Mobile Communications (KECE425)

Lecture Note 5

03-17-2014

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Summary

- Outage probability
  - Area outage probability
- Multi-path fading

Outage Probability

- Carrier-to-noise ratio

$$\Gamma = \frac{\text{Carrier power}}{\text{Noise power}}$$

  - Thermal noise outage probability

$$O_N = \Pr[\Gamma < \Gamma_{\text{th}}]$$

- Carrier-to-interference ratio

$$\Lambda = \frac{\text{Carrier power}}{\text{Interference power}}$$

  - Co-channel interference outage probability

$$O_I = \Pr[\Lambda < \Lambda_{\text{th}}]$$

- Overall outage due to both thermal noise and co-channel interference

$$O = \Pr[\Gamma < \Gamma_{\rm th} \text{ or } \Lambda < \Lambda_{\rm th}]$$

- Edge noise outage probability

$$\begin{aligned} O_N(R) &= P(\Omega_{p({\rm dBm})(R)} < \Omega_{\rm th(dBm)}) \\ &= \int_{-\infty}^{\Omega_{\rm th(dBm)}} \frac{1}{\sqrt{2\pi}\sigma_\Omega} \exp\left\{ -\frac{(x-\mu_{\Omega_{p({\rm dBm})}}(R)^2}{2\sigma_\Omega^2} \right\} dx \end{aligned}$$

$$= Q\left(\frac{M_{\rm shad}}{\sigma_\Omega} \right)$$

where $M_{\rm shad} = \mu_{\Omega_{p({\rm dBm})}} - \Omega_{\rm th(dBm)}$ is called *Shadow margin*.

- Example
  - Suppose that we wish to have $O_N(R) = 0.1$. Determine the Shadow margin M_{shad}.

- Solution
  - We solve

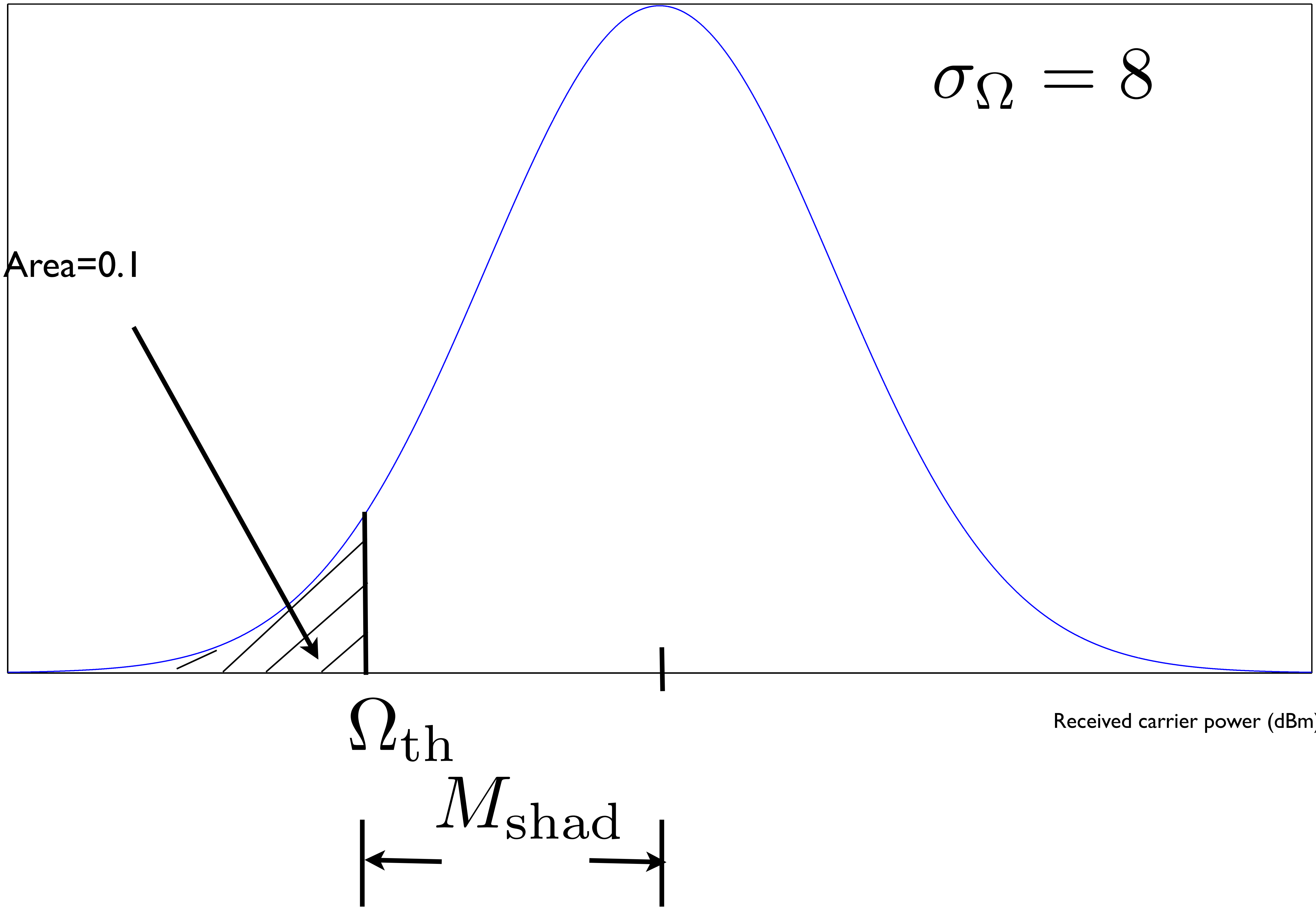
$$0.1 = Q\left(\frac{M_{\text{shad}}}{\sigma_\Omega}\right),$$

    which gives

$$\frac{M_{\text{shad}}}{\sigma_\Omega} = Q^{-1}(0.1) = 1.28$$

  - For $\sigma_\Omega = 8$ dB, the required shadow margin is

$$M_{\text{shad}} = 1.28 \times 8 = 10.24\,\text{dB}$$


$\sigma_\Omega = 8$
Area=0.1
$\Omega_{\rm th}$
Received carrier power (dBm)
$M_{\rm shad}$

Area Outage Probability

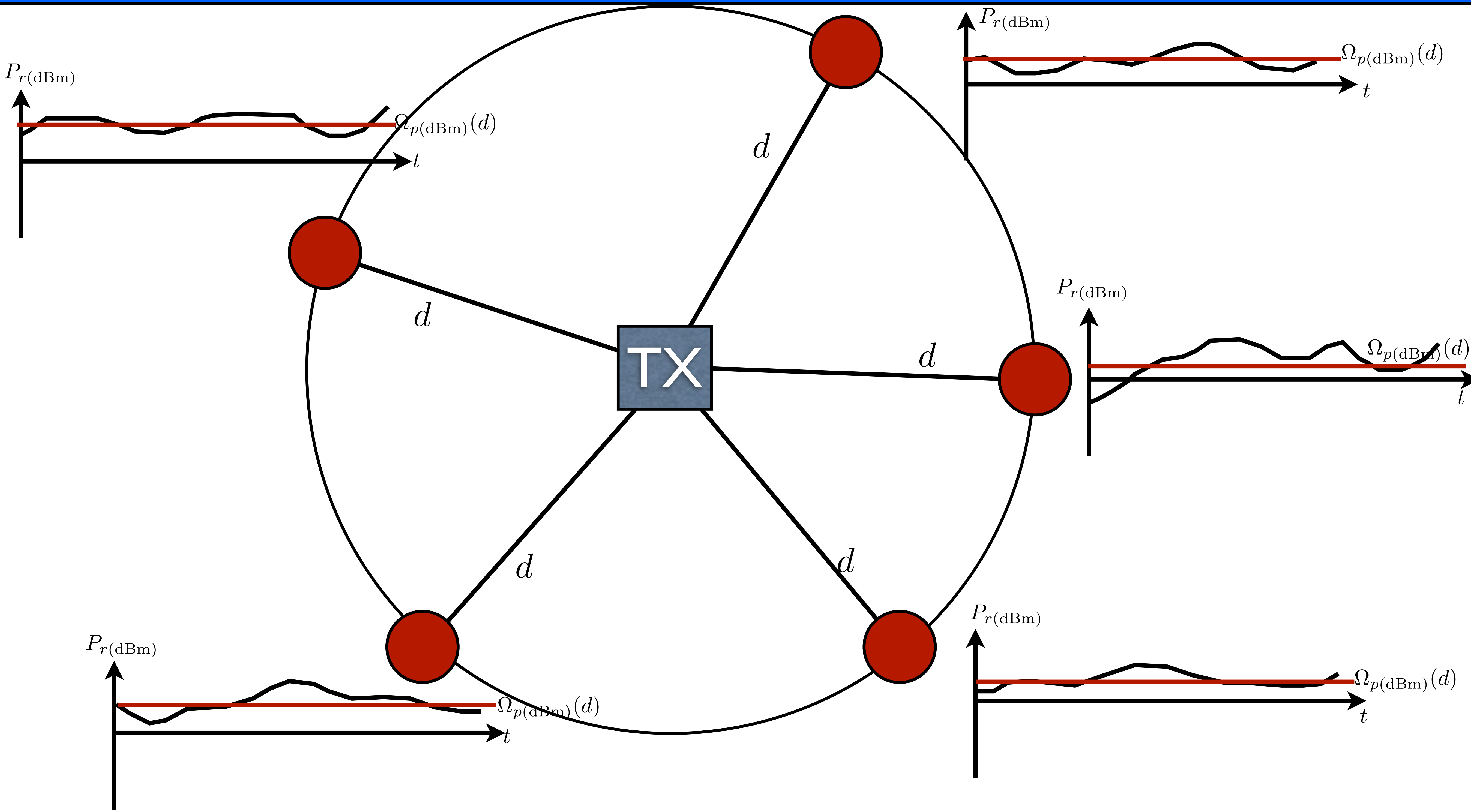
- Area outage probability averaged over area of a cell:

$$
\begin{aligned}
O_N &= \frac{1}{\pi R^2}\int_0^R O(r) 2\pi r \, dr \\
&= Q(X) - \exp\{XY + Y^2/2\} Q(X+Y)
\end{aligned}
$$

where

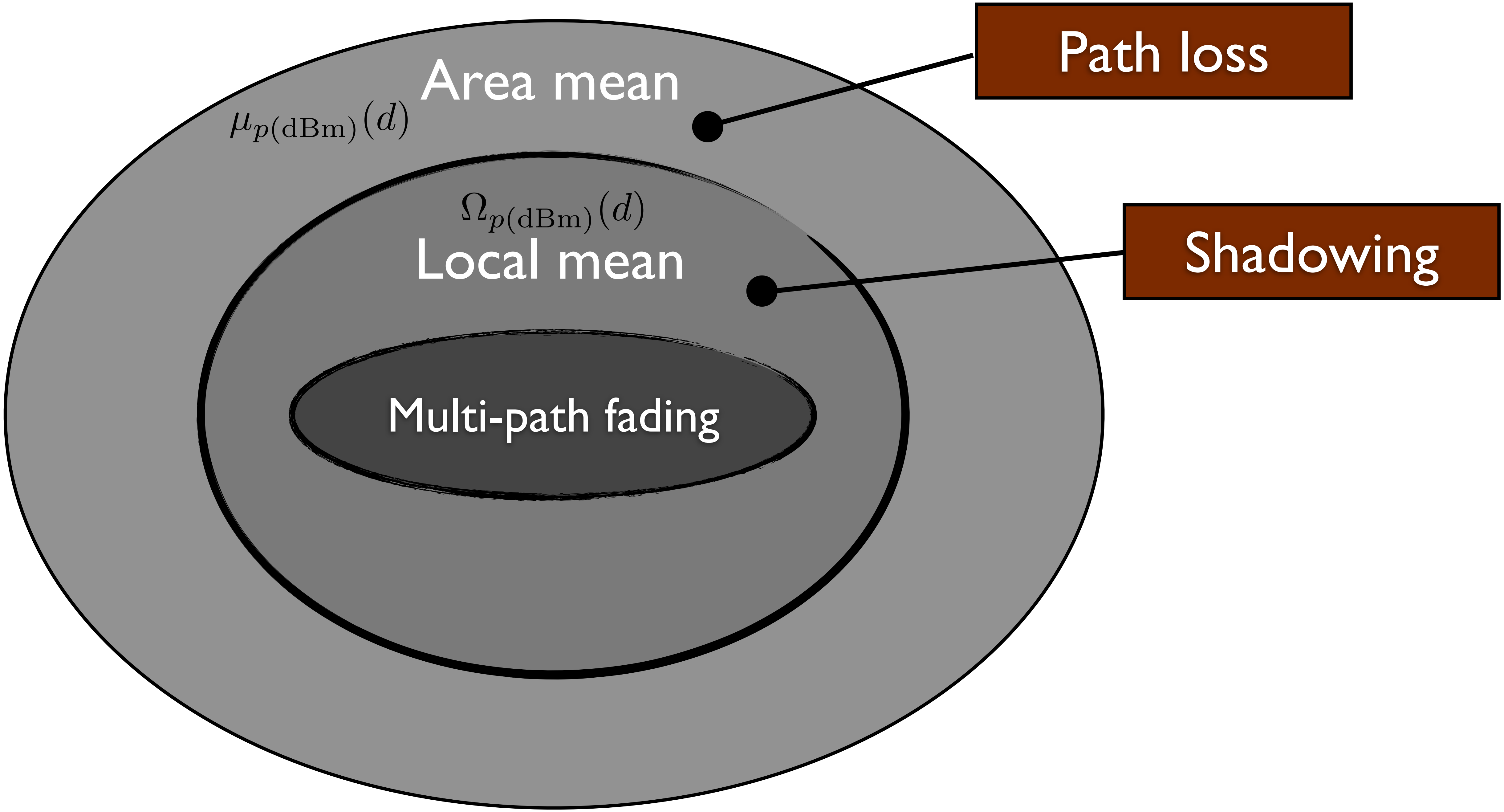
$$
\begin{aligned}
X &= \frac{M_{\text{shad}}}{\sigma_\Omega} \\
Y &= \frac{2\sigma_\Omega}{\beta\zeta} \\
\zeta &= \frac{10}{\ln 10}
\end{aligned}
$$

Wireless Channel Model



Wireless Channel Models

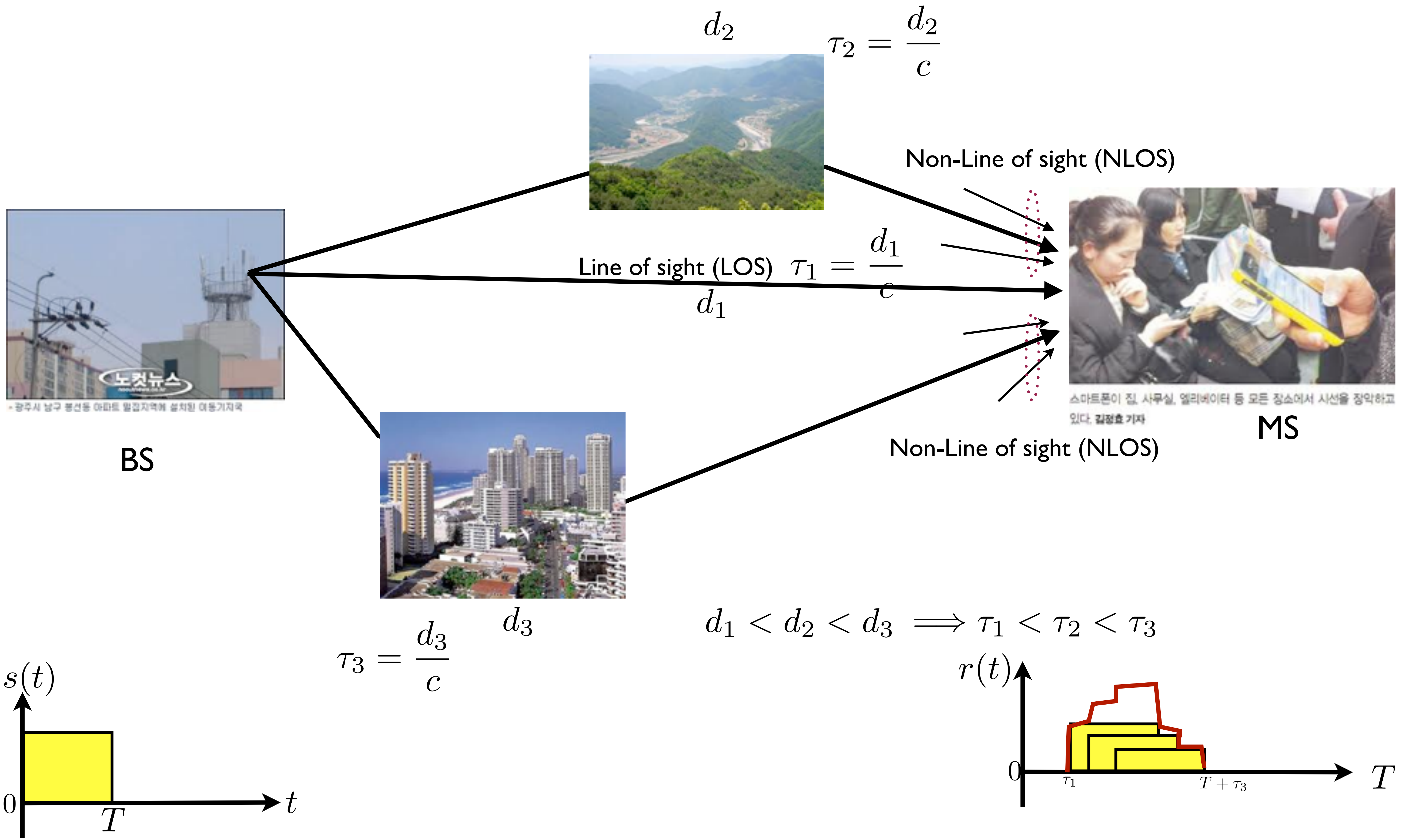
- Large scale channel model
  - Path loss model
  - Shadowing
- Small scale channel model
  - Multi-path fading


Area mean
$\mu_{p(\mathrm{dBm})}(d)$
Path loss
$\Omega_{p(\mathrm{dBm})}(d)$
Local mean
Shadowing
Multi-path fading

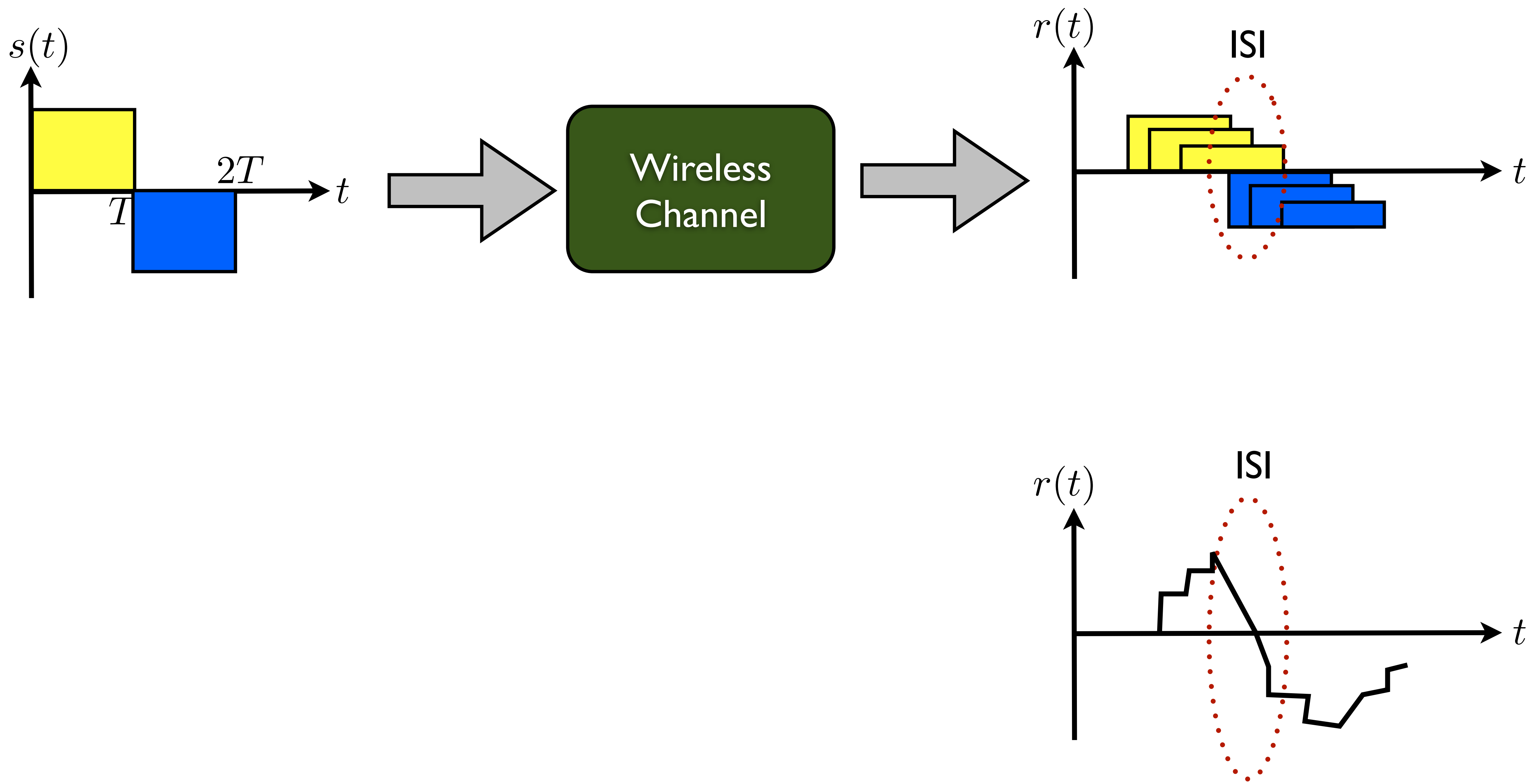
Multi-path Fading

- Categorization of multi-path fading
    - Depending on the vehicle speed
        - Fast fading vs. slow fading
    - Depending on the signal bandwidth (or data rate)
        - Frequency flat fading vs. frequency selective fading

Multi-Path Phenomenon

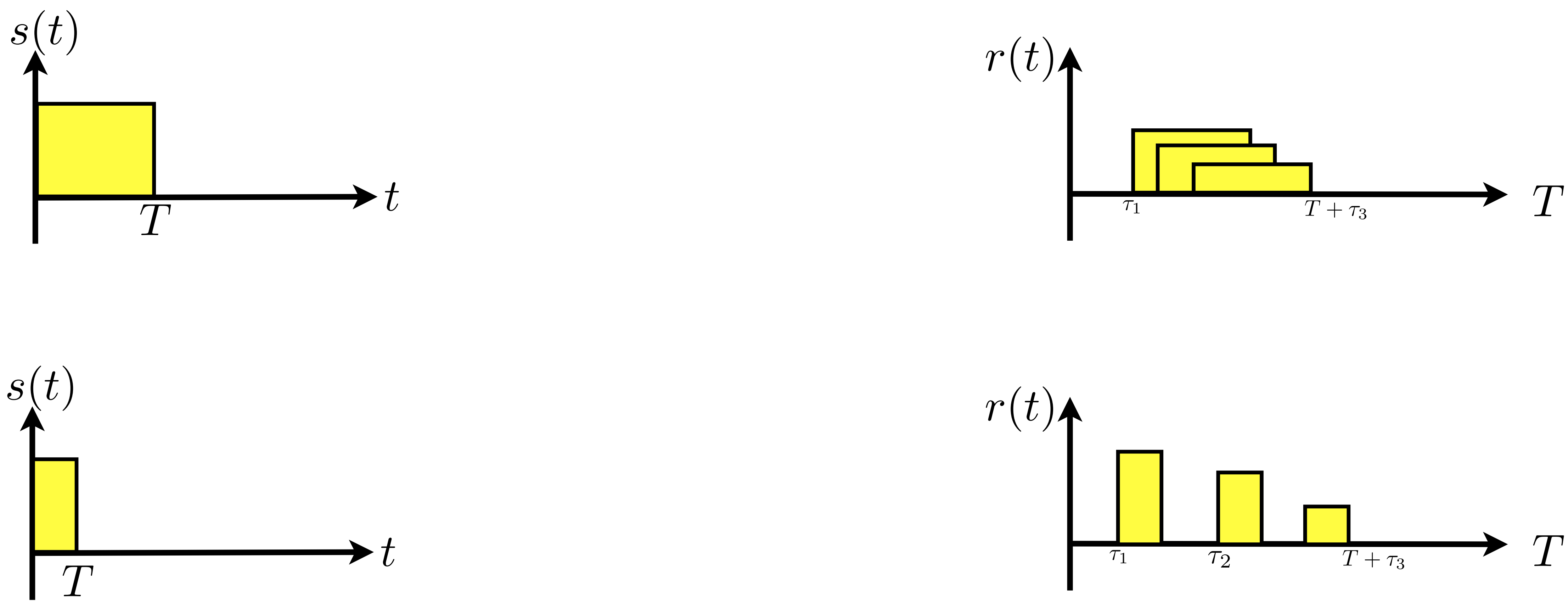


Inter-Symbol Interference due to Multi-Path Fading



Effect of Data Rate (or Bandwidth)

- Smaller time duration of the transmitted signal => higher data rate



ISI is getting severer when the data rate gets higher.

Wireless Channels

Wireless Channel

Received signal power

- *Time-varying random signal*
- *multi-path signal*

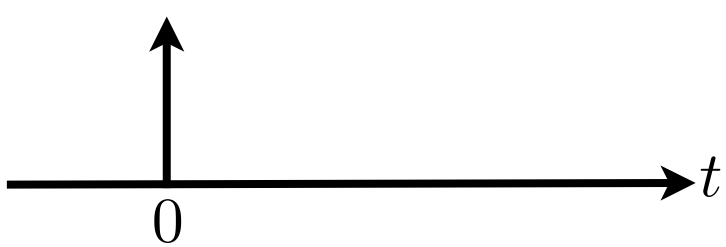
Impulse Response of the Wireless Channels

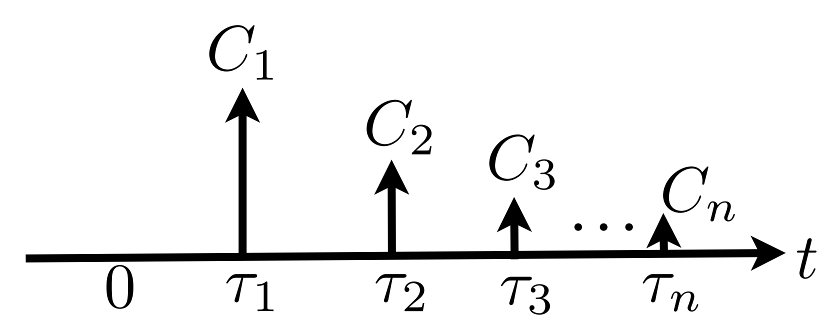
Equivalent low-pass signal and system representation

$s(t) = \Re[\tilde{s}(t)e^{j2\pi f_c t}]$ → **Wireless Channels** $h(t) = \Re[\tilde{h}(t)e^{j2\pi f_c t}]$ → $r(t) = \Re[\tilde{r}(t)e^{j2\pi f_c t}]$

$$r(t) = s(t) * h(t) \qquad \tilde{r}(t) = \tilde{s}(t) * \tilde{h}(t)$$

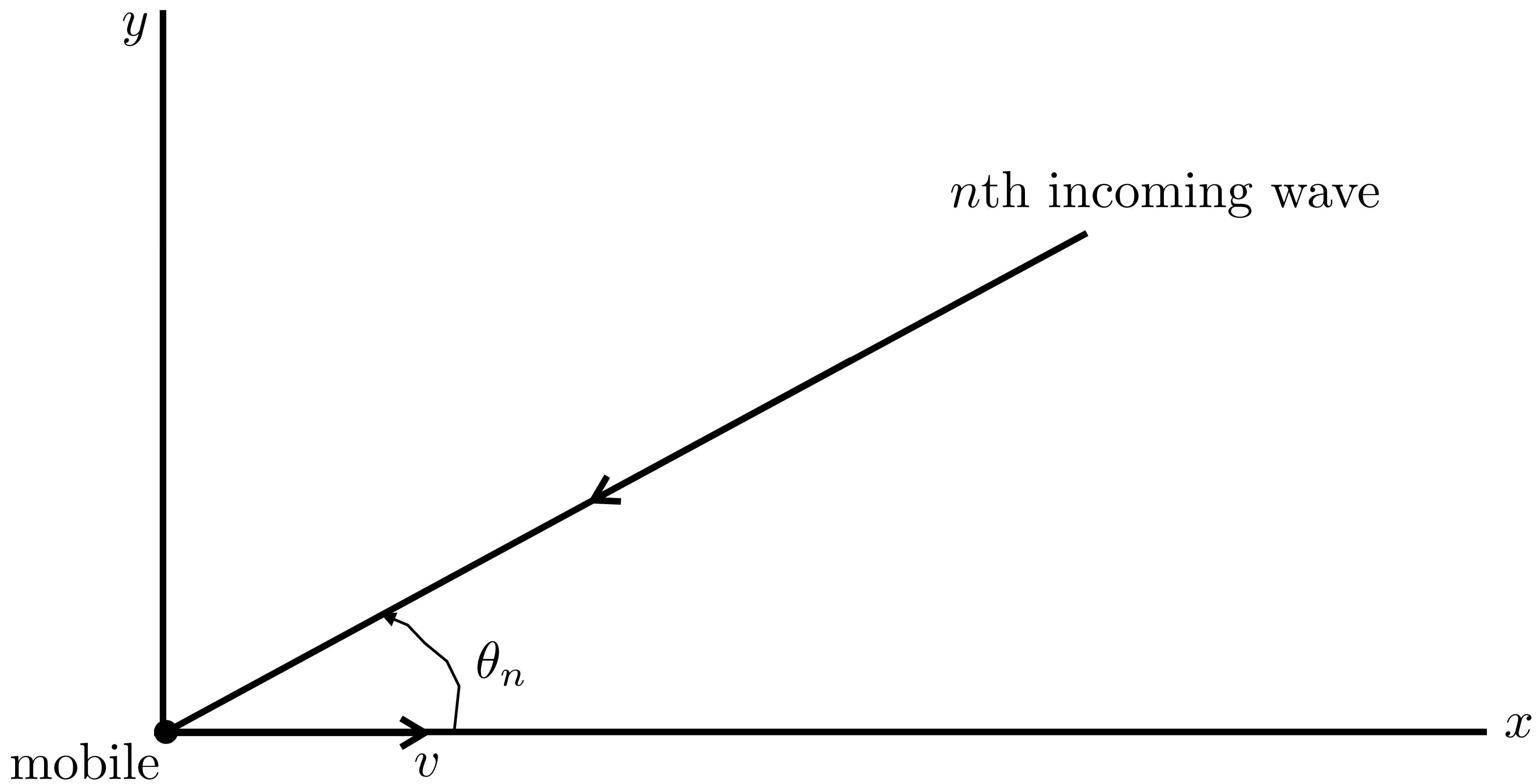
- only if the channel can be modeled as linear-time-invariant system
- However, the wireless channel is time-variant.





In practice, there are many multiple paths arriving at the receiver with random amplitude and phase.

Doppler Effect



Doppler frequency

$$f_{D,n} = f_m \cos\theta_n \ \text{Hz}$$

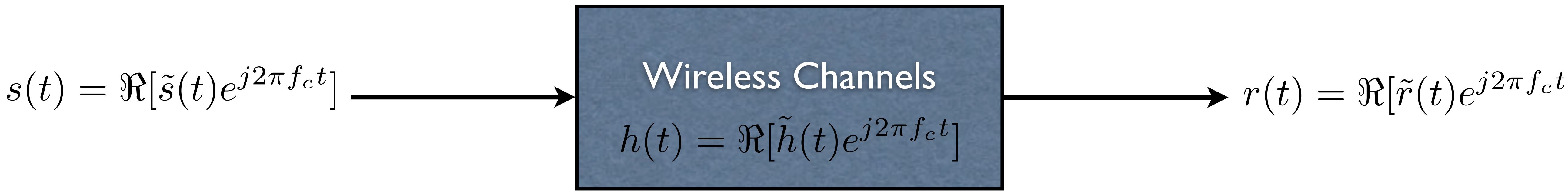
where $f_m = \dfrac{v}{\lambda_c}$ is maximum Doppler frequency.

Frequency Shift Due to Doppler Effect

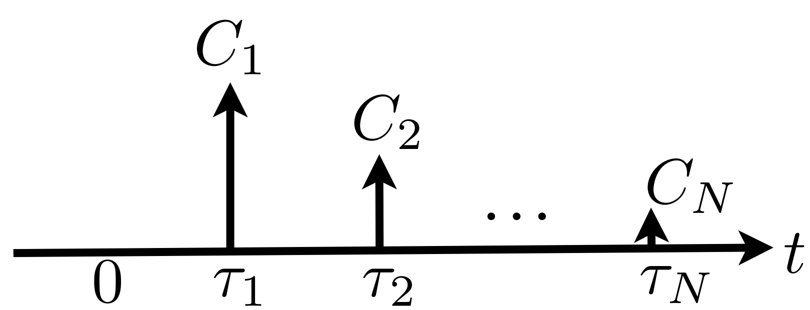
Example: $f_c = 2\ \mathrm{GHz}$ $\lambda_c = 15\ \mathrm{cm}$

Vehicle speed (km/hr)	Maximum Doppler freq. (Hz)	Frequency shift
3	5.56	2GHz + 5.56 Hz
30	55.56	2GHz + 55.56 Hz
60	111.11	2GHz + 111.11 Hz
120	222.22	2GHz + 222.22 Hz
300	555.56	2GHz + 555.56 Hz

- Doppler frequency shift: $f_c + f_{D,n} = f_c + f_m \cos\theta_n$

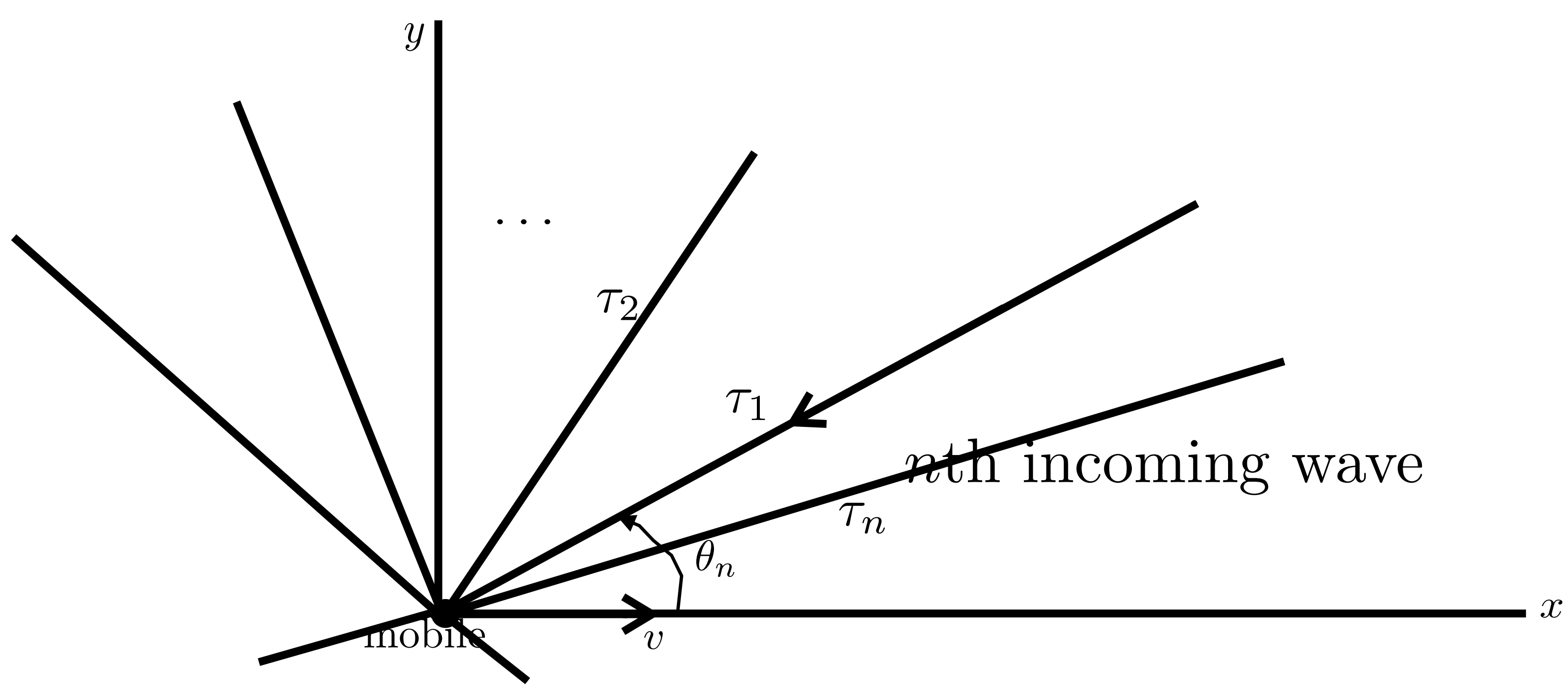


$$h(t, \tau)$$



τ_n and C_n are random variable

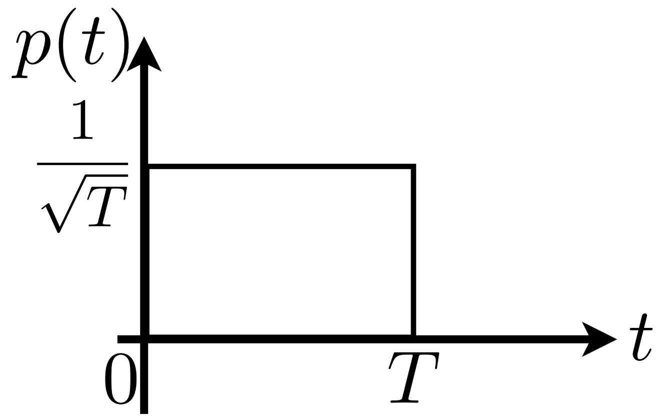
- C_n and τ_n are the amplitude and time delay, respectively associated with the nth propagation path.



- Transmitted signal:

$$s(t) = a_m p(t) \cos(2\pi f_c t + \theta_m), \quad m = 1, 2, ..., M$$

where $p(t)$ is the pulse shape for the symbol duration.



- Equivalent lowpass signal form:

$$s(t) = \Re\left[\tilde{s}(t)e^{j2\pi f_c t}\right]$$

where $\tilde{s}(t) = a_m p(t) e^{j\theta_m}$

- Received signal is the sum of a total of N multi-path signals:
  - First incoming wave: $C_1 a_m p(t-\tau_1)\cos(2\pi(f_c+f_{D,1})(t-\tau_1)+\theta_m)$
  - Second incoming wave: $C_2 a_m p(t-\tau_2)\cos(2\pi(f_c+f_{D,2})(t-\tau_2)+\theta_m)$

$$\vdots$$

  - nth incoming wave: $C_n a_m p(t-\tau_n)\cos(2\pi(f_c+f_{D,n})(t-\tau_n)+\theta_m)$

$$\vdots$$

  - Nth incoming wave: $C_N a_m p(t-\tau_N)\cos(2\pi(f_c+f_{D,N})(t-\tau_N)+\theta_m)$