Communication Systems II

[KECE322_01]

<2012-2nd Semester>

Lecture #17

2012. 11. 5

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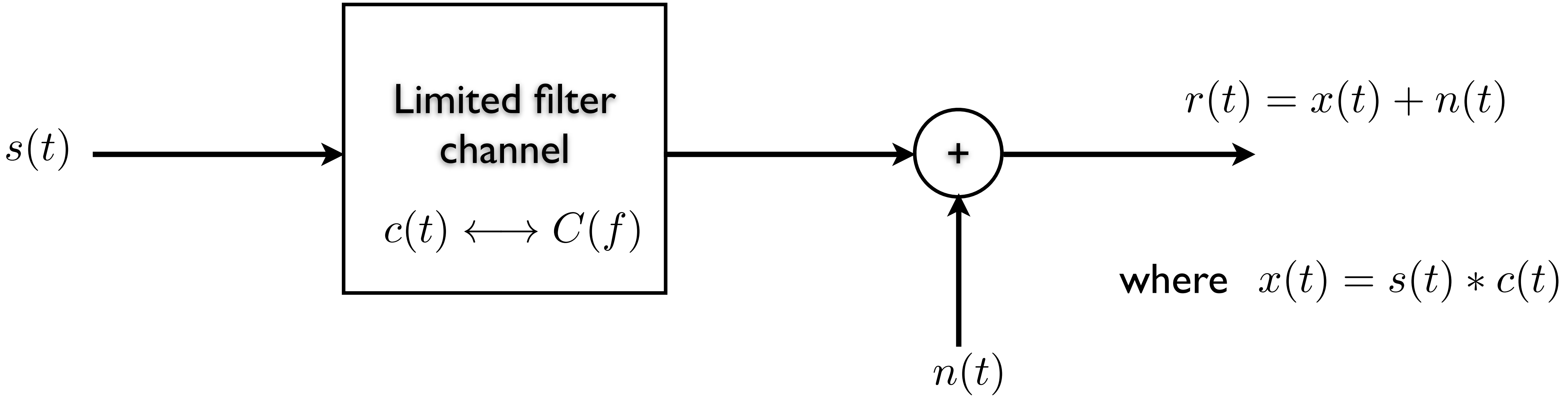
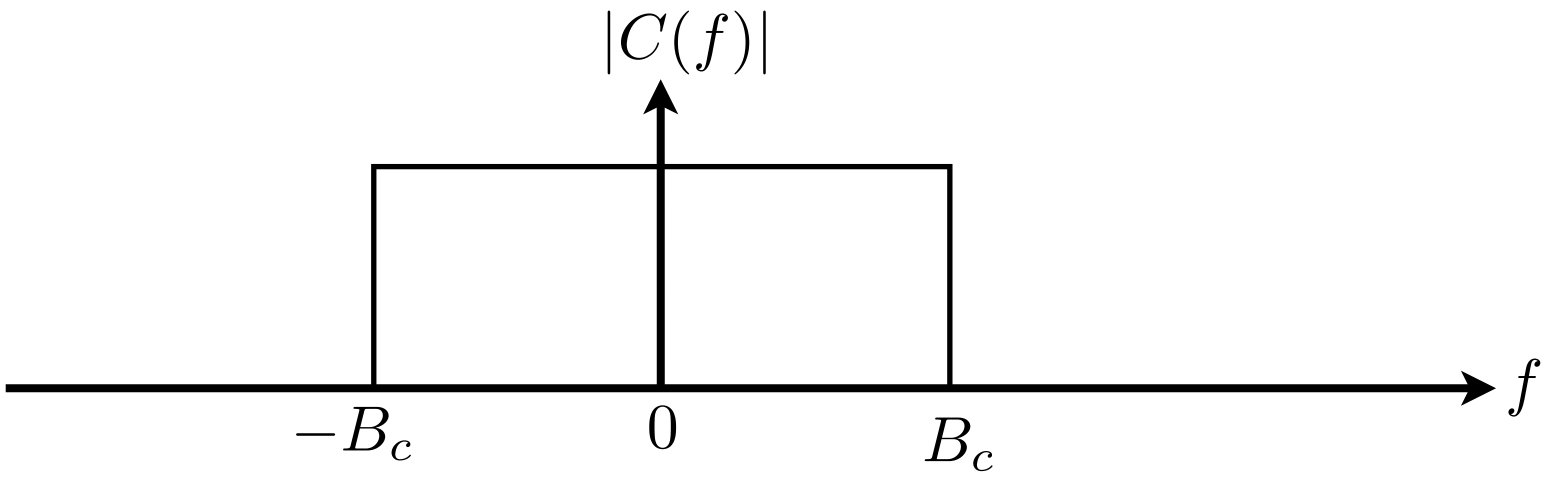
Outline

- Digital transmission through bandlimited channels
- Digital PAM transmission through bandlimited baseband channels
- Signal design for bandlimited channels

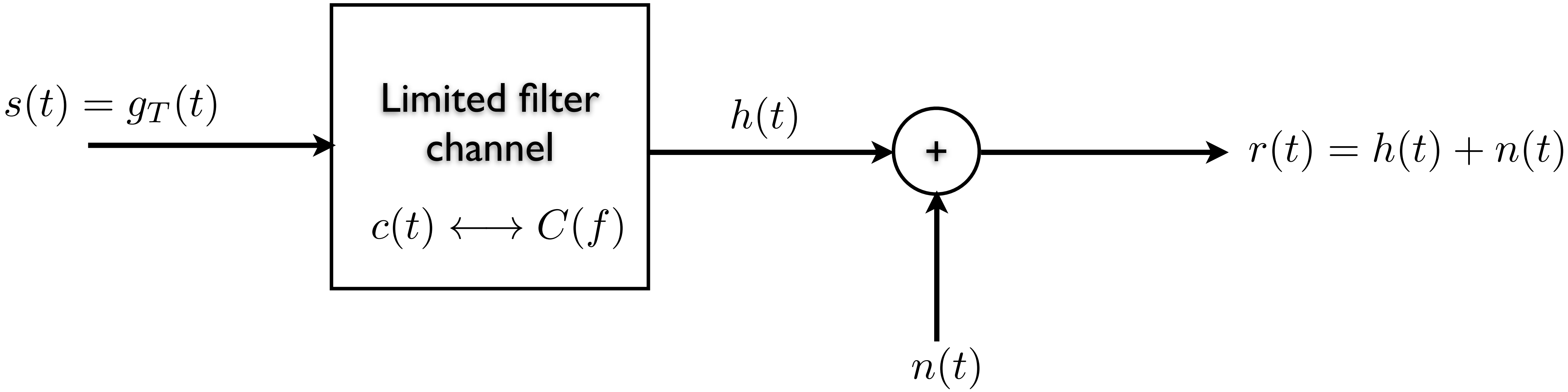
Bandlimited Channel Model



where $x(t) = s(t) * c(t)$



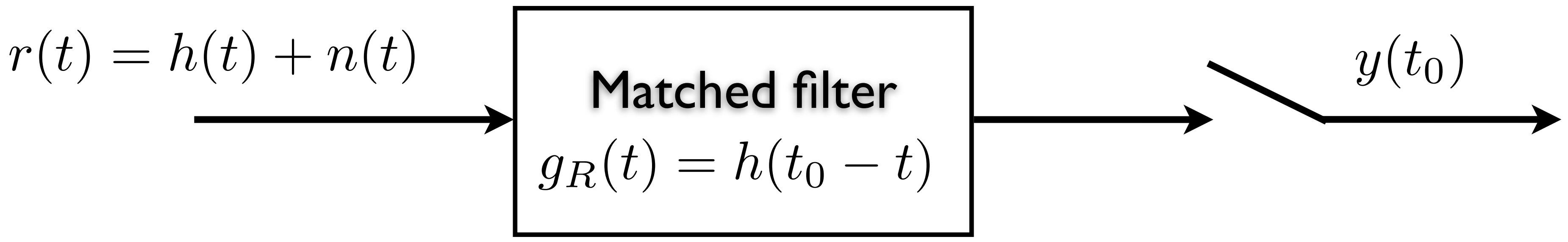
- Suppose $s(t) = g_T(t)$



$$h(t) = \int_{-\infty}^{\infty} c(\tau)g(t-\tau)\, d\tau = c(t) * g_T(t)$$

$$H(f) = C(f)G_T(f)$$

- Receiver with matched filter



$r(t) = h(t) + n(t)$

Matched filter

$g_R(t) = h(t_0 - t)$

$y(t_0)$

$$t_0 : \quad T \text{ and time delay synchronized sampling time}$$

Frequency response of the matched filter

$$G_R(f) = \mathcal{F}[g_R(t)] = \mathcal{F}[h(t_0 - t)] = H^*(f)e^{-j2\pi f t_0}$$

Signal component at the output of the matched filter

$$y_s(t_0) = \int_{-\infty}^{\infty} |H(f)|^2 \, df \ = \mathcal{E}_h$$

Noise component at the output of the matched filter is a zero mean and a power spectral density

$$S_n(f) = \frac{N_0}{2}|H(f)|^2$$

Noise power at the output of the matched filter has a variance

$$\sigma_n^2 = \int_{-\infty}^{\infty} S_n(f)\, df = \frac{N_0}{2} \int_{-\infty}^{\infty} |H(f)|^2\, df = \frac{N_0 \mathcal{E}_h}{2}$$

SSR at the output of the matched filter

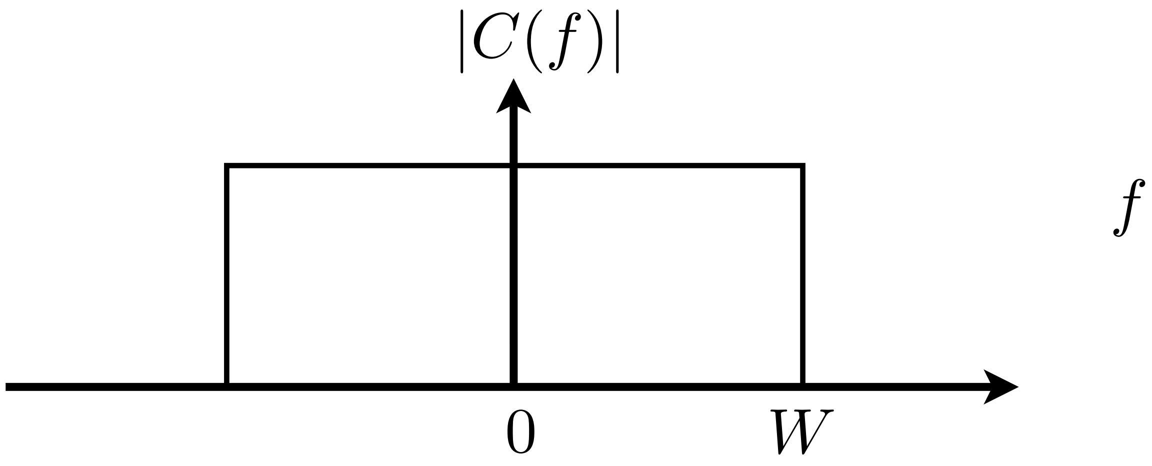
$$\left(\frac{S}{N}\right)_o = \frac{\mathcal{E}_h^2}{N_0 \mathcal{E}_h / 2} = \frac{2\mathcal{E}_h}{N_0}$$

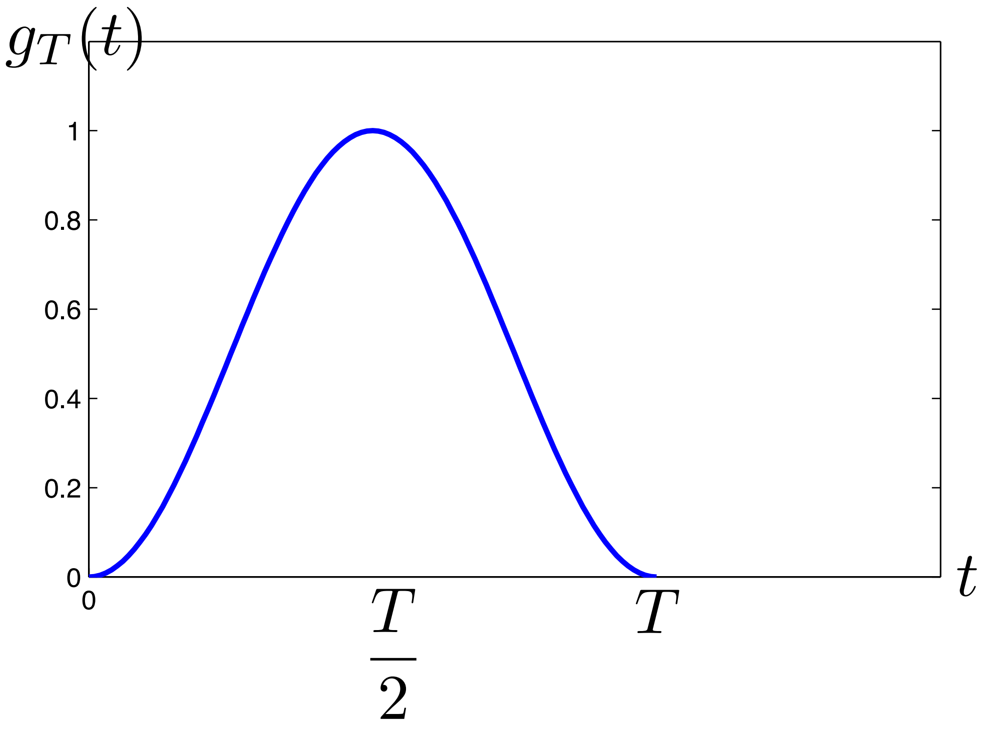
Note that for the implementation of the matched filter at the receiver, the channel impulse response $c(t)$ must be known to the receiver.

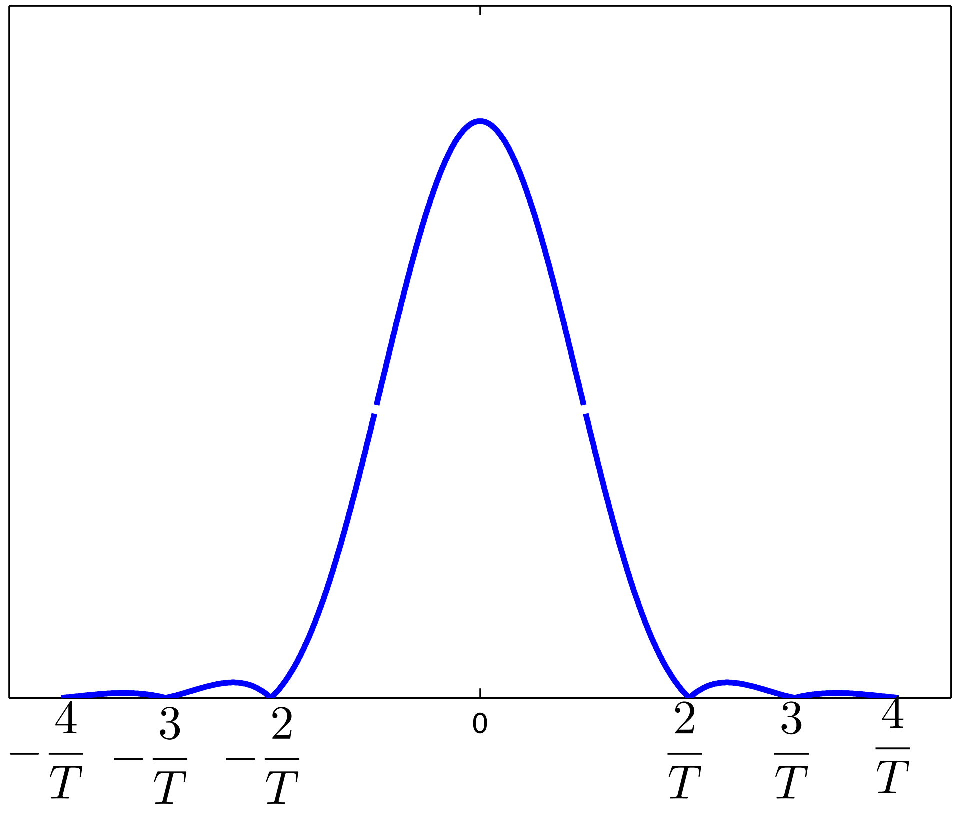
Example

$$g_T(t) = \frac{1}{2}\left[1 + \cos\frac{2\pi}{T}\left(t - \frac{T}{2}\right)\right], \qquad 0 \le t \le T$$

$$G_T(f) = \frac{T}{2}\frac{\sin \pi f T}{\pi f T(1 - f^2T^2)} e^{-j2\pi f T} = \frac{T}{2}\frac{\mathrm{sinc}\pi f T}{(1 - f^2T^2)} e^{-j2\pi f T}$$







$$H(f) = C(f)G_T(f) = \begin{cases} G_T(f), & |f| \leq W \\ 0, & \text{otherwise} \end{cases}$$

Signal component at the output of the matched to $H(f)$

$$\begin{aligned} \mathcal{E}_h = \int_{-W}^{W} |G_T(f)|^2 \, df &= \frac{1}{(2\pi)^2} \int_{-W}^{W} \frac{(\sin(\pi f T))^2}{\pi^2 f^2 T^2 (1 - f^2T^2)^2} \, df \\ &= \frac{T}{(2\pi)^2} \int_{-WT}^{WT} \frac{\sin^2 \pi\alpha}{\alpha^2(1-\alpha^2)^2} \, d\alpha \end{aligned}$$

Variance of the noise component

$$\sigma_n^2 = \frac{N_0}{2} \int_{-W}^{W} |G_T(f)|^2 \, df = \frac{N_0 \mathcal{E}_h}{2}$$

- Hence, the output SNR is

$$\left(\frac{S}{N}\right)_0 = \frac{2\mathcal{E}_h}{N_0}$$

- The amount of signal energy at the output of the matched filter depends on the value of the channel bandwidth W when the signal pulse duration is fixed. The maximum value of $\mathcal{E}_h$ is obtained as $W \to \infty$, that is,

$$\max \mathcal{E}_h = \int_{-\infty}^{\infty} |G_T(f)|^2 \, df = \int_0^T g_T^2(t) \, dt = \mathcal{E}_g$$

  - To maximize the received SNR, we must make sure that the spectrum of the transmitted signal waveform $g_T(t)$ is limited to the bandwidth of the channel.
  - The impact of the channel bandwidth limitation is felt when we consider the transmission of a sequence of signal waveforms.

PAM Transmission through Bandlimited Baseband Channels

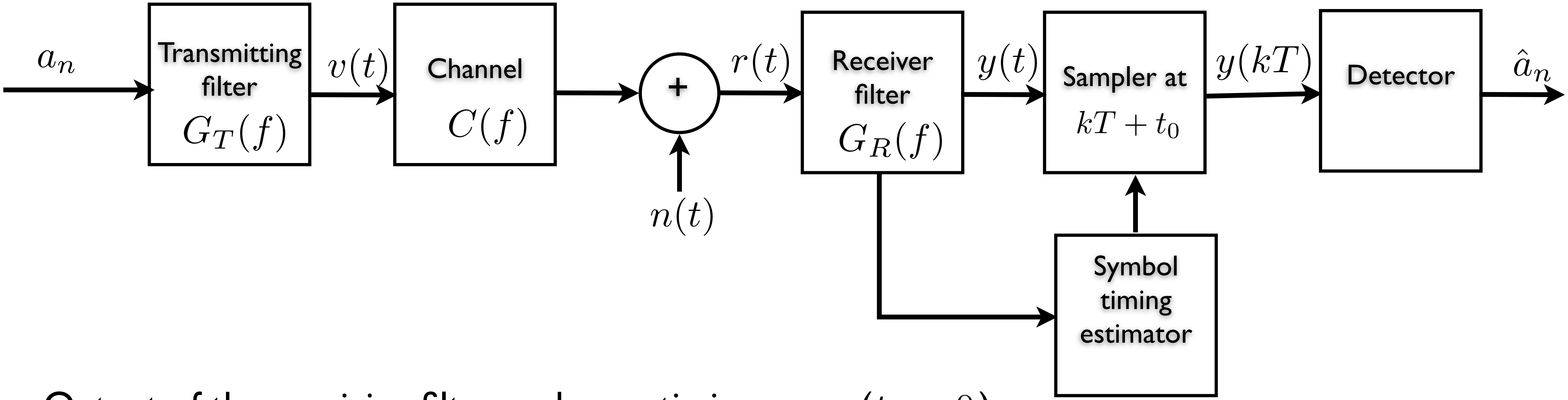
- PAM transmit signals

$$v(t) = \sum_{n=-\infty}^{\infty} a_n g_T(t - nT),$$

where $T = \dfrac{k}{R_b}$ is the symbol interval, R_b is the bit rate and $\{a_n\}$ is a sequence of the amplitude levels corresponding to the sequence of k-bit blocks of information bits.

- Received signals

$$r(t) = \sum_{n=-\infty}^{\infty} a_n h(t - nT) + n(t)$$



Output of the receiving filter under no timing error ($t_0 = 0$):

$$y(t) = \sum_{n=-\infty}^{\infty} a_n x(t - nT) + w(t)$$

where $\quad x(t) = h(t) * g_R(t) = g_T(t) * c(t) * g_R(t)$

Sampled signal

$$y(mT) = \sum_{n=-\infty}^{\infty} a_n x(mT - nT) + w(mT)$$

or equivalently,

$$y_m \quad = \sum_{n=-\infty}^{\infty} a_n x_{m-n} + w_m = x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

$$y_m = x_0 \underbrace{a_m}_{\text{desired symbol}} + \underbrace{\sum_{n \neq m} a_n x_{m-n}}_{\text{Inter-Symbol interference (ISI)}} + w_m$$

Note that

$$x(t) = h(t) * g_R(t)$$

if the receiving filter $g_R(t)$ is matched to $h(t)$, then

$$\begin{aligned} x(0) \triangleq x_0 &= \int_{-\infty}^{\infty} h(\lambda)h(\lambda)\, d\lambda \\ &= \int_{-\infty}^{\infty} h^2(t)\, dt \\ &= \int_{-\infty}^{\infty} |H(f)|^2\, df = \int_{-W}^{W} |G_T(f)|^2 |C(f)|^2\, df = \mathcal{E}_h \end{aligned}$$

Signal Design for Bandlimited Channels

- ISI signal

$$y_m \quad = \quad x_0 a_m + \sum_{n \neq m} a_n x_{m-n} + w_m$$

- Bandlimited channel model

$$C(f) = \begin{cases} C_0 e^{-j2\pi f t_0}, & |f| \leq W \\ 0, & |f| > W \end{cases}$$

- Output of the receiving filter

$$X(f) = G_T(f)C(f)G_R(f) = G_T(f)C(f)C_0 e^{-j2\pi f t_0}$$

  - Assuming $C_0 = 1$ and $t_0 = 0$

$$X(f) = G_T(f)C(f), \qquad |f| \leq W$$

- Zero ISI condition

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

  - which is called *Nyquist condition* for zero ISI.

- Nyquist condition for zero ISI
  - A necessary and sufficient condition for $x(t)$ to satisfy

$$x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

is that its Fourier transform $X(f)$ must satisfy

$$\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) = T.$$

- Proof

$$x(t) = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t}\, df.$$

At the sampling instants $t = nT,$ it becomes

$$\begin{aligned} x(nT) &= \int_{-\infty}^{\infty} X(f) e^{j2\pi f nT}\, df \\ &= \sum_{m=-\infty}^{\infty} \int_{-(2m-1)/2T}^{(2m+1)/2T} X(f) e^{j2\pi f nT}\, df \\ &= \sum_{m=-\infty}^{\infty} \int_{-1/2T}^{1/2T} X\left(f + \frac{m}{T}\right) e^{j2\pi f nT}\, df \\ &= \int_{-1/2T}^{1/2T} \left[\sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)\right] e^{j2\pi f nT}\, df \\ &= \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df, \end{aligned}$$

where $$Z(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right)$$

$$Z(f) = \sum_{m=-\infty}^{\infty} X\left(f + \frac{m}{T}\right) \qquad\qquad x(nT) = \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df,$$

- $Z(f)$ is periodic function with period $\frac{1}{T}$; therefore it can be expanded in terms of its Fourier series coefficients $\{z_n\}$ as

$$Z(f) = \sum_{n=-\infty}^{\infty} z_n e^{j2\pi n f T}$$

where

$$z_n = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} Z(f) e^{-j2\pi n f T}\, df.$$

Compare the following two:

$$z_n = T \int_{-\frac{1}{2T}}^{\frac{1}{2T}} Z(f) e^{-j2\pi n f T}\, df, \quad \text{and} \quad x(nT) = \int_{-1/2T}^{1/2T} Z(f) e^{j2\pi f nT}\, df,$$

Then we have: $z_n = Tx(-nT)$

- Zero ISI condition

$$z_n = \begin{cases} T, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

since $z_n = Tx(-nT)$ and zero ISI condition is $x(nT) = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}.$

- which yields

$$Z(f) = T,$$

or equivalently