Reflection on NCTM standards from Assessment Perspectives

What does it mean to "do" mathematics?

① Problem solving
② Reasoning and proof
③ Communication
④ Connections
⑤ Representation

1. Problem Solving

- Exercise vs. problem
- Exercise is a routine task
- Problem is a task for which there is no immediate solution
- Finding a circumference can be a problem for a sixth grader but an exercise for a high school sophomore
- **Problem solving** is the process by which an individual attempts to find a solution to a *non-routine* mathematical question

NCTM(2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

Problem solving example

A certain farmer in Florida has an orange grove. In his grove are 120 trees. Each tree ordinarily produces 650 oranges. He is interested in raising his orange production and knows that because of lost space and sunlight, every additional tree that he plants will cause a reduction of 5 oranges from each tree. What is the maximum number of oranges that he will be able to produce in his grove, and how many trees will he need to reach this maximum?

NCTM(2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

2. Reasoning and proof

- Students should be encouraged to reason mathematically by being challenged with "**why**" and "**how**" questions
- Why fractions are divided by inverting the last fraction and multiplying? Can you visualize the process?
- How do you know that?

NCTM(2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.

3. Communication

- "Doing" mathematics should include the process of communicating with others as a critical component.
- Students need to justify their conclusions, **communicate** them to others, and respond to the arguments of others
- They should be able to explain, describe, and clearly communicate solutions and strategies that lead to the answers

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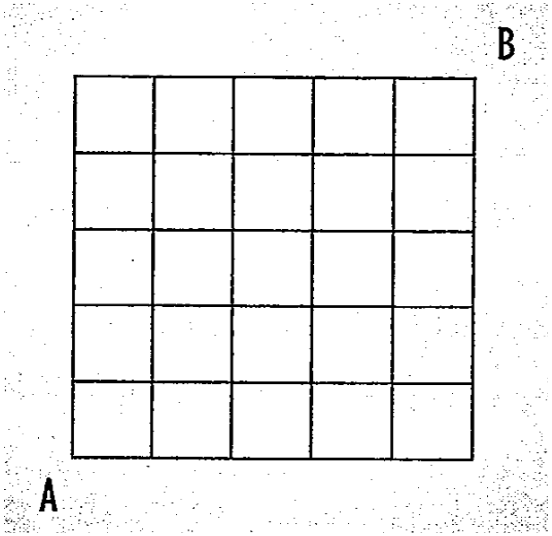
4. Connections

- How to use activities and examples that help students to see how the various areas of mathematics are related?
- How many paths there are to move from point A to point B?



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Principles and Standards for School Mathematics

Connections should be made in the mathematics classroom to help students

1. Recognize and use connections among mathematical ideas
2. Understand how mathematical ideas interconnect and build on one another to produce a coherent whole
3. Recognize and apply mathematics in contexts outside of mathematics

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NCTM example 1

Recognize and use connections ideas

RECIPE A
2 cups cranberry juice
3 cups sparkling water

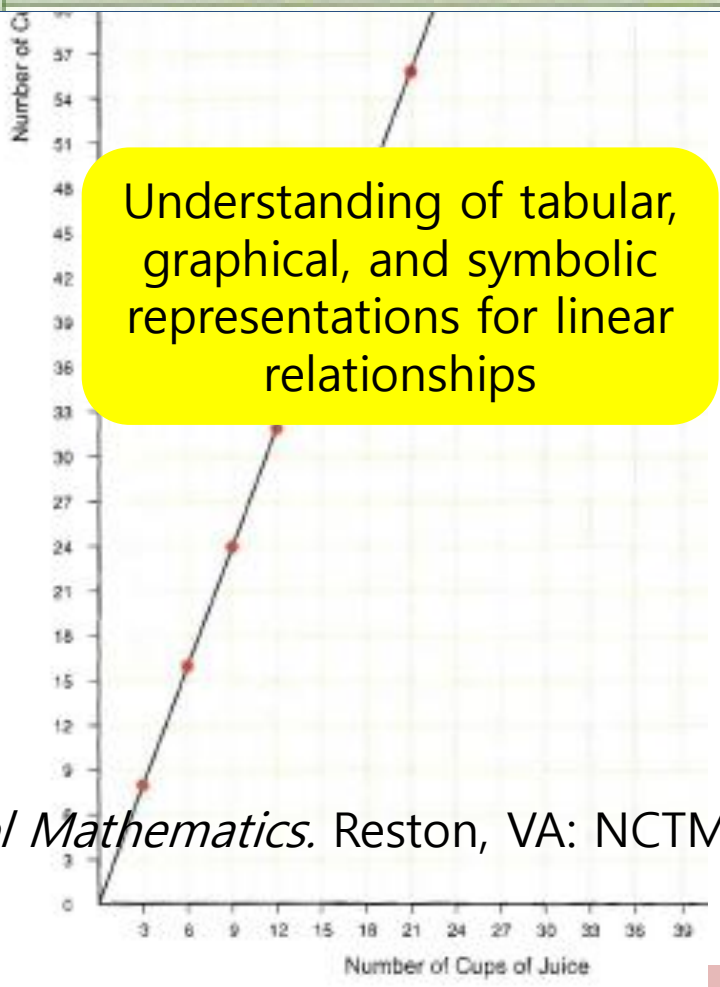
RECIPE B
4 cups cranberry juice
8 cups sparkling water

RECIPE C
3 cups cranberry juice
5 cups sparkling water

RECIPE D
1 cup cranberry juice
4 cups sparkling water

1. Which recipe will make punch that has the strongest cranberry flavor? Explain your answer.
2. Which recipe will make punch that has the weakest cranberry flavor? Explain your answer.
3. The band director says that 120 cups of punch are needed. For each recipe, how many cups of cranberry juice and how many cups of sparkling water are needed? Explain your answer.

Fig. **6.39.**
A graph of the numbers of cups of juice and punch reveals a linear relationship.



Understanding of tabular, graphical, and symbolic representations for linear relationships

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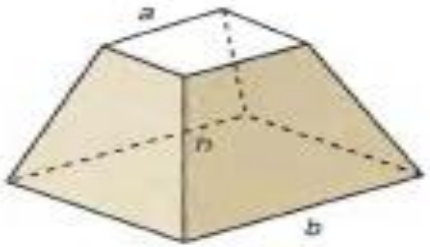
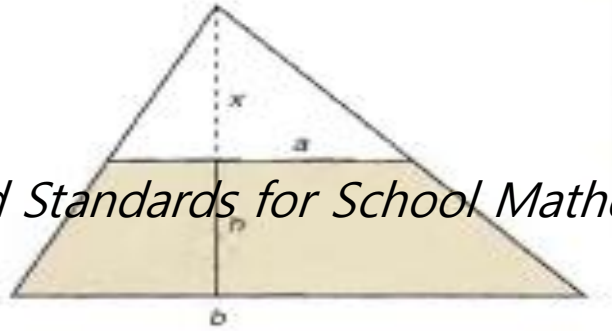
NCTM example 2

Understand how mathematical ideas interconnect and build on one another to produce a coherent whole

Fig. 3.7.

Connections between methods for finding the volume of a truncated pyramid and for finding the area of a trapezoid





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NCTM example 3

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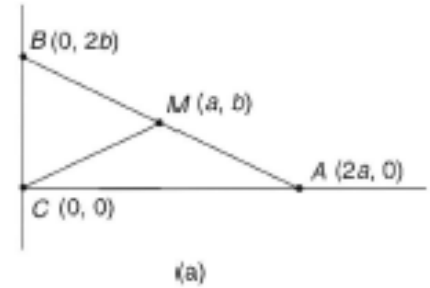
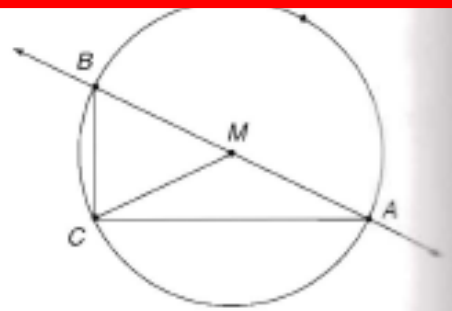
Stage for abstracting and generalizing some work for making more connections



(a)



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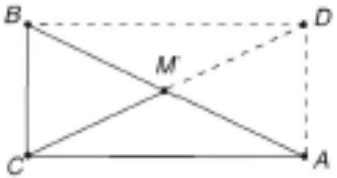

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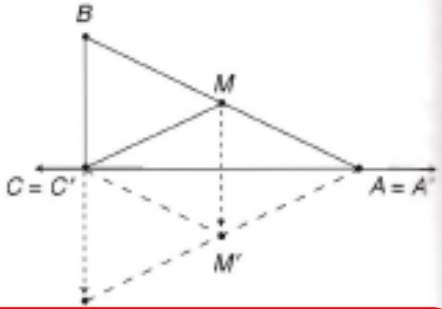
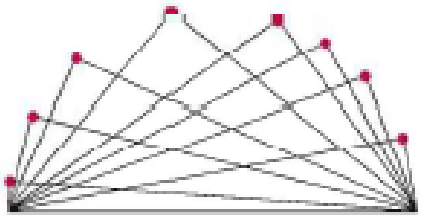
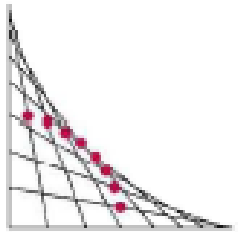


Dynamic representations of right triangles (from Goldenberg, Lewis, and O'Keefe [1992, p. 257])



(a) A trace of the locus of the vertices of right angles with a common hypotenuse



(b) A trace of the locus of midpoints of hypotenuses of fixed length in triangles with a common right angle

Different approaches using coordinate geometry, Euclidean geometry, and transformational geometry

5. Representation

- How to choose a particular representation depending on the context of a problem and what information we need?

a. $y = x^2 + 2x - 15$

b. $y = (x-3)(x+5)$

c. $Y + 16 = (x+1)^2$

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A Big Question

Can you analyze your classes through the lens of the five mathematical processes that should permeate every assessment in the classroom?

Questions about Developing Assessment Items

Do you think which process is most important?

① Problem solving
② Reasoning and proof
③ Communication
④ Connections
⑤ Representation

What kinds of problem solving can you think about?

① Routine vs. non-routine problems
② One-step vs. two or more step problems
③ One context vs. integrated problems

What kinds of reasoning can you think about?

① Induction
② Deduction
③ Inference
④ Integration (Gestalt)
⑤ Abstraction
⑥ Simplification
⑦ Generalization
⑧ Specification

What kinds of communication can you think about?

① To oneself (organize and use through the language of mathematics)
② To others (peers, teachers, and others)
③ Analyze and evaluate thinking of others (Speaking, Listening, Writing, Reading)

What kinds of connections can you think about?

① Other areas of the mathematics
② Other areas of the curriculum
③ The real world
(Vertical and horizontal mathematizations)

What kinds of representations can you think about?

① Creation and uses
② Selection and application
③ Translations

First Miscellaneous Item

- How is the treatment of the topic, that you group selected, related to the NCTM's *principles and standards for School Mathematics*? In particular, what roles do problem solving, reasoning, communication, connections, and representation play in the treatment of this topic?
- See an analysis example for connections
- By **September 23rd**

First Miscellaneous Item

- Please submit a copy of the textbook that you selected
- Can you discuss a **hierarchical order** of the statements of the NCTM process you selected?
- Under each category, you should develop **AT LEAST two different examples** from your perspectives with your explanations of **similarities** and **differences** between them
- If you would not find out a good example in the textbook, you group should find out an example through the use of other textbooks or develop a creative item from your group idea

References

- NCTM(2000). *Principles and Standards for School Mathematics.* Reston, VA: NCTM.