



Doing Mathematics Through NCTM Standards

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In teaching and learning mathematics in classrooms

- Goals and objectives
- Conceptual frameworks and structures
- Teaching methods
- **Assessments**

(Assessment and instruction must be integrated)

In the future society

- Holistic knowledge rather than separate
- Active engagement rather than passive
- Process rather than result
- Multi-dimensional contents rather than one-dimensional contents

(성태제, 권오남, 1999)

성태제, 권오남(1999). 수학과 학업성취도 평가를 위한 수행평가의 과제와 전망. 대한수학교육학회 학교 수학, 1(1), 217-234

Why do we need to care about **mathematical essays?**

- Assessment should support the learning of important mathematics and furnish useful information to both teachers and students (NCTM, 2000)
- What kinds of mathematical knowledge and performance are valued?
- Formative assessment rather than summative assessment

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

The **Assessment Standards** for School Mathematics (NCTM, 1995)

- Reflect the mathematics that students should know and be able to do
- Enhance mathematics learning
- Promote equity
- Be an open process
- Promote valid inference
- Be a coherent process

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

1. Mathematics Standard

(Assessment should reflect the math all students need to know)

- What mathematics is reflected in the assessment?
- What efforts are made to ensure that the mathematics is significant and correct?
- How does the assessment engage students in realistic and worthwhile mathematical activities?
- How does the assessment elicit the use of mathematics that it is important to know and be able to do?
- How does the assessment fit within a framework of mathematics to be assessed?
- What inferences about students' mathematical knowledge, understanding, thinking processes, and dispositions can be made from the assessment?

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

2. Learning Standard

(Assessment should enhance mathematics learning)

- How does the assessment contribute to each student's learning of mathematics?
- How does the assessment relate to instruction?
- How does the assessment allow students to demonstrate what they know and what they can do in novel situations?
- How does the assessment build on each student's understanding, interests, and experiences?
- How does the assessment involve students in selecting activities, applying performance criteria, and using results?
- How does the assessment provide opportunities for students to evaluate, reflect on, and improve their own work—that is, to become independent learners?

3. Equity Standard

(Assessment should promote equity)

- What opportunities has each student had to learn the mathematics being assessed?
- How does the design of the assessment enable all students to exhibit what they know and can do?
- How is the role of students' backgrounds and experiences recognized in judging their responses to the assessment?
- How do scoring guides accommodate unanticipated but reasonable responses?
- How have the effects of bias been minimized throughout the assessment?
- To what sources can differences in performance be attributed?

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

4. Openness Standard

(Assessment should be an open process)

- How do students become familiar with the assessment process and with the purposes, performance criteria, and consequences of the assessment?
- How are teachers and students involved in choosing tasks, setting criteria, and interpreting results?
- What access do those affected by the assessment have to tasks, scoring goals, performance criteria, and samples of students' work that have been scored and discussed?
- How is the assessment process itself open to evaluation and modification?
- How is the public involved in the assessment process?

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

5. Inferences Standard

(Assessment should promote valid inferences about learning)

- What evidence about learning does the assessment provide?
- How is professional judgment used in making inferences about learning?
- How sensitive is the assessor to the demands the assessment makes and to unexpected responses?
- How is bias minimized in making inferences about learning?
- What efforts are made to ensure that scoring is consistent across students, scorers, and activities?
- What multiple sources of evidence are used for making inferences, and how is the evidence used?
- What is the value of the evidence for each use?

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

6. Coherence Standard

(Assessment should be a coherent process)

- How is professional judgment used to ensure that the various parts of the assessment process form a coherent whole?
- How do students view the connection between instruction and assessment?
- How does the assessment match its purposes with its uses?
- How does the assessment match the curriculum and instructional practice?
- How can assessment practice inform teachers as they make curriculum decisions and determine their instructional practices?

NCTM(1995). Assessment Standards for School Mathematics. Reston, VA: NCTM.

During the semester,
We are going to focus on

- How to develop **assessment items**
- How to make **evaluation standards (rubrics)** based on the assessment items

Performance Assessment

- Not only results, but also performing process
- Constructed-response assessment (CA)
- Mathematical essay (ME)
- Oral assessment
- Self evaluation
- Peer evaluation
- Projects

강옥기, 이장주, 이환철(2011). 수리논술의 이론과 실제. 서울: 경문사.

Constructed-response **A**ssessment (서술형평가) vs. **M**athematical **E**ssay (수리논술)?

- PA: 학생 스스로의 지식이나 기능 등을 나타내도록 하는 평가 (CA, ME)
- Essay (논술): 어떤 문제 상황에 대하여 자신의 견해나 주장을 논리적으로 진술하는 것 (강옥기 외, 2011)
- ME: 어떤 문제 상황에 대하여 자신의 견해나 주장을 수학을 근거로 하여 논리적으로 진술하는 것 (강옥기 외, 2011)

강옥기, 이장주, 이환철(2011). 수리논술의 이론과 실제. 서울: 경문사.

How to evaluate students' doing mathematics in assessment in order to appeal to their curiosity and intellect simultaneously?

If students can be actively engaged in "**doing**" mathematics, they may be motivated enough to perform their best in the classroom and on assignments

What does it mean to “do” mathematics?

- ① Problem solving
- ② Reasoning and proof
- ③ Communication
- ④ Connections
- ⑤ Representation

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Principles and Standards for School Mathematics

- Problem solving** should enable all students
- Build new mathematical knowledge through problem solving
 - Solve problems that arise in mathematics and in other contexts
 - Apply and adapt a variety of appropriate strategies to solve problems
 - Monitor and reflect on the process of mathematical problem solving

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Principles and Standards for School Mathematics

Through emphasis on **reasoning and proof** in the classroom, all students will

- Recognize reasoning and proof as fundamental aspects of mathematics
- Make and investigate mathematical conjectures
- Develop and evaluate mathematical arguments and proofs
- Select and use various types of reasoning and methods of proof

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Principles and Standards for School Mathematics

- Communication** should be stressed so that students
- Organize and consolidate their mathematical thinking through communication
 - Communicate their mathematical thinking coherently and clearly to peers, teachers, and others
 - Analyze and evaluate the mathematical thinking and strategies of others
 - Use the language of mathematics to express mathematical ideas precisely

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Principles and Standards for School Mathematics

Connections should be made in the mathematics classroom to help students

- Recognize and use connections among mathematical ideas
- Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
- Recognize and apply mathematics in contexts outside of mathematics

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

Principles and Standards for School Mathematics

- Use of **representation** should enable students to
- Create and use representations to organize, record, and communicate mathematical ideas
 - Select, apply, and translate among mathematical representations to solve problems
 - Use representations to model and interpret physical, social, and mathematical phenomena

NCTM(2000). *Principles and Standards for School Mathematics*. Reston, VA: NCTM.

References

- 강옥기, 이장주, 이환철(2011). 수리논술의 이론과 실제. 서울: 경문사.
- 성태제, 권오남(1999). 수학과 학업성취도 평가를 위한 수행평가의 과제와 전망. 대한수학교육학회 학교 수학, 1(1), 217-234.
- NCTM(1995). *Assessment Standards for School Mathematics*. Reston, VA: NCTM.
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