KECE321 Communication Systems I

(Haykin Sec. 3.9 - Sec. 4.2)

Lecture #13, April 30, 2012

Prof. Young-Chai Ko

Announcement

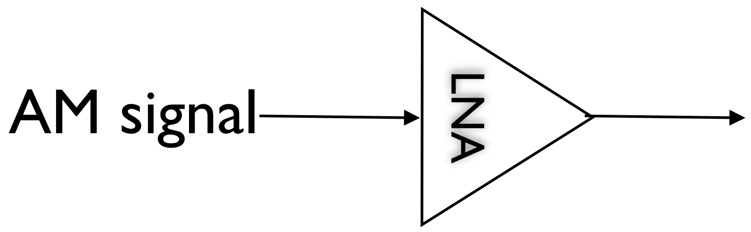
- No class on May 7, Monday
- Supplementary class: May 11, Friday
  - 4:00 - 5:15 PM

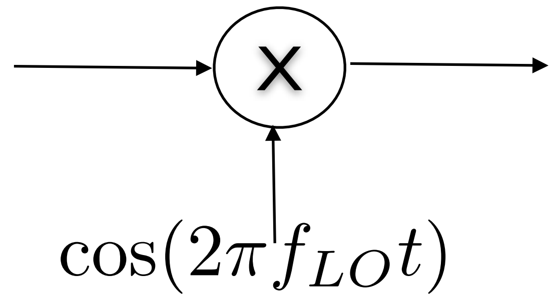
Summary

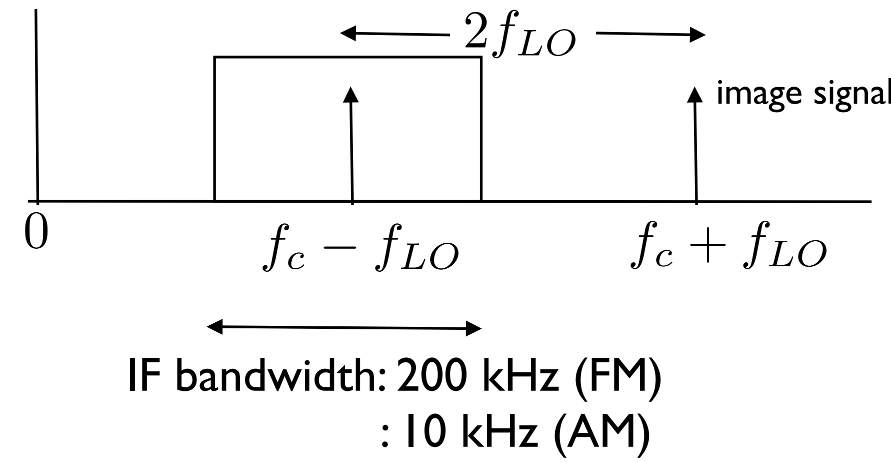
- Superheterodyne receiver
- Frequency-division multiplexing
- Time-division multiplexing
- Code-division multiplexing
- Angle modulation
  - Basics
  - Properties of angle-modulated waves

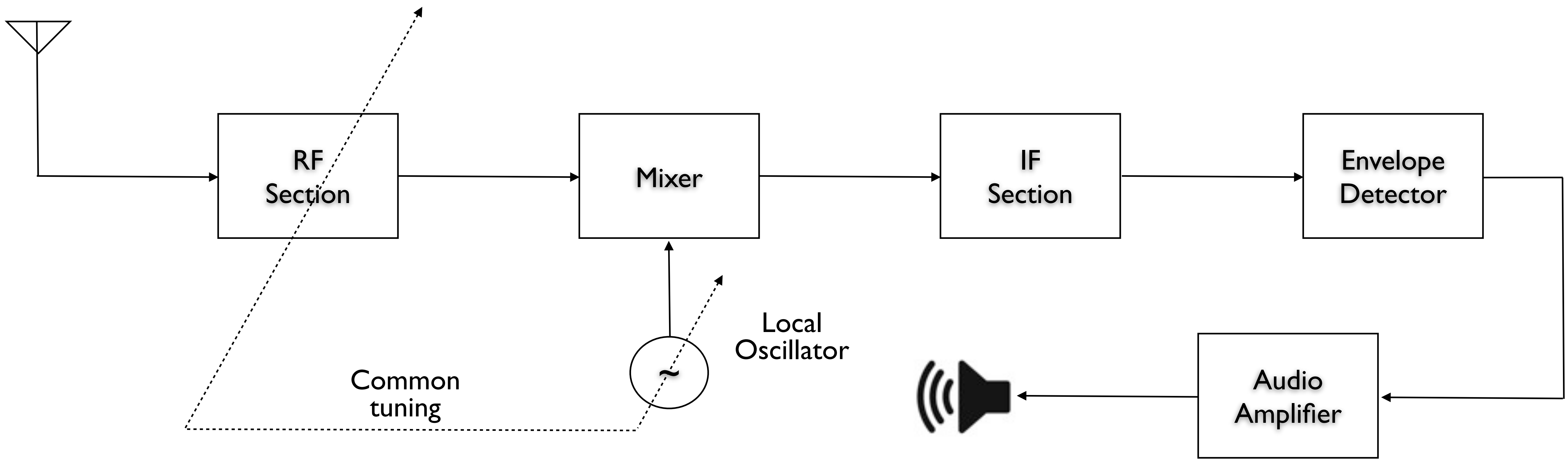
Superheterodyne Receiver

- Functions in the receiver for broadcasting system
  - Carrier-frequency tuning
  - Filtering
  - Amplification









f_c	f_{LO}	f_{IF}
89.1 MHz	78.4 MHz	10.7 MHz
91.9 MHz	81.2 MHz	10.7 MHz
93.1 MHz	92.4 MHz	10.7 MHz

Frequency Division Multiplexing

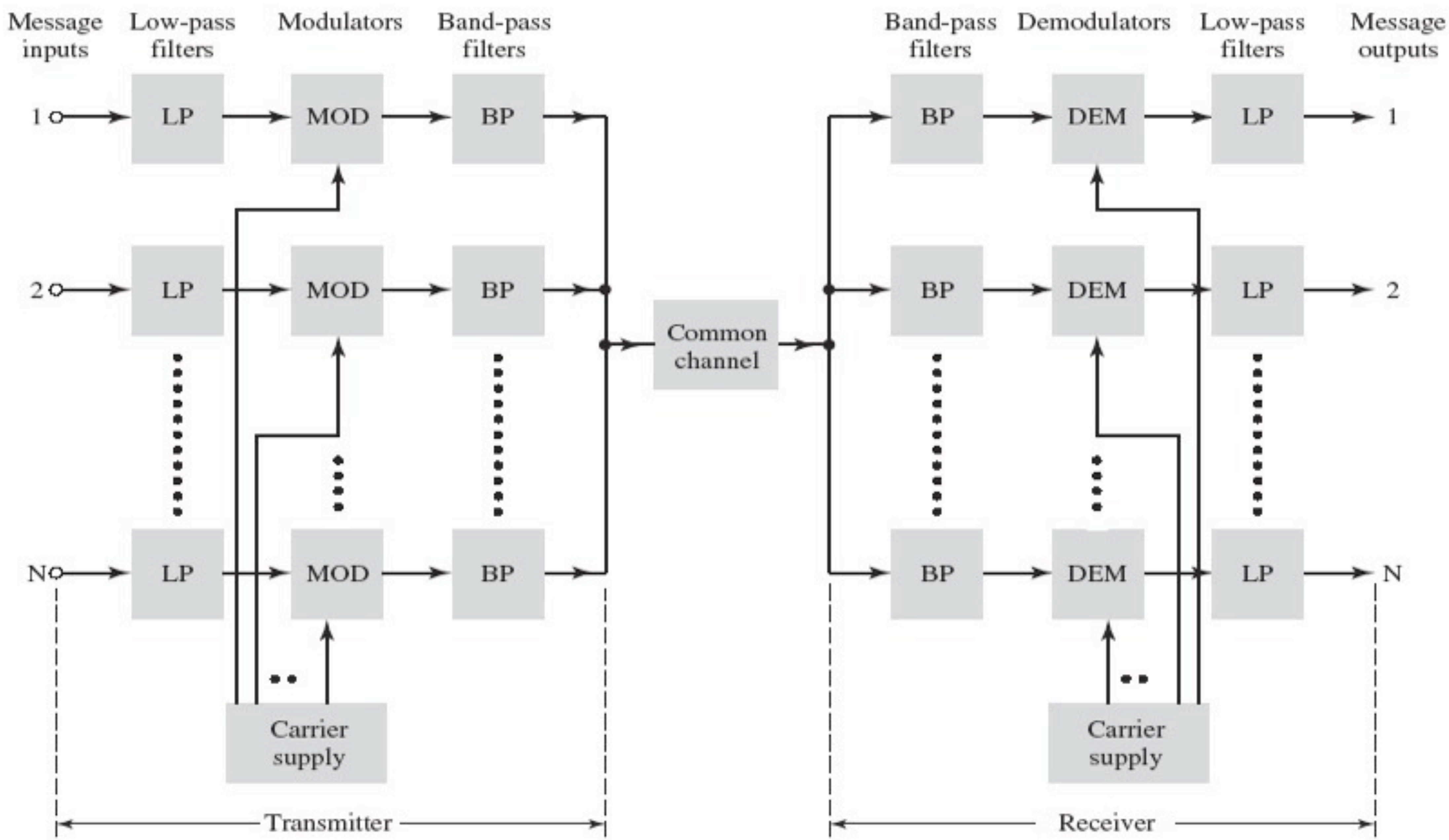


FIGURE 3.29 Block diagram of frequency-division multiplexing (FDM) system.

[Ref: Haykin Textbook]

Time Division Multiplexing

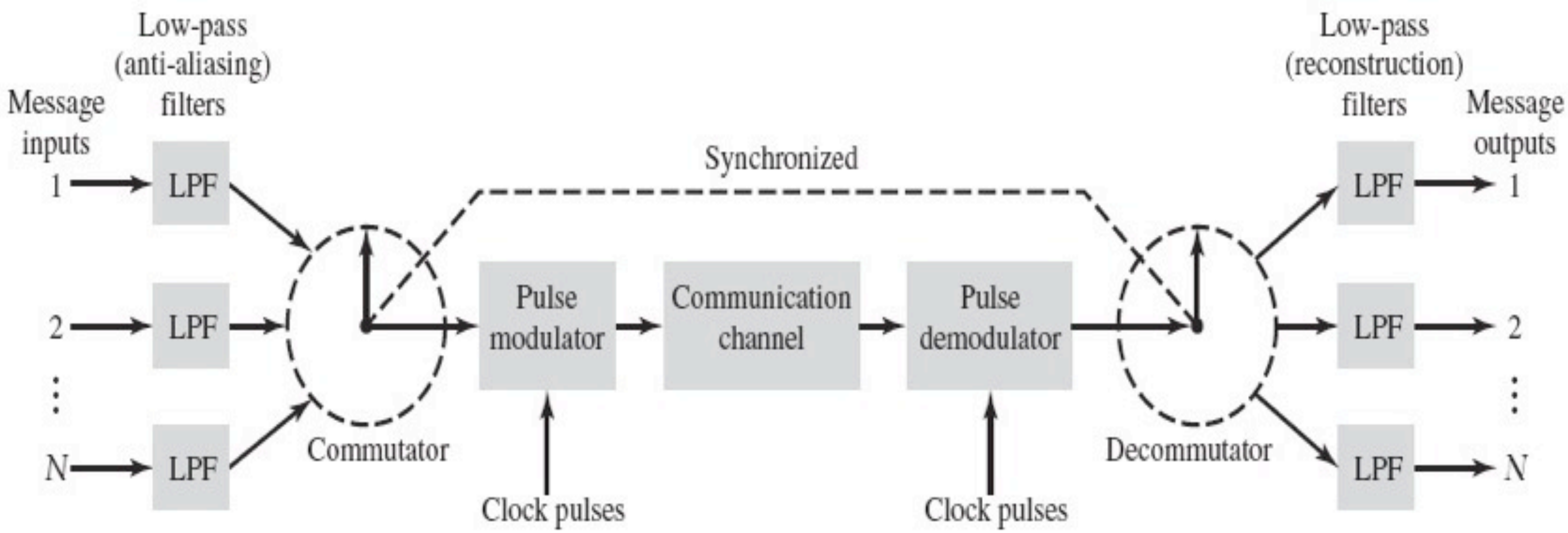


FIGURE 5.21 Block diagram of TDM system.

[Ref: Haykin Textbook]

Angle Modulation

- Basic definition of angle modulation

$$s(t) = A_c \cos[\theta_i(t)] = A_c \cos[2\pi f_c t + \phi_c]$$

  - Phase modulation (PM) if

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

  - Frequency modulation (FM) if

$$\theta_i(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

Basic Definition

- Angle modulated wave

$$s(t) = A_c \cos[\theta_i(t))]$$

- Average frequency in hertz

$$f_{\Delta t} = \frac{\theta(t + \Delta t) - \theta(t)}{2\pi \Delta t}$$

- Instantaneous frequency of the angle modulated signal

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t) = \frac{1}{2\pi} \frac{d\theta_i(t)}{dt}$$

  - Thus

$$\theta_i(t) = 2\pi f_c t + \phi_c, \qquad \text{for } m(t) = 0$$

- Phase modulation (PM):
  - a form of angle modulation in which instantaneous angle is varied linearly with with the message signal

$$\theta_i(t) = 2\pi f_c t + k_p m(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p m(t)\right]$$

$$k_p : \text{ phase sensitivity factor}$$

- Frequency modulation (FM):
  - a form of angle modulation in which the instantaneous frequency is varied linearly with the message signal

$$f_i(t) = f_c + k_f m(t)$$

$$\theta_i(t) = 2\pi \int_0^t f_i(t)\, d\tau = 2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$$

k_f : frequency sensitivity factor

TABLE 4.1 *Summary of Basic Definitions in Angle Modulation*

	Phase modulation	*Frequency modulation*	*Comments*
Instantaneous phase $\theta_i(t)$	$2\pi f_c t + k_p m(t)$	$2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau$	A_c: carrier amplitude f_c: carrier frequency $m(t)$: message signal k_p: phase-sensitivity factor k_f: frequency-sensitivity factor
Instantaneous frequency $f_i(t)$	$f_c + \frac{k_p}{2\pi}\frac{d}{dt} m(t)$	$f_c + k_f m(t)$	
Modulated wave $s(t)$	$A_c \cos[2\pi f_c t + k_p m(t)]$	$A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(\tau)\, d\tau\right]$	

[Ref: Haykin & Moher, Textbook]

Properties of Angle-Modulated Wave

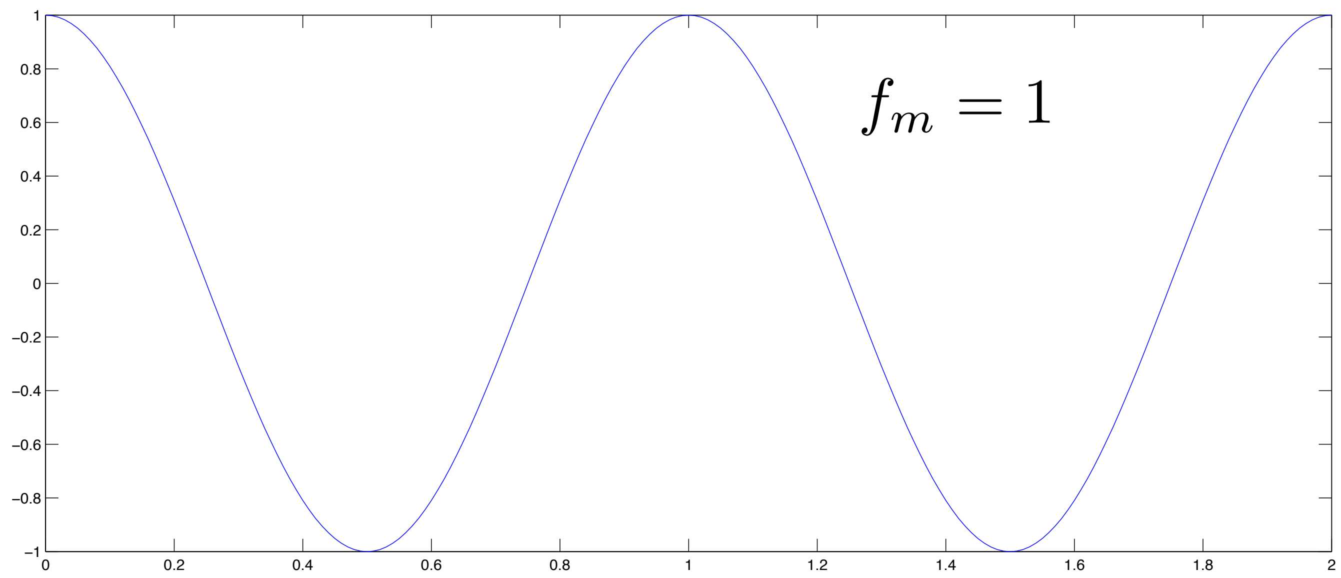
- Property 1: Constancy of transmitted wave
  - The amplitude of PM and FM waves is maintained at a constant value equal to the carrier amplitude for all time.
  - The average transmitted power of angle-modulated wave is a constant

$$P_{av} = \frac{1}{2}A_c^2$$

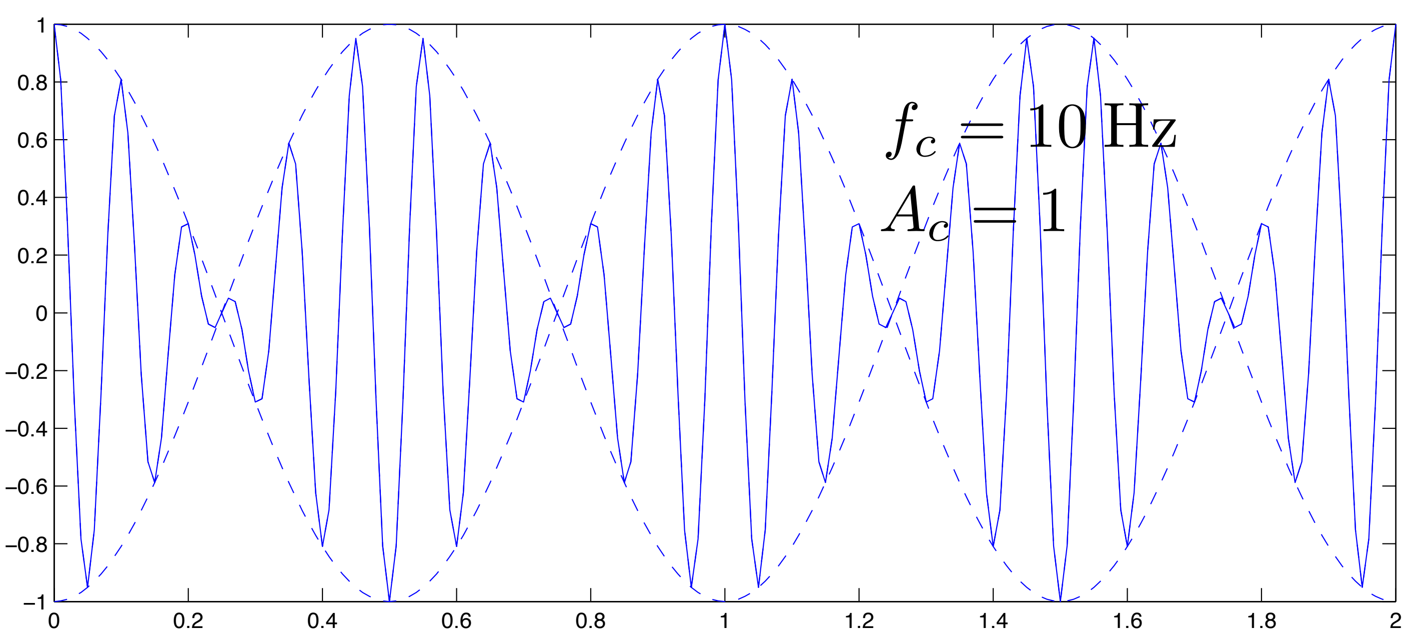
$$P_{av} = \frac{1}{T}\int_T [A_c \cos(\theta_i(t))]^2 \, dt = \frac{1}{2}A_c^2$$

Example:

Message signal: $m(t) = \cos(2\pi f_m t)$

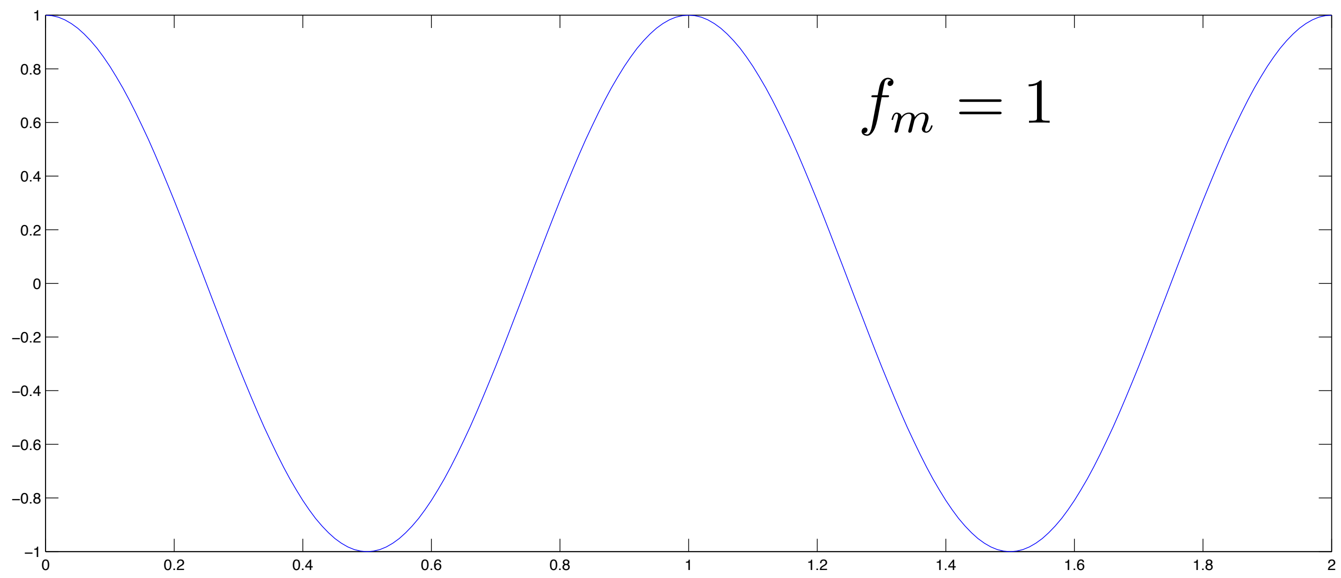


DSB-SC signal: $A_c m(t) \cos(2\pi f_c t)$



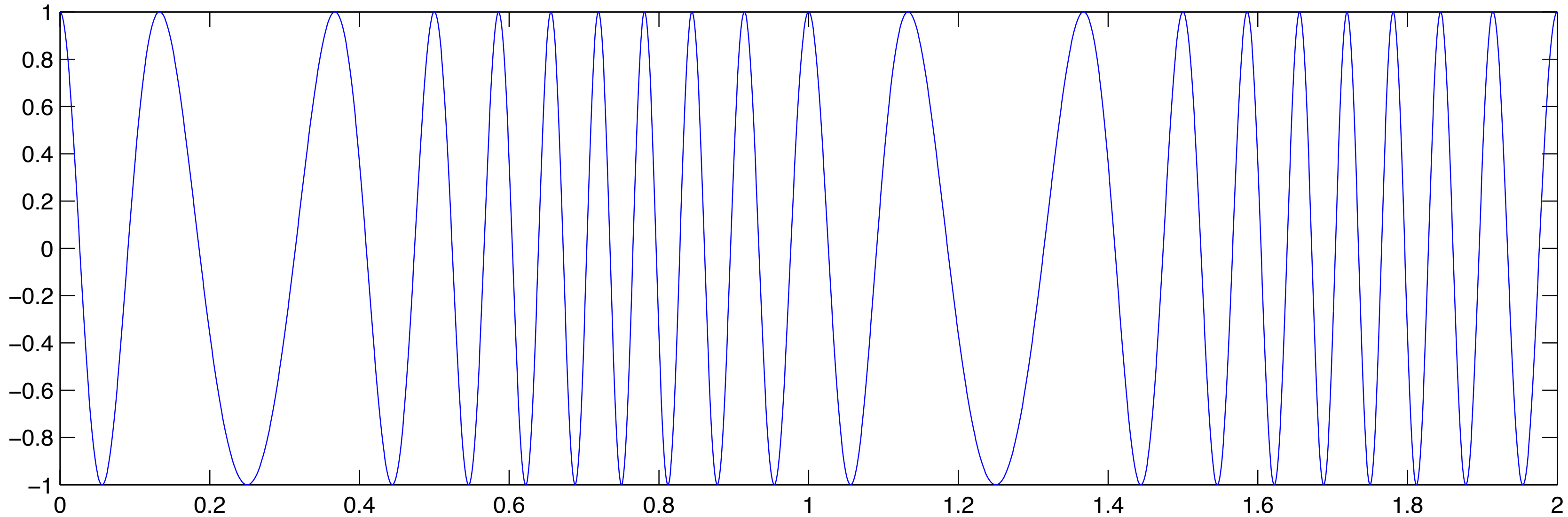
Example:

Message signal: $m(t) = \cos(2\pi f_m t)$



$f_m = 1$

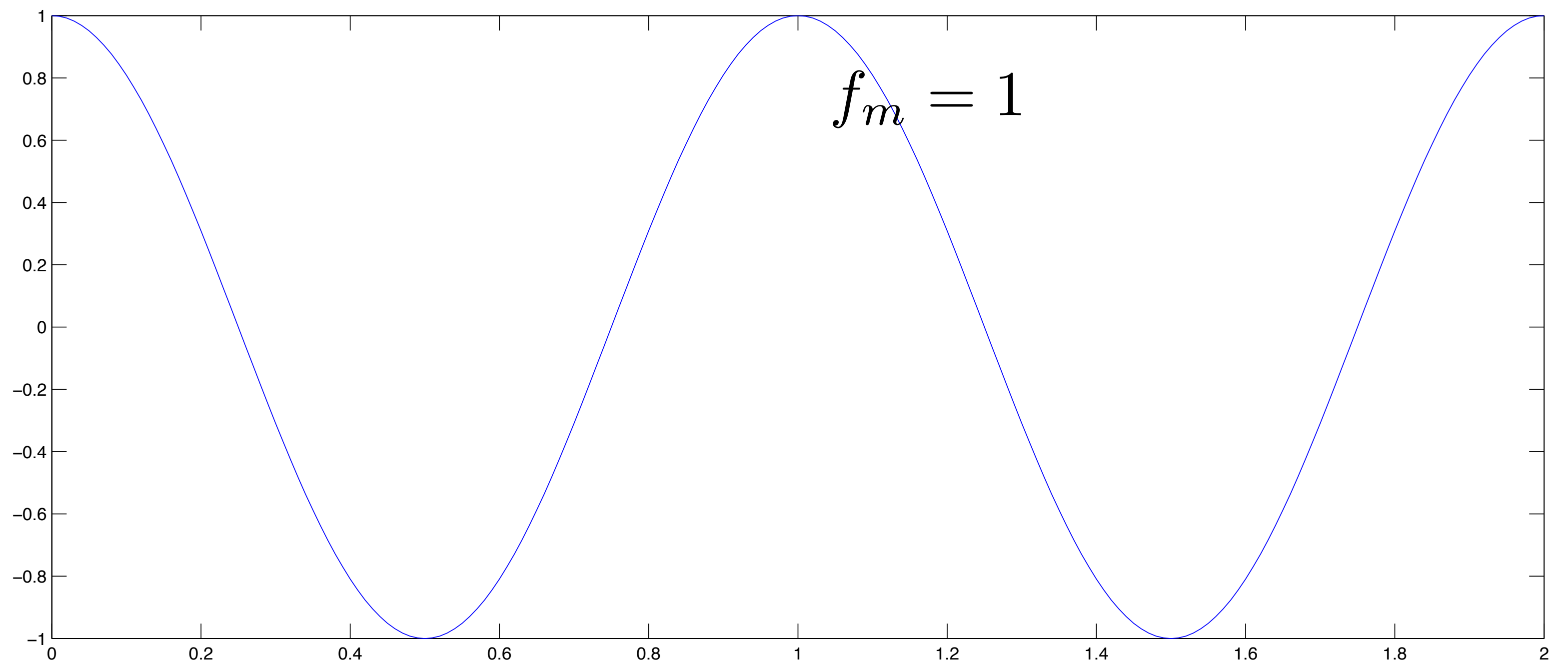
PM signal: $A_c \cos(2\pi f_c t + k_p m(t)) = A_c \cos(2\pi f_c t + k_p \cos(2\pi f_m t))$



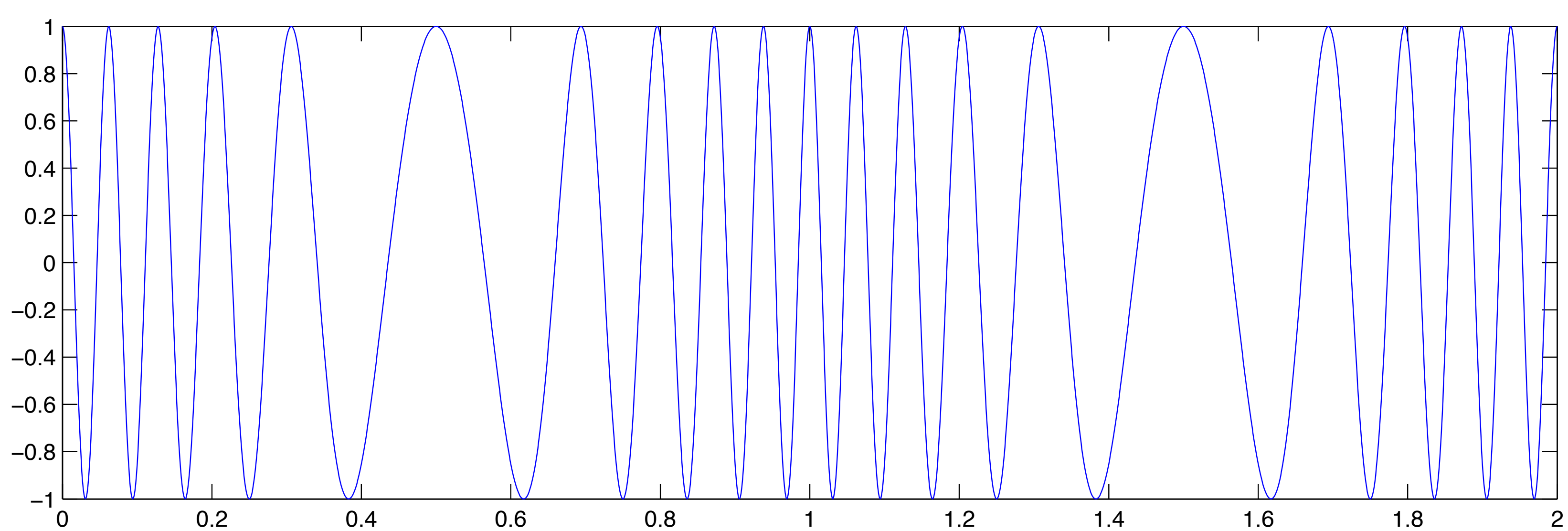
$k_p = 2\pi$

Example:

Message signal: $m(t) = \cos(2\pi f_m t)$



FM signal: $A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t \cos(2\pi f_m \tau)\, d\tau\right] = A_c \cos\left[2\pi f_c t + 2\pi k_f \sin(2\pi f_m t)\right]$



$k_f = 1$

- Property 2: Nonlinearity of the modulation process

$$m(t) = m_1(t) + m_2(t)$$

$$s(t) = A_c \cos\left[2\pi f_c t + k_p(m_1(t) + m_2(t))\right]$$

$$s_1(t) = A_c \cos(2\pi f_c t + k_p m_1(t)), \qquad s_2(t) = A_c \cos(2\pi f_c t + k_p m_2(t))$$

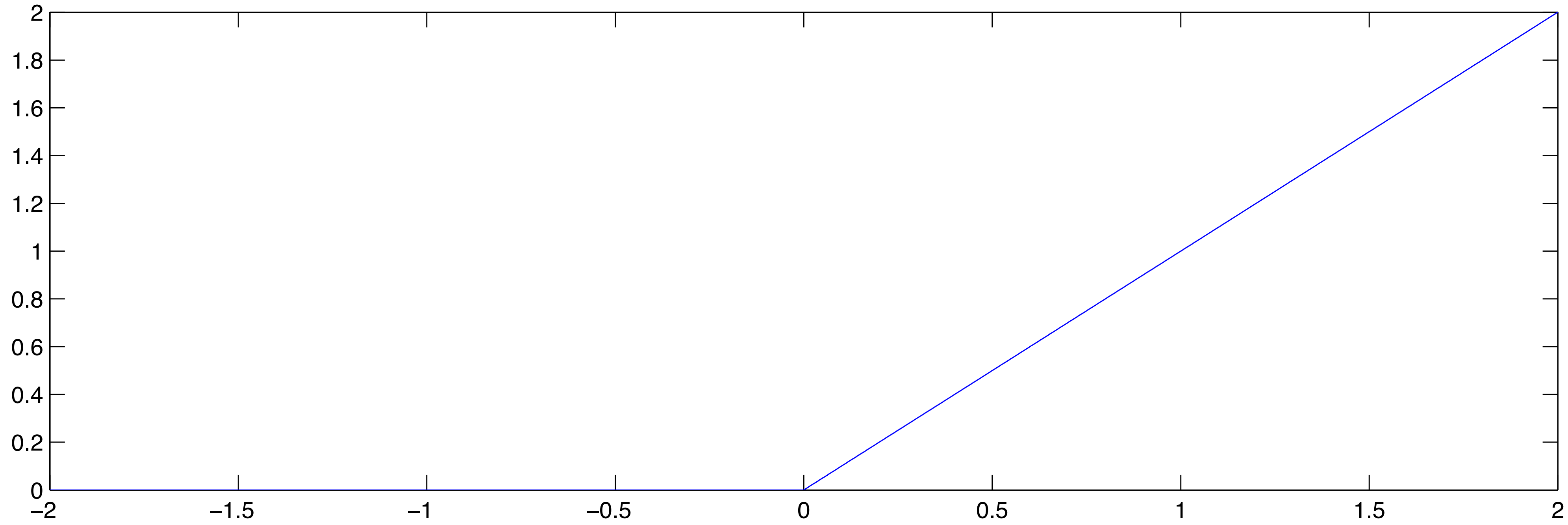
$$s(t) \neq s_1(t) + s_2(t)$$

- Property 3: Irregularity of zero-crossing
- Property 4: Visualization difficulty of message waveform
- Property 5: Tradeoff between increased transmission bandwidth for improved noise performance

Example of Zero-Crossing

- Consider the message signal given as

$$m(t) = \begin{cases} at, & t \geq 0 \\ 0, & t < 0 \end{cases}$$



$a = 1$

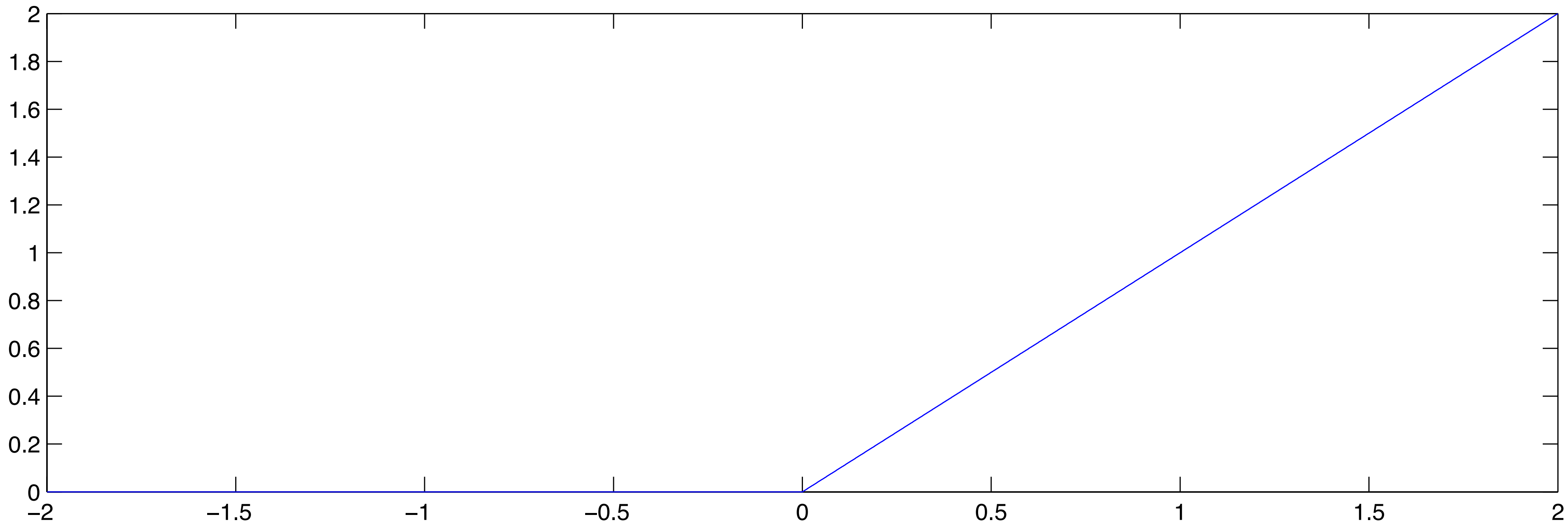
- PM signal

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

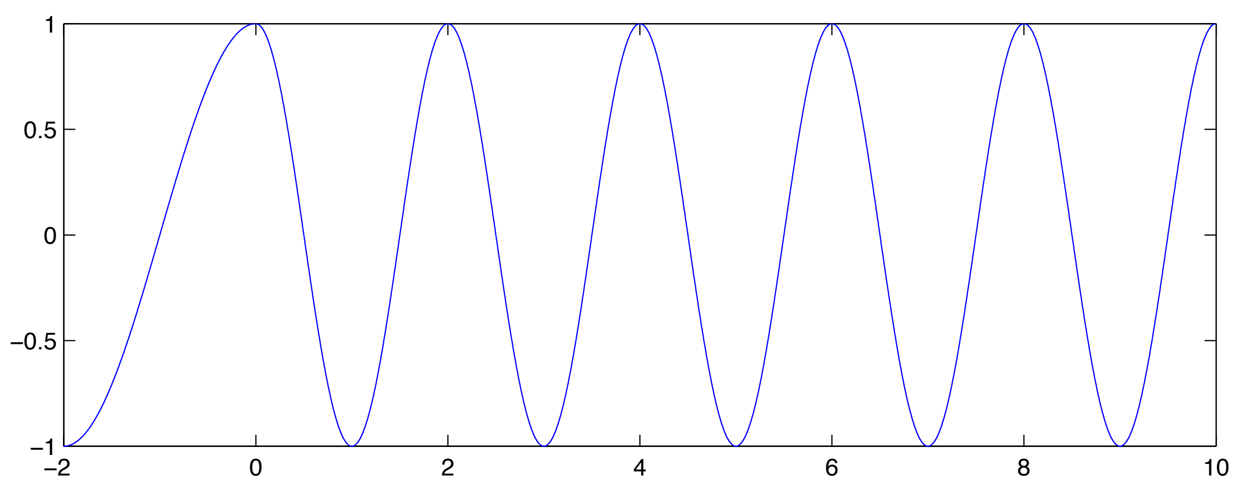
- FM signal

$$2\pi k_f \int_0^t a\tau \, d\tau = \pi k_f a t^2$$

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$



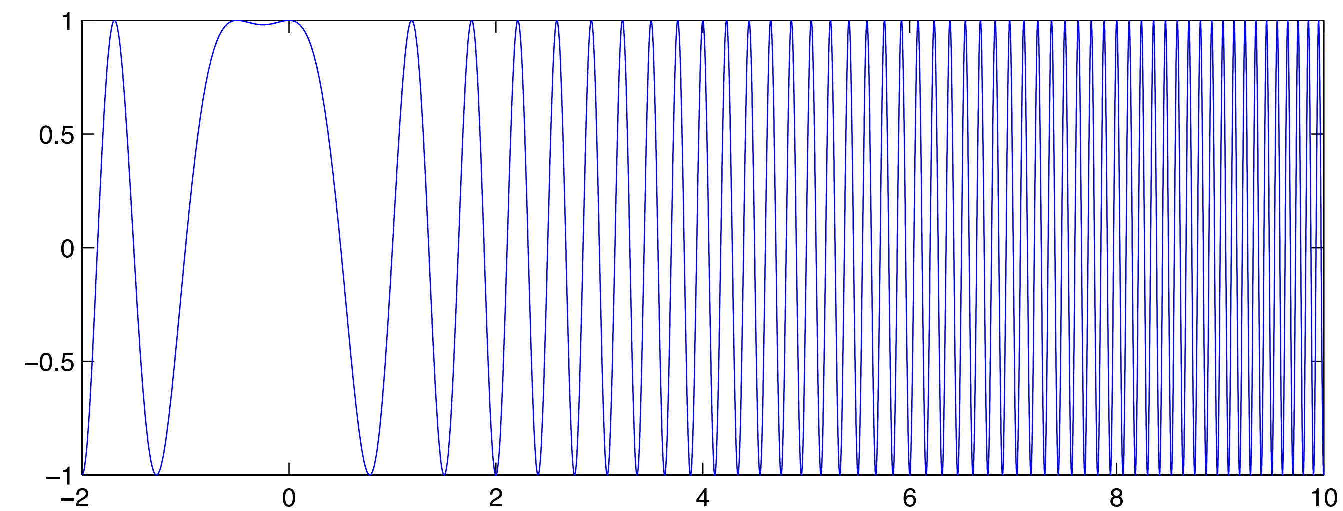
PM for $k_p = \pi/2$



$$s(t) = A_c \cos(2\pi f_c t + \frac{\pi}{2} t)$$

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + \frac{\pi}{2} t) = f_c + \frac{1}{4}$$

FM for $k_f = 1$



$$s(t) = A_c \cos(2\pi f_c t + \pi t^2), \qquad t \geq 0$$

$$f_i(t) = \frac{1}{2\pi} \frac{d}{dt} (2\pi f_c t + \pi t^2) = f_c + t$$

- PM signal

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + k_p a t), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

  - PM signal is zero at the instance of time t_n

$$2\pi f_c t_n + k_p a t_n = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, \ldots$$

  - Solving for t_n gives

$$t_n = \frac{\frac{1}{2} + n}{2f_c + \frac{k_p}{\pi} a} = \frac{1}{2} + n, \qquad n = 0, 1, 2, \ldots$$

$$f_c = 1/4 \text{ [Hz] and } a = 1 \text{ volt/s}$$

- FM signal:

$$s(t) = \begin{cases} A_c \cos(2\pi f_c t + \pi k_f a t^2), & t \geq 0 \\ A_c \cos(2\pi f_c t), & t < 0 \end{cases}$$

- Zero crossing at

$$2\pi f_c t_n + \pi k_f a t_n^2 = \frac{\pi}{2} + n\pi, \qquad n = 0, 1, 2, \ldots$$

- Solving for t_n gives

$$t_n = \frac{1}{a k_f} \left(-f_c + \sqrt{f_c^2 + a k_f \left(\frac{1}{2} + n \right)} \right), \qquad n = 0, 1, 2, \ldots$$

$$t_n = \frac{1}{4} \left(-1 + \sqrt{9 + 16n} \right), \qquad n = 0, 1, 2, \ldots$$

$$f_c = 1/4 \text{ [Hz] and } a = 1 \text{ volt/s}$$