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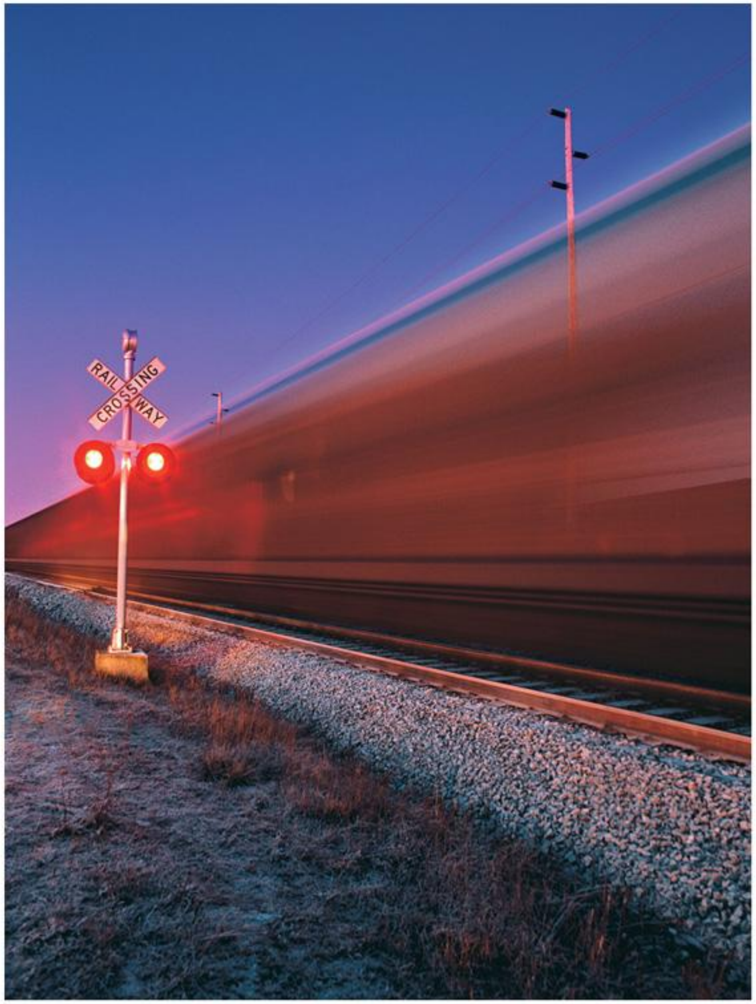
  1. University Physics, Bauer and Westfall, McGraw-Hill, 2011.
  2. Principles of Physics, Halliday, Resnick, and Walker, Wiley, 8^{th} and 9^{th} Ed.
- The rest is made by me.

Chapter 2
Motion in a straight line



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Displacement

Velocity

Acceleration

Motion with constant acceleration

물체의 운동을 어떻게 이해할까?

kinematics: 운동의 원인과 무관하게 물체의 운동을 기술하는 방법

I. 어떤 물리량으로 기술할까? (2장)

displacement, velocity, acceleration

II. 이 물리량들을 어떻게 기술할까? (1장)

vector

III. 운동이 일어나는 원인은 무엇인가?

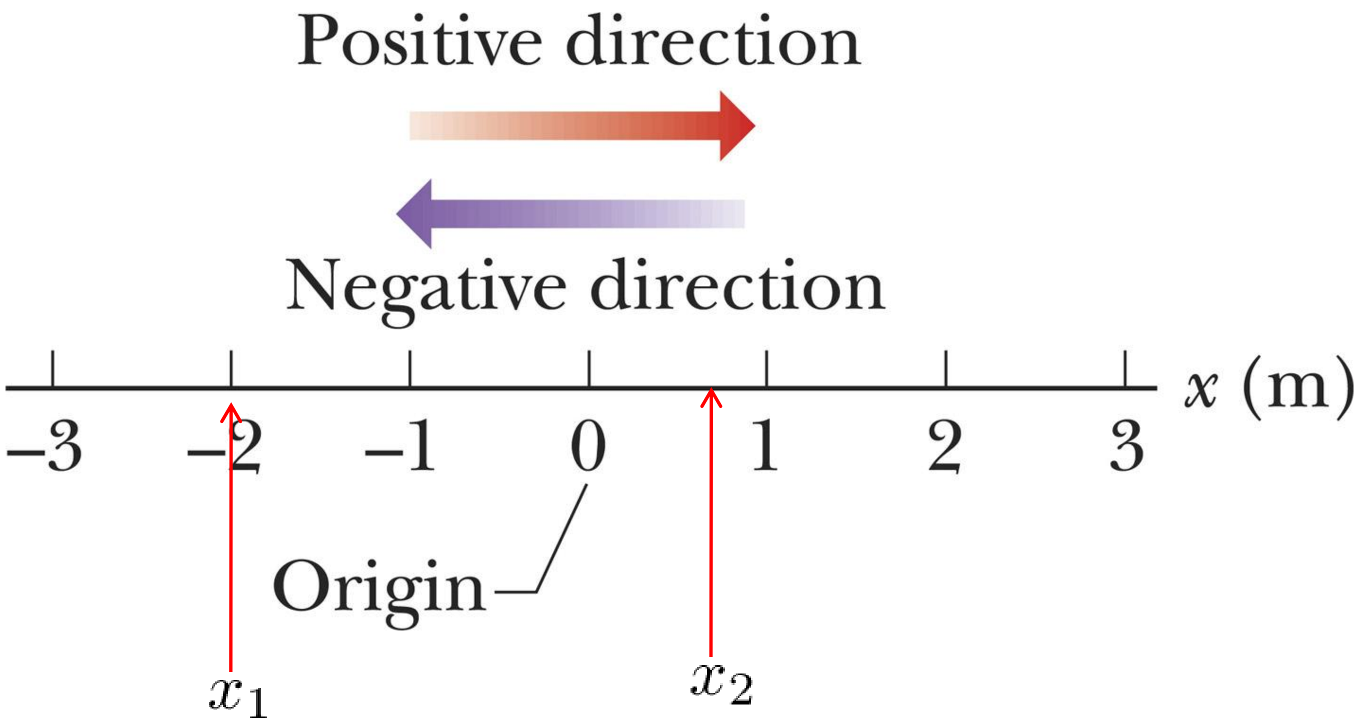
Newton의 운동법칙(4장), Gravitation(12장)

운동을 나타내는 물리량

물리학 용어	일상 용어	차원
Displacement	distance location	L
velocity	speed	LT^{-1}
acceleration		LT^{-2}

displacement

location: x



displacement: $\Delta x = x_2 - x_1$

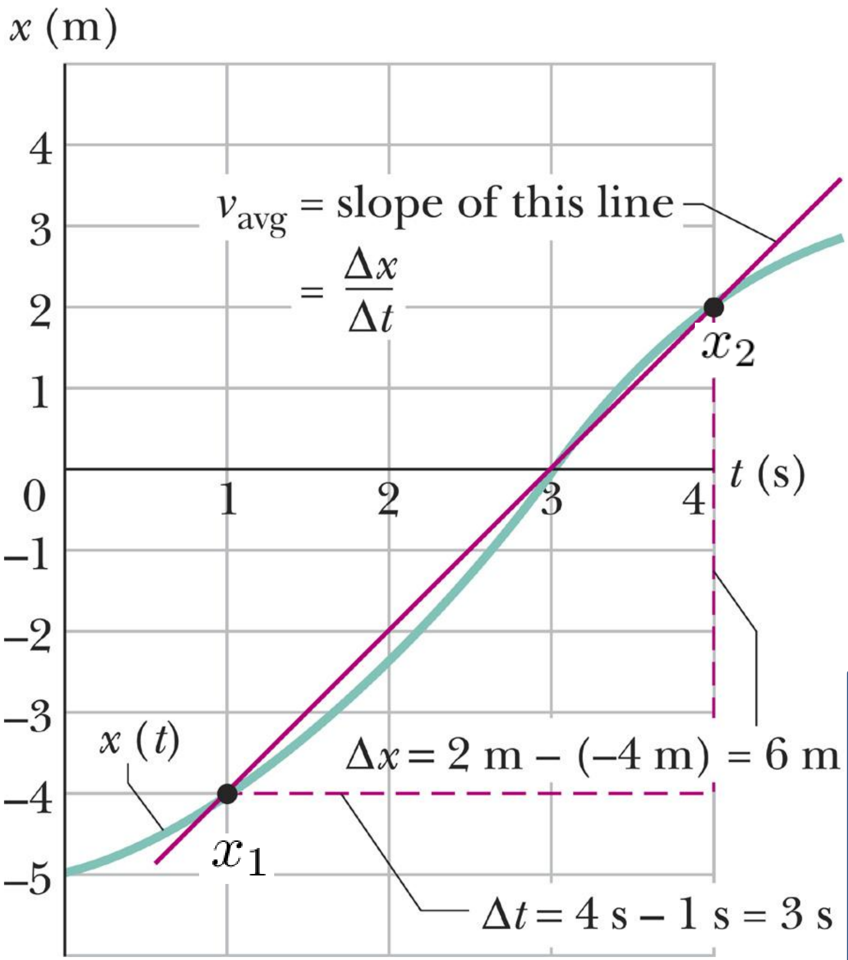
Velocity, acceleration

$$\Delta x = x_B - x_A$$

$$v_{av} = \frac{\Delta x}{\Delta t} = \frac{x_B - x_A}{t_B - t_A}$$

$$a_{av} = \frac{\Delta v}{\Delta t}$$

Average velocity, instantaneous velocity



Average velocity

$$v_{\rm avg} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}$$

Average speed

$$s_{\rm avg} = \frac{\text{total distance}}{\Delta t}$$

Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{x(t+\Delta t) - x(t)}{\Delta t} = \frac{dx}{dt}$$

acceleration

Average acceleration

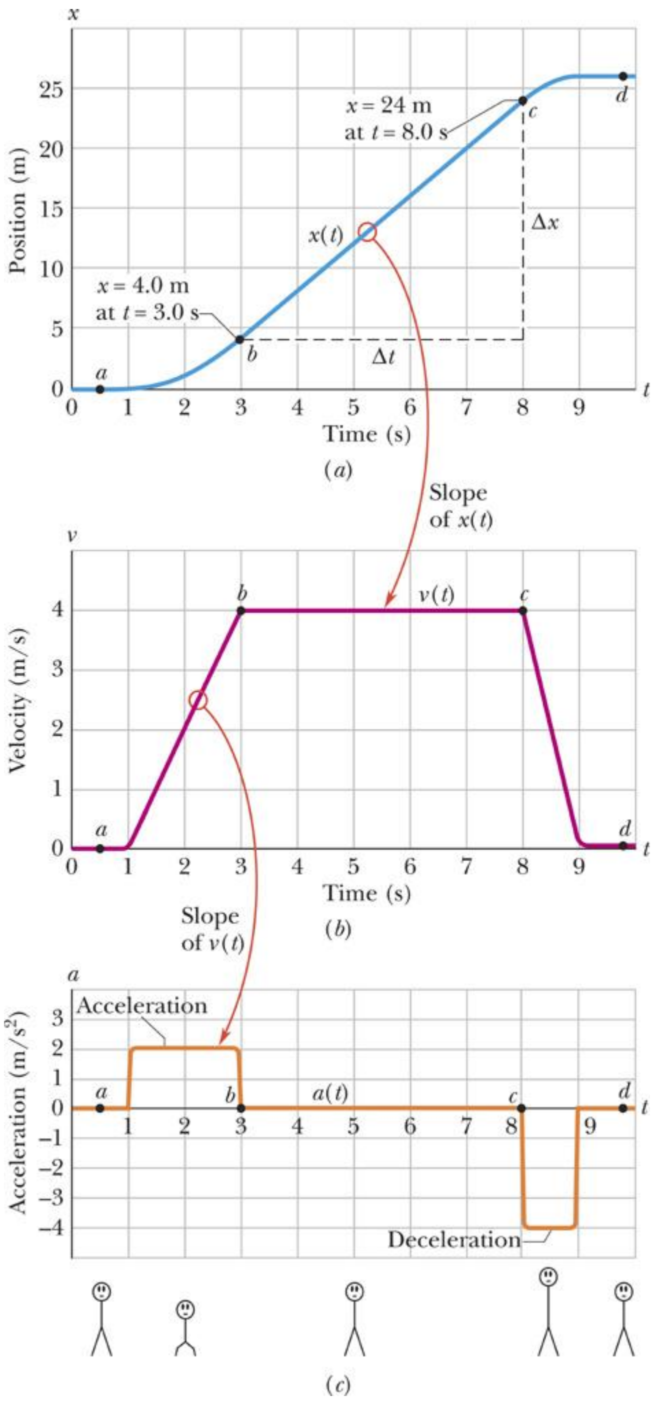
$$a_{\mathrm{avg}} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

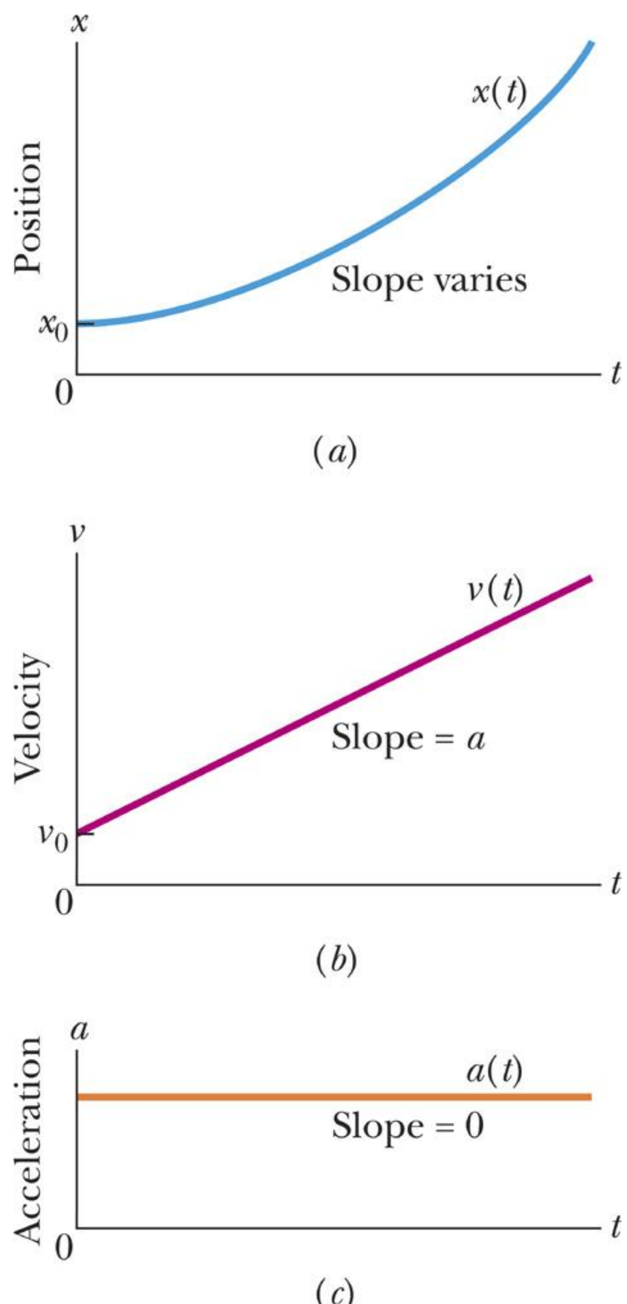
Instantaneous acceleration

$$a = \lim_{\Delta t \to 0} \frac{v(t + \Delta t) - v(t)}{\Delta t} = \frac{dv}{dt}$$

$$a = \frac{dv}{dt} = \frac{d^2 x}{dt^2}$$

Constant acceleration





Constant acceleration

Equations for Motion with Constant Acceleration[a]

Equation	Missing Quantity
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2}at^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v_0 + v)t$	a
$x - x_0 = vt - \frac{1}{2}at^2$	v_0

[a]Make sure that the acceleration is indeed constant before using the equations in this table.

공식의 유도

$$v = v_0 + at, \quad (x - x_0),$$

$$a = a_{av} = \frac{v - v_0}{t - 0} \qquad v = v_0 + at$$

$$x - x_0 = v_0 t + \frac{1}{2}at^2, \quad (v)$$

$$v_{av} = \frac{v + v_0}{2} \qquad X - X_0 = v_{av}\, t$$

$$= \frac{1}{2}(v + v_0)t$$

$$= \frac{1}{2}(2v_0 + at)t$$

$$= v_0 t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0), \quad (t)$$

$$v = v_0 + at \longrightarrow t = \frac{v - v_0}{a}$$

$$x - x_0 = v_0 t + \frac{1}{2} a t^2$$

$$= v_0 t + \frac{1}{2} a \frac{(v - v_0)^2}{a^2} = v_0 \frac{v - v_0}{a} + \frac{(v - v_0)^2}{2a}$$

$$2a(x - x_0) = 2v_0(v - v_0) + (v - v_0)^2 = v^2 - v_0^2$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t, \quad (a)$$

$$a = \frac{v - v_0}{t} \qquad x - x_0 = v_0 t + \frac{1}{2} t \frac{v - v_0}{t}$$

$$= \frac{t}{2}(v + v_0)$$

$$v^2 = v_0^2 + 2a(x - x_0), \quad (t)$$

$$x - x_0 = \frac{1}{2}(v_0 + v)t, \quad (a)$$

$$x - x_0 = vt - \frac{1}{2}at^2, \quad (v_0)$$

미적분을 이용하여 본 등가속도 운동

$$v = \frac{dx}{dt} \qquad x = \int_0^t dt' \, v$$

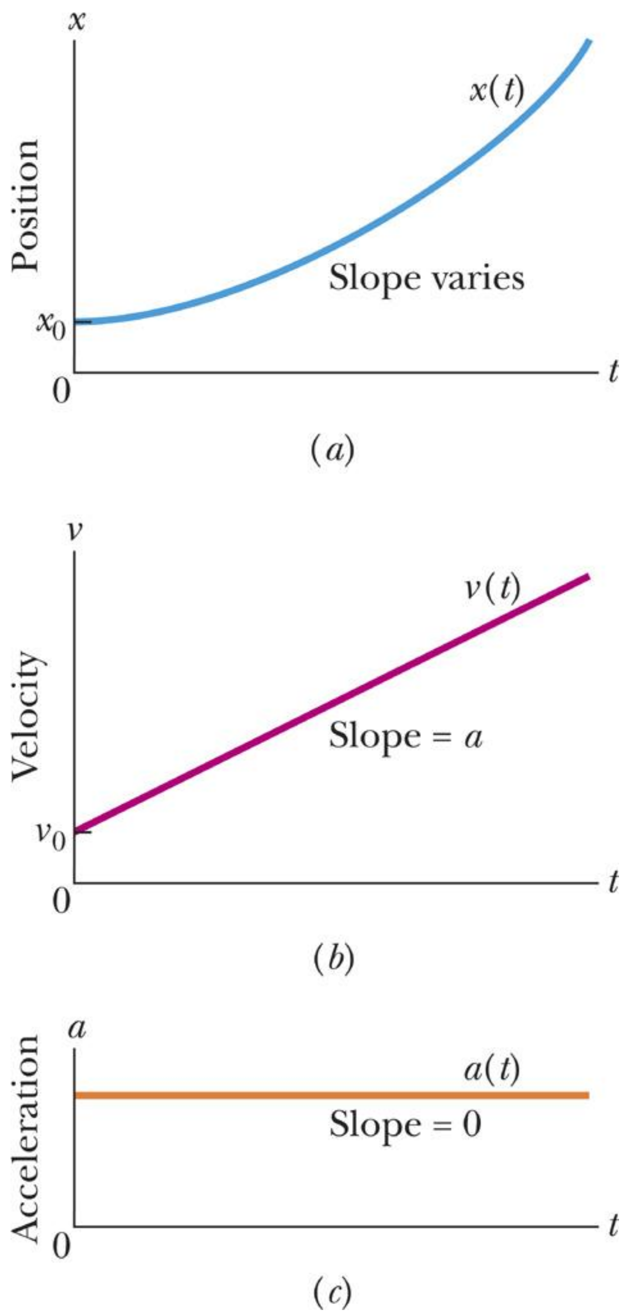
~~$v = v_0$~~

$$a = \frac{dv}{dt} \qquad \int_0^t dt' \, a = at = v + C$$

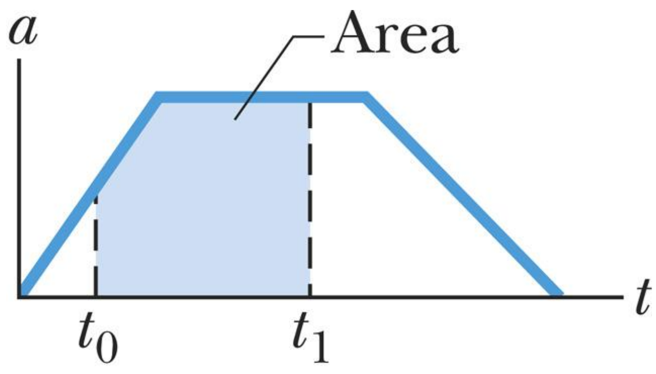
$$v = v_0 + at$$

$$x - x_0 = \int_0^t dt' (v_0 + at')$$

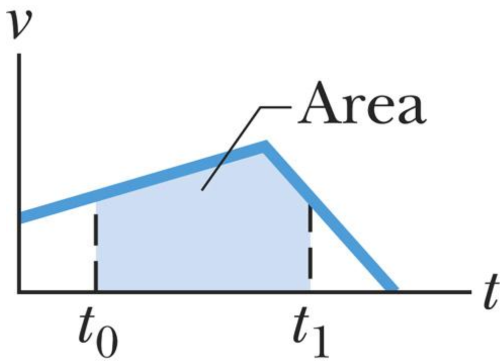
$$= v_0 t + \frac{1}{2} a t^2$$


x
Position
x(t)
Slope varies
x0
0
t
(a)
v
Velocity
v(t)
Slope = a
v0
0
t
(b)
a
Acceleration
a(t)
Slope = 0
0
t
(c)

그래프를 이용한 운동 해석

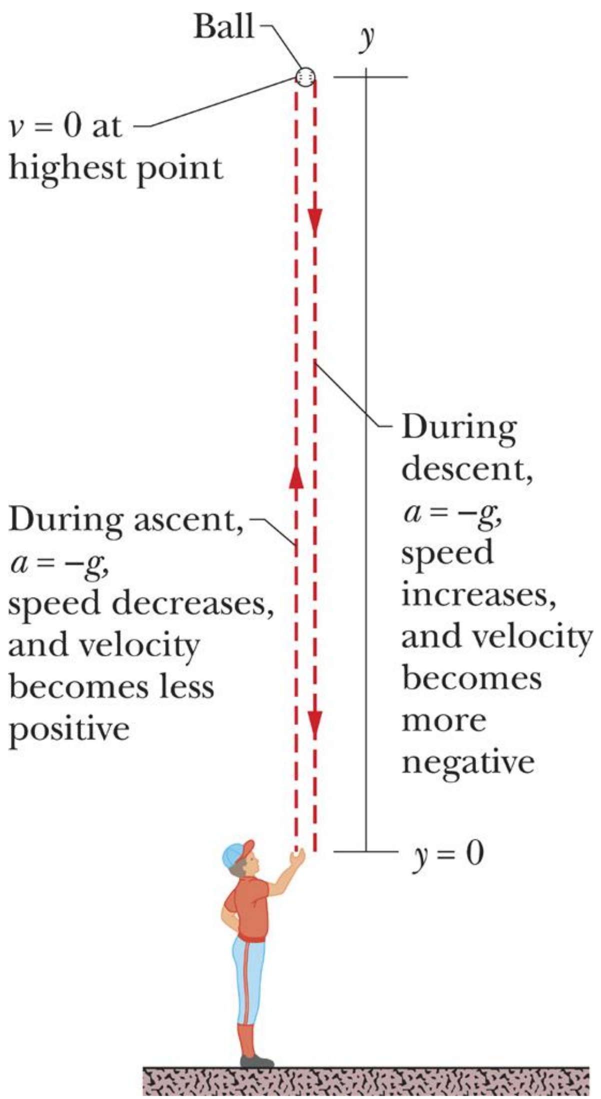


(a)



(b)

Problem 1: 공 던지기



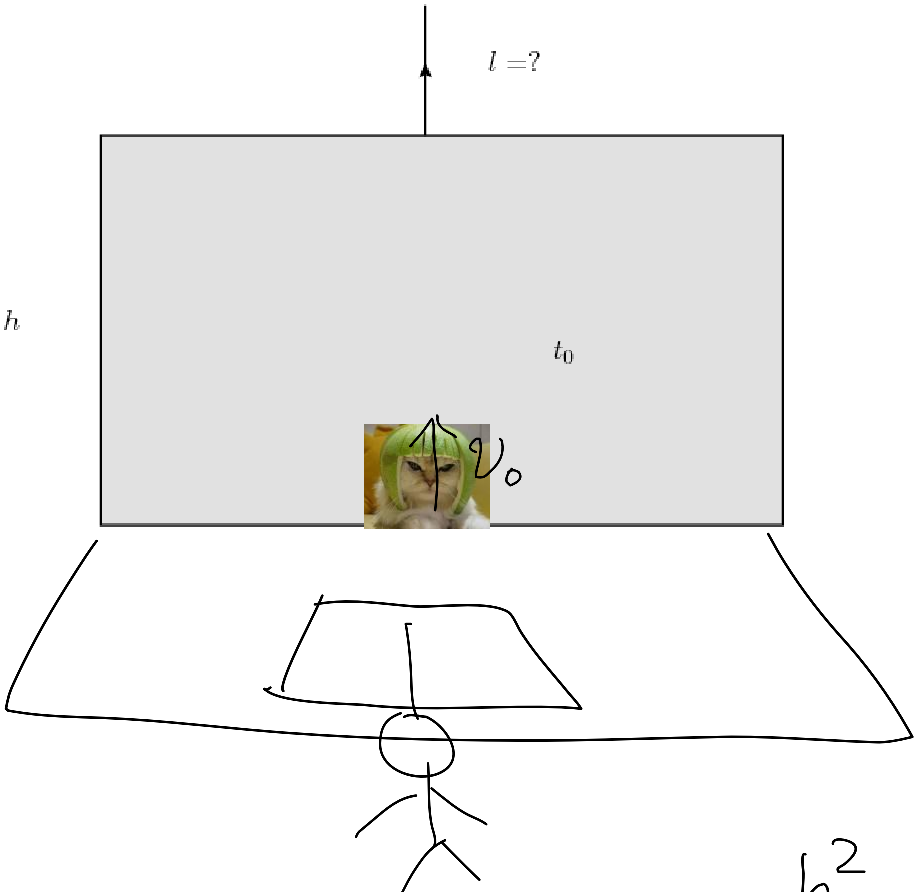
$$v = v_0 - gt = 0 \qquad t = \frac{v_0}{g}$$

$$v^2 - v_0^2 = -2gh$$

$$h = \sqrt{\frac{v_0^2}{2g}}$$

$$\sqrt{2gh} = \sqrt{2g\frac{v_0^2}{2g}} = v_0$$

Problem 2: 창 밖의 고양이



$$h = v_0 t_0 - \frac{1}{2} g t_0^2$$

$$v_0 = \frac{h + \frac{1}{2} g t_0^2}{t_0}$$

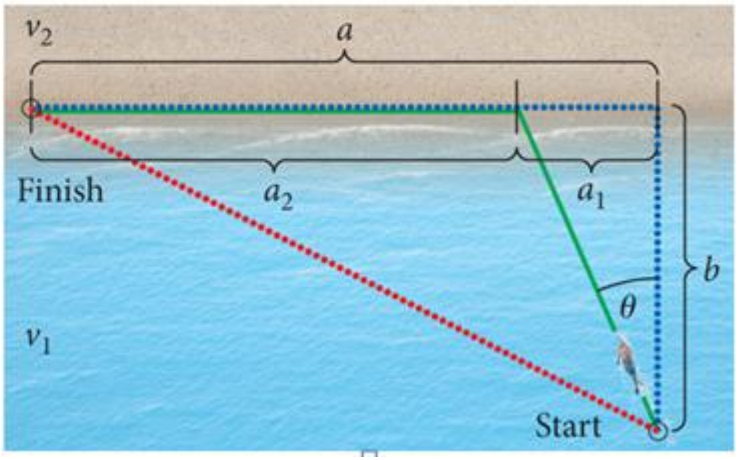
$$+ v_0^2 = +2g(l+h)$$

$$l = \frac{v_0^2}{2g} - h$$

$$= \frac{1}{2g} \frac{(h + \frac{1}{2} g t_0^2)^2}{t_0^2} - h$$

$$= \frac{h^2}{2gt_0} - \frac{h}{2} + \frac{g t_0^2}{8}$$

Problem 3: Aquathlon



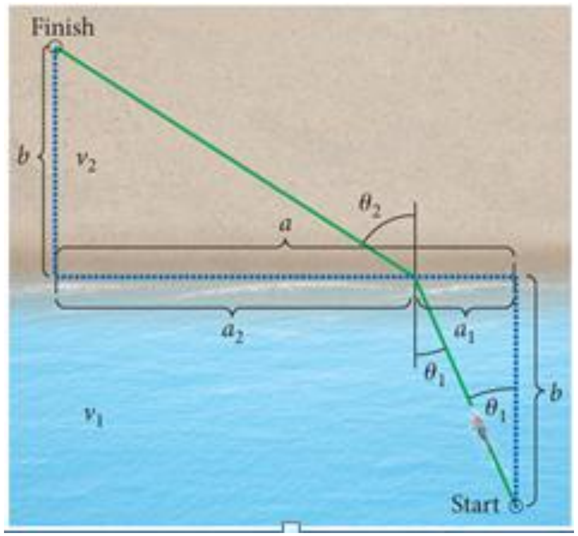
$$t = \frac{\sqrt{b^2 + a_1^2}}{v_1} + \frac{a - a_1}{v_2}$$

$$\frac{dt}{da_1} = \frac{1}{v_1}\frac{a_1}{\sqrt{a_1^2 + b_1^2}} - \frac{1}{v_2} = 0$$

$$\frac{a_1}{\sqrt{a_1^2 + b_1^2}} = \frac{v_1}{v_2} = \sin\theta$$

$$\sin\theta = \frac{v_1}{v_2}$$

Problem 4: 이효리 구하기



$$t = \frac{\sqrt{(a-a_1)^2+b^2}}{v_2} + \frac{\sqrt{a_1^2+b^2}}{v_1}$$

$$\frac{dt}{da_1} = \frac{-a_2}{v_2\sqrt{a_2^2+b^2}} + \frac{a_1}{v_1\sqrt{a_1^2+b^2}} = 0$$

$$\frac{\sin\theta_2}{v_2} = \frac{\sin\theta_1}{v_1}$$

$$\frac{\sin\theta_1}{v_1} = \frac{\sin\theta_2}{v_2}$$

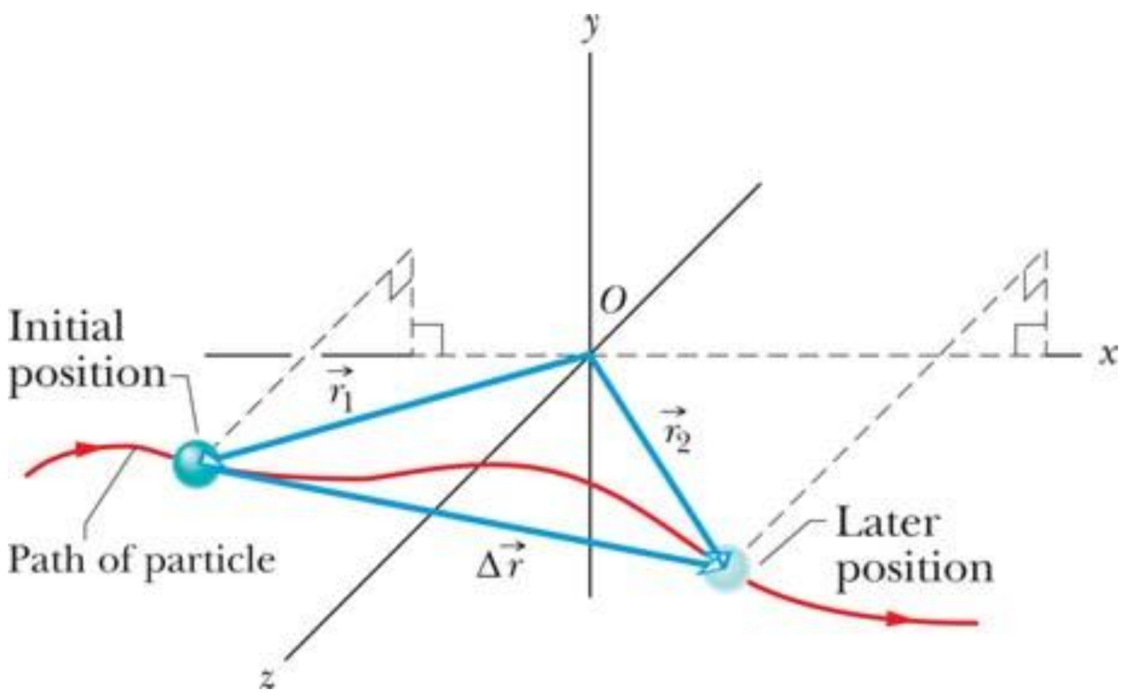
$$\frac{v_1}{v_2} = \frac{\sin\theta_1}{\sin\theta_2}$$

$$\frac{n_1}{n_2}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta$$

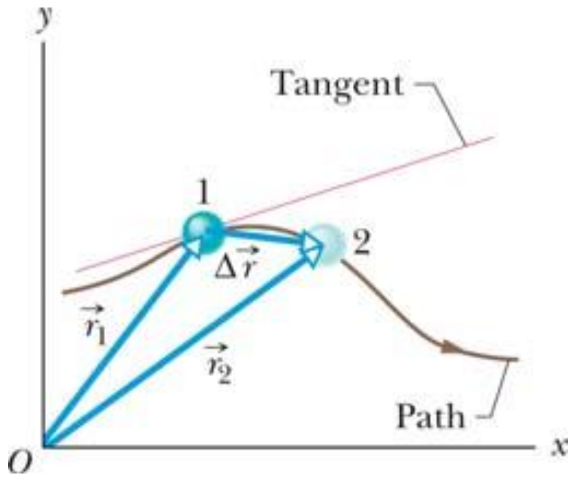
Chapter 3 Motion in 2 & 3 D

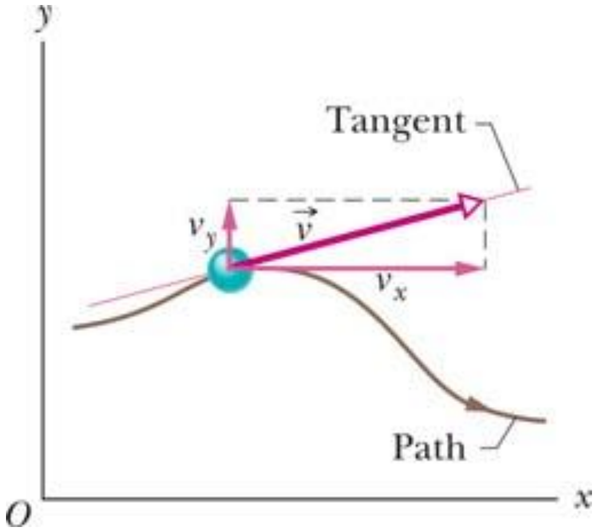
Displacement and position



Position: $\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ Displacement: $\Delta\mathbf{r} = \mathbf{r}_2 - \mathbf{r}_2$

velocity, acceleration





$$\mathbf{v} \equiv \lim_{\Delta t \to 0} \frac{\Delta \mathbf{r}}{\Delta t} = \frac{d\mathbf{r}}{dt}$$

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}\left(x\mathbf{i} + y\mathbf{j} + z\mathbf{k}\right)$$

$$= \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j} + \frac{dz}{dt}\mathbf{k} + \left(x\frac{d\mathbf{i}}{dt} + \cdots \right)$$

$$= v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k}$$

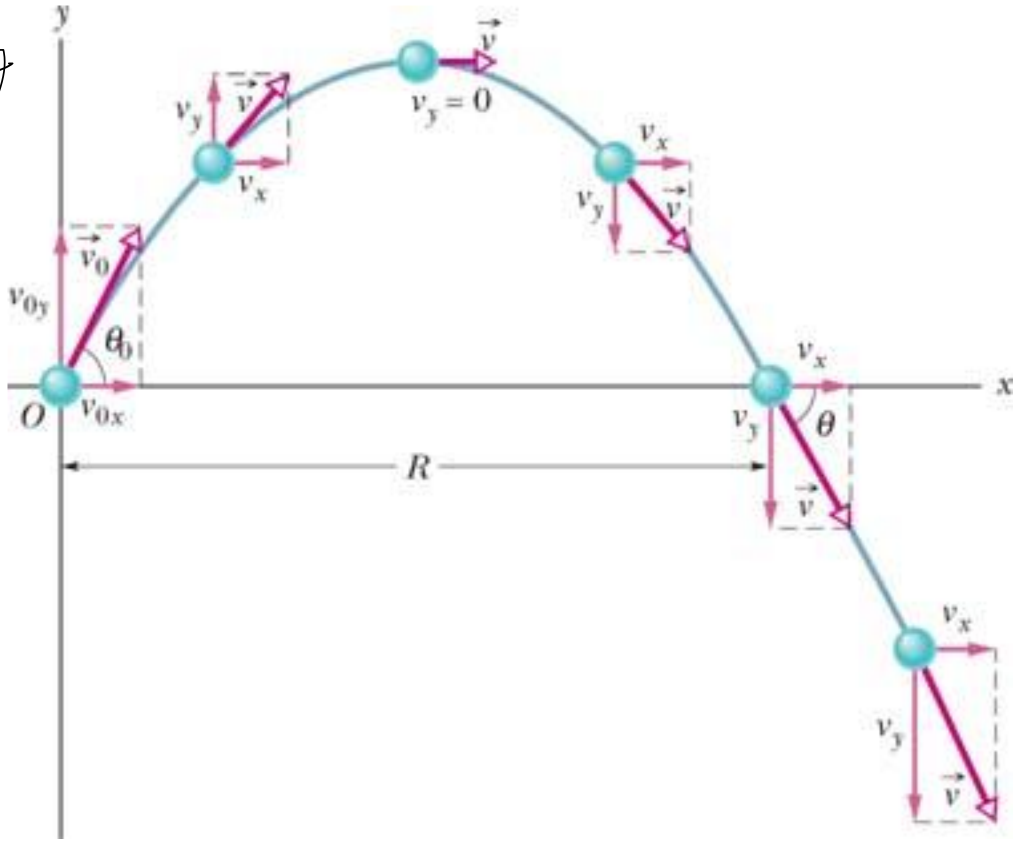
$$\mathbf{a} = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$

Projectile motion

$$v_{0y} = v_0 \sin\theta_0$$

$$\mathbf{v_0} = v_{0x}\mathbf{i} + v_{0y}\mathbf{j}$$

$$v_{0x} = v_0 \cos\theta_0$$

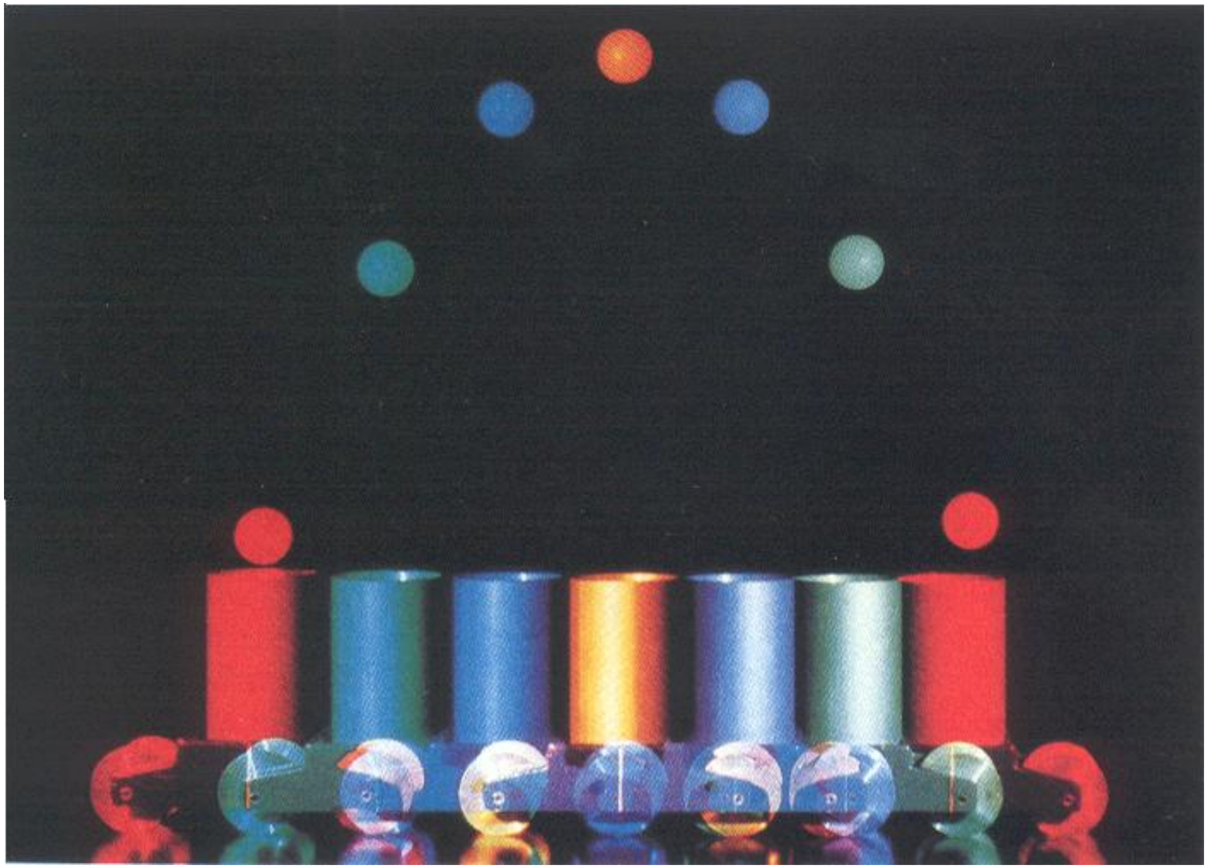


$$\mathbf{v} = v_x\mathbf{i} + v_y\mathbf{j}$$

Horizontal distance

path

Horizontal motion

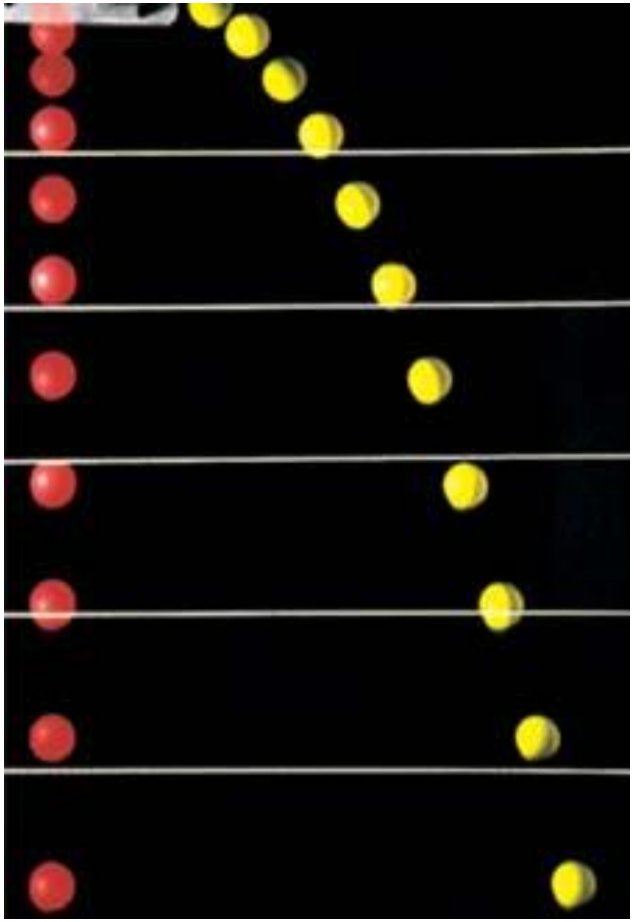


$$\mathbf{a}_x = \mathbf{0}$$

$$x = x_0 + v_{0x}t = v_{0x}t$$

$$v_x = v_{0x} = v_0 \cos\theta_0$$

Vertical motion



$$\mathbf{a}_y = \mathbf{g} = -g\mathbf{j}$$

$$v_y = v_{0y} - gt = v_0 \sin\theta_0 - gt$$

$$y = y_0 + v_{0y}t - \tfrac{1}{2}gt^2$$
$$= v_{0y}t - \tfrac{1}{2}gt^2$$

$$v_x = v_0 \cos\theta_0 \qquad v_y = v_0 \sin\theta_0 - gt$$

$$x = v_{0x} t \qquad y = v_{0y} t - \tfrac{1}{2} g t^2$$

$$t = \frac{v_0 \sin\theta_0}{g}$$

Eq. of the path

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \tfrac{1}{2} g \left(\frac{x}{v_{0x}}\right)^2$$

$$= \left(\frac{v_{0y}}{v_{0x}}\right) x - \left(\frac{g}{2v_{0x}^2}\right) x^2 = ax(R-x)$$

$$\therefore \quad y = ax - bx^2$$

포물선

Horizontal distance

$$x = v_0 \cos\theta_0 t = R, \qquad t = \frac{2 v_0 \sin\theta_0}{g}$$

$$y = v_0 \sin\theta_0 t - \tfrac{1}{2} g t^2 = 0$$

$$R = v_0 \cos\theta_0 \left(\frac{2 v_0 \sin\theta_0}{g}\right)$$

$$\therefore \quad R = \frac{v_0^2}{g} \sin(2\theta_0)$$

$$\theta_0 = 45^\circ \rightarrow R_{max}$$