Quantum Mechanics II

Assignment 3

Due: October 15 (Tuesday), 2013

1. Consider N identical spin-1/2 electrons confined in the two-dimensional box with each side L.
   (a) Find the Fermi energy E_F.
   (b) Find the total ground-state energy E.
   (c) Find the Fermi wave number k_F.

2. In Problem 1, we have the fixed-end boundary conditions in which the wave function at the boundary of the box becomes zero. Instead, suppose that we have periodic boundary conditions, that is, $\psi(x+L, y, z) = \psi(x, y, z)$, $\psi(x, y+L, z) = \psi(x, y, z)$ and $\psi(x, y, z+L) = \psi(x, y, z)$. Compute the same quantities in this different boundary condition.

3. The Hamiltonian for an isotropic harmonic oscillator is given by
$$H = \frac{\mathbf{p}^2}{2m} + \frac{1}{2}m\omega^2\mathbf{r}^2. \tag{1}$$
   Suppose that there are N spin-1/2 electrons in this system in its ground state. Find the Fermi energy, the total energy and the Fermi wave number.

4. In class we computed the Fermi energy of a gas of fermions in the nonrelativistic case. Using the same logic, compute the Fermi energy of a gas of relativistic, massless fermions with its energy-momentum relation $E = pc$.

5. Suppose that there are two electrons in a one-dimensional box with size L. Construct the energy eigenfunctions explicitly for the ground state, the first excited state. Before attacking this problem, consider which coordinate systems reflect the symmetry of the system better.