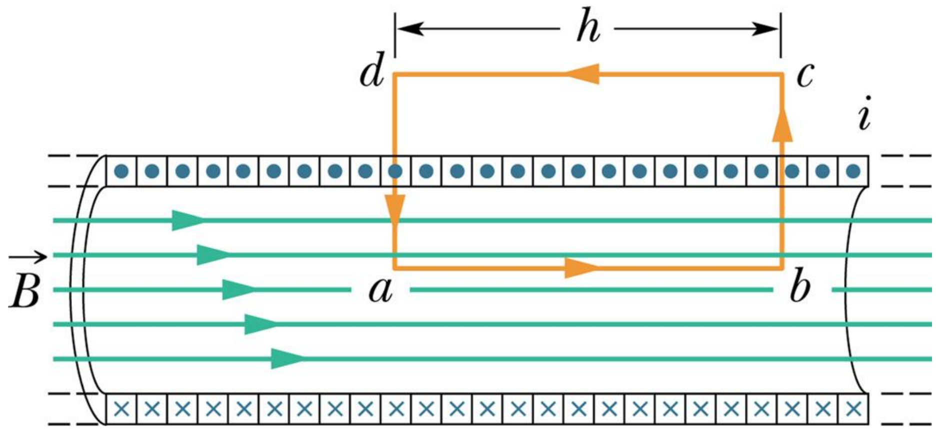
$$L = \frac{N\Phi_B}{i}$$

$$1 \text{ henry} = 1 \text{ H} = 1 \text{ T} \cdot \text{m}^2/\text{A}$$

Induction

- self induction
- mutual induction

Inductance of a solenoid



$$N\Phi_B = (nh)(BA) \qquad L = \frac{N\Phi_B}{i} = \frac{nl\mu_0 i n A}{i} = \mu_0 n^2 h A$$

$$B = \mu_0 i n$$

Inductance per unit length $\quad \dfrac{L}{h} = \mu_0 n^2 A$

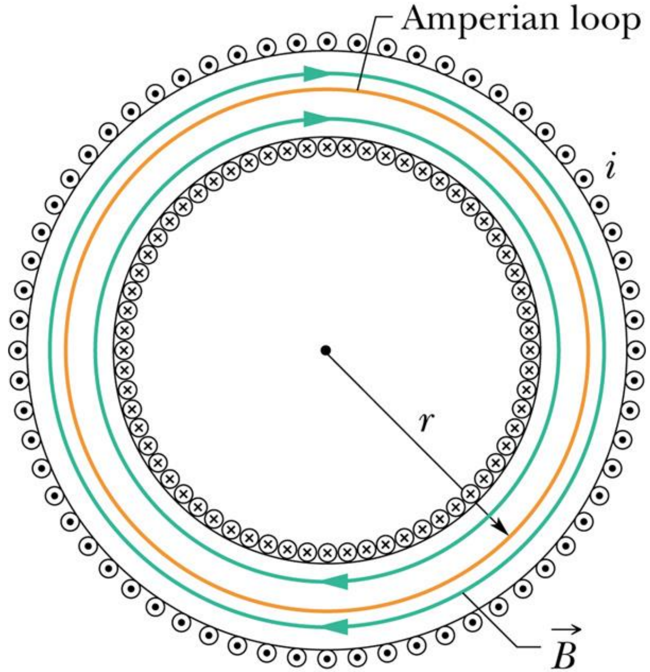
$$C = \epsilon_0 \mathcal{L} \quad L = \mu_0 \mathcal{L}$$

$$\mu_0 = 4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}$$
$$= 4\pi \times 10^{-7}\ \mathrm{H/m}$$

Inductance of a toroid

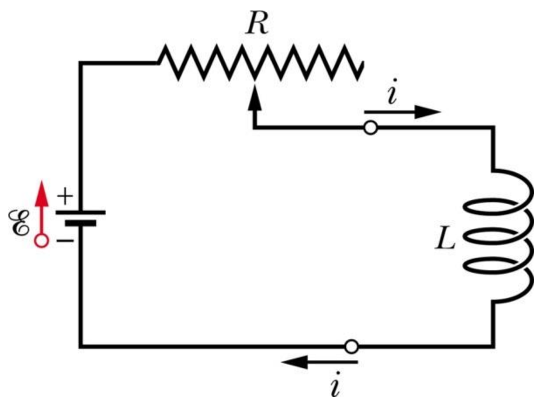
(a)

$$B = \frac{\mu_0 i N}{2\pi}\frac{1}{r}$$



(b)

Self inductance

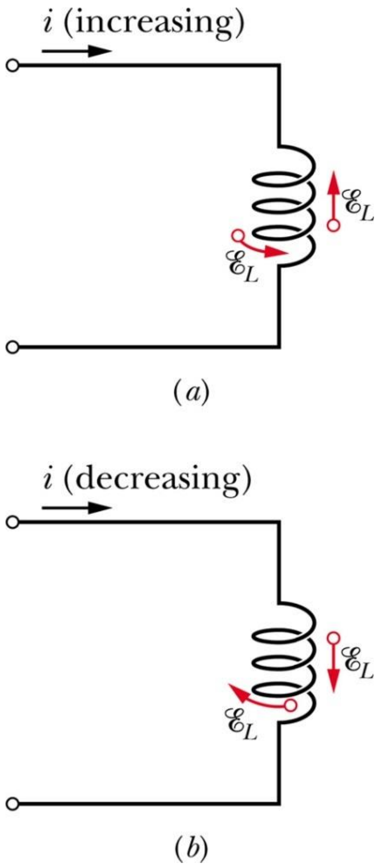


EMF $\mathcal{E}_L$ is generated in all the current loops with changing currrents.

self inductance

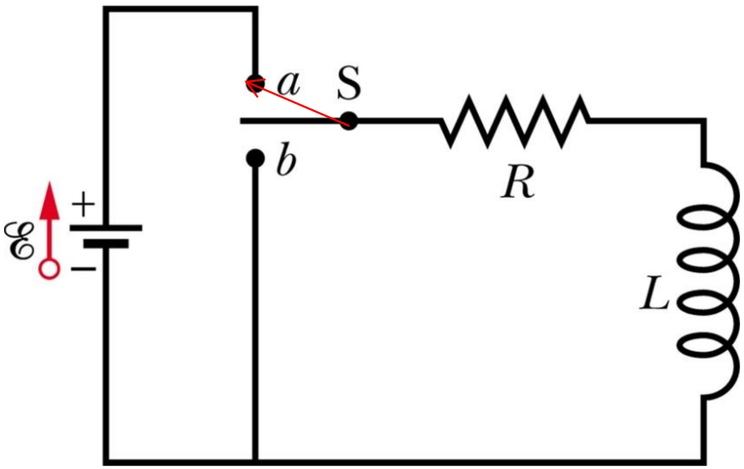
$$N\Phi_B = Li$$

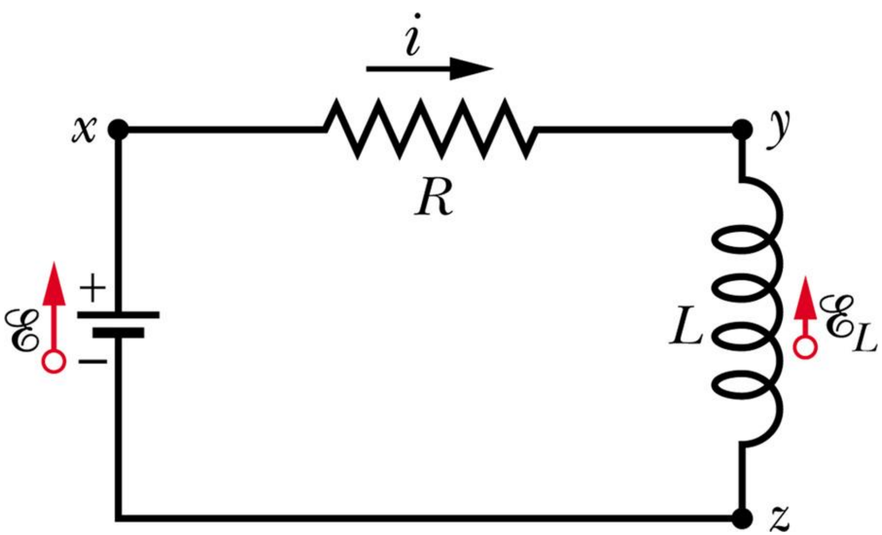
$$\mathcal{E}_L = -\frac{d(N\Phi_B)}{dt}$$



$$\mathcal{E}_L = -L\frac{di}{dt}$$

RL circuit





$$-iR - L\frac{di}{dt} + \mathcal{E} = 0$$

$$L\frac{di}{dt} + Ri = \mathcal{E}$$

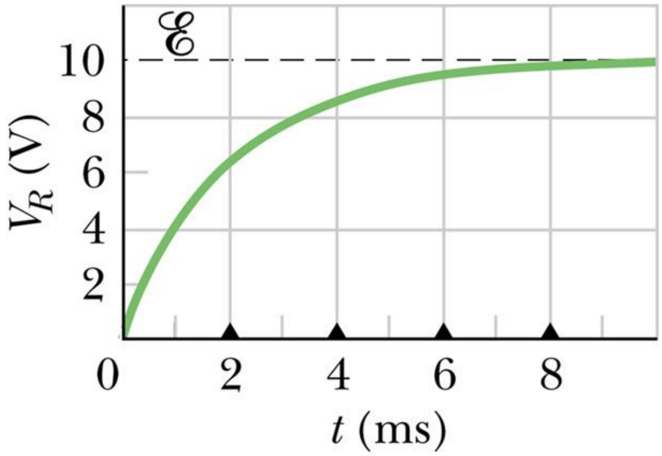
$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

time constant $\tau_L = \frac{L}{R}$

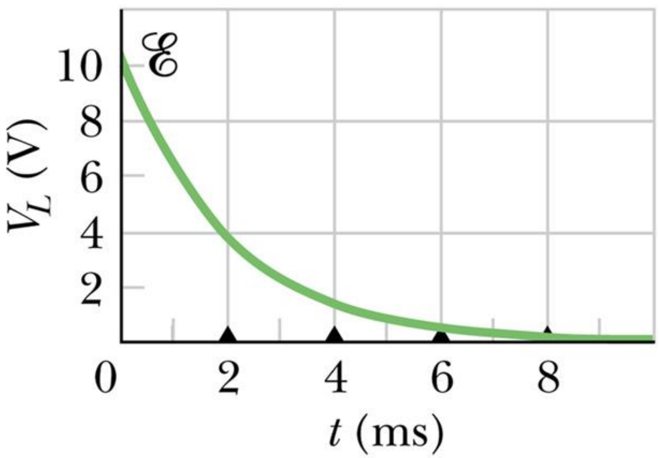
$$\mathrm{H}/\Omega = \mathrm{H}/\Omega \frac{\mathrm{V}\cdot\mathrm{s}}{\mathrm{H}\cdot\mathrm{A}}\frac{\Omega\cdot\mathrm{A}}{\mathrm{V}} = \mathrm{s}$$

$$\mathcal{E}_L = -L\frac{di}{dt}$$

$$V = iR$$

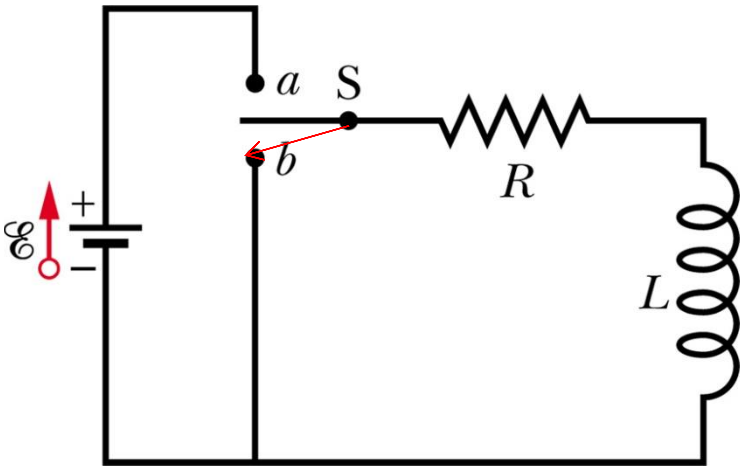


(a)



(b)

$$i(t) = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$



$$L\frac{di}{dt} + iR = 0$$

$$i(t) = \frac{\mathcal{E}}{R} = e^{-t/\tau_L} = i_0 e^{-t/\tau_L}$$